**1** The functions f and g are defined by

$$f : x \to 3x + 4, \qquad x \in \mathbb{R}$$

$$g : x \to \frac{2}{x+3}, \qquad x \in \mathbb{R},\ x \neq 3$$

(a) Evaluate fg(1) **(2)**

(b) Solve the equation gf($x$) = 6 **(4)**

a/ $g(1) = \frac{2}{1+3} = \frac{1}{2}$

$f\left(\frac{1}{2}\right) = 3\left(\frac{1}{2}\right) + 4$

$= \frac{11}{2}$

b/ $gf(x) = \frac{2}{3x+4+3}$

$= \frac{2}{3x+7}$

$\frac{2}{3x+7} = 6$

$2 = 6(3x+7)$

$2 = 18x + 42$

$-40 = 18x$

$x = -\frac{20}{9}$

**2** The function f is defined by

$$f(x) = x^2 + 2x + 1, \quad x \in \mathbb{R}, \quad x \geq -1$$

(a) State the range of f **(1)**

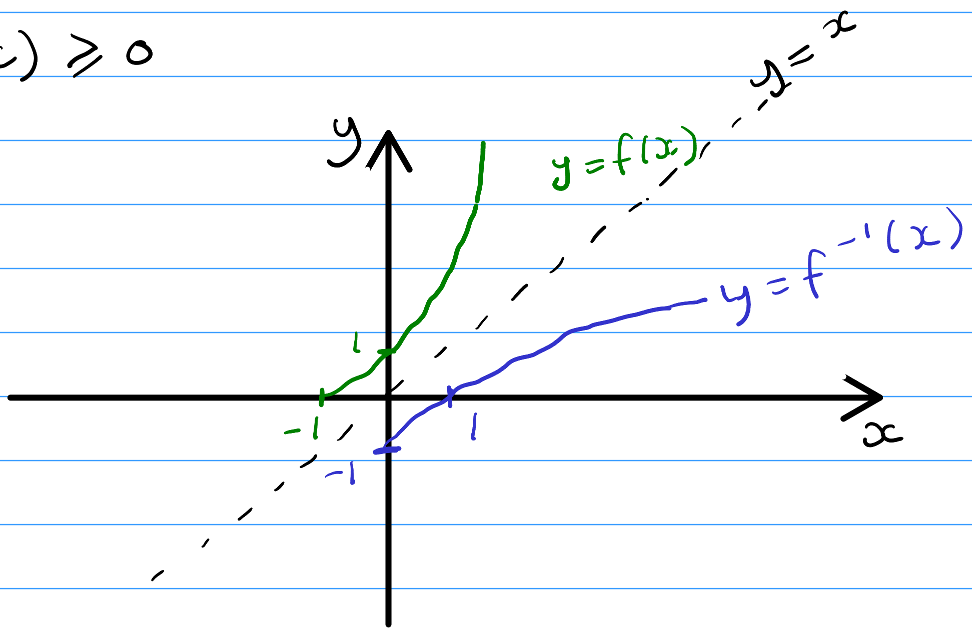
(b) Sketch the graphs of f($x$) and f$^{-1}$($x$) on the same diagram **(3)**

(c) Find an expression for f$^{-1}$($x$) and state its domain **(4)**

a/ $f(x) = (x+1)^2$

$f(x) \geq 0$

b/




c/ $y = x^2 + 2x + 1$

$y = (x+1)^2$

$x = (y+1)^2$

$\sqrt{x} = y + 1$

$\sqrt{x} - 1 = y$

$\underline{f^{-1}(x) = \sqrt{x} - 1}$ $\quad\quad$ $\underline{x \geq 0}$

**3** The function f is defined by

$$f(x) = 2 + \ln(2x - 1), \quad x \in \mathbb{R}, \quad x > 0.5$$

(a) Find the exact value of ff(1) **(2)**

(b) Find an expression for $f^{-1}(x)$ **(3)**

a/ $f(1) = 2 + \ln(1)$

$= 2$

$f(2) = 2 + \ln 3$

b/ $y = 2 + \ln(2x - 1)$

$x = 2 + \ln(2y - 1)$

$x - 2 = \ln(2y - 1)$

$e^{x-2} = 2y - 1$

$e^{x-2} + 1 = 2y$

$$y = \frac{e^{x-2} + 1}{2}$$

$$f^{-1}(x) = \frac{e^{x-2} + 1}{2}$$

**4** The functions f and g are defined by

$$f : x \to e^x, \qquad x \in \mathbb{R}$$

$$g : x \to 2x + \ln x, \qquad x \in \mathbb{R},\ x > 0$$

(a) Write down the range of f **(1)**

(b) Find an expression for the composite function gf **(2)**

(c) Write down the range of gf **(1)**

a/ $f(x) > 0$

b/ $2e^x + \ln e^x$

$\underline{\underline{2e^x + x}}$

c/ $gf(x) \in \mathbb{R}$

**5** The function f is defined by

$$f(x) = \frac{1}{x+2}, \qquad x \in \mathbb{R},\ x \neq -2$$

(a) Write down the range of f($x$) **(2)**

(b) Find an expression for $f^{-1}(x)$ and state its domain **(3)**

$$g(x) = x^2 - 5, \qquad x \in \mathbb{R}$$

(c) Solve $fg(x) = \frac{1}{2}$ **(3)**

a/ $f(x) \in \mathbb{R}, \quad f(x) \neq 0$

b/ $y = \dfrac{1}{x+2}$

$x = \dfrac{1}{y+2}$

$y + 2 = \dfrac{1}{x}$

$y = \dfrac{1}{x} - 2$

$f^{-1}(x) = \dfrac{1}{x} - 2 \qquad x \in \mathbb{R},\ x \neq 0$

$$fg(x) = \frac{1}{x^2 - 5 + 2}$$

$$= \frac{1}{x^2 - 3}$$

$$\frac{1}{x^2 - 3} = \frac{1}{2}$$

$$x^2 - 3 = 2$$

$$x^2 = 5$$

$$x = \pm\sqrt{5}$$

**6** The function f is defined by

$$f(x) = x^2 + 4x + 1, \quad x \in \mathbb{R}$$

(a) Find the range of f($x$) **(3)**

(b) Explain why the function f($x$) does not have an inverse **(1)**

a/ $(x+2)^2 - 4 + 1$

$(x+2)^2 - 3$ Min point $(-2, -3)$

$$f(x) \geq -3$$

b/ It is a many to one function. (Many to one functions do not have an inverse)

7 (a) Sketch the graph with equation

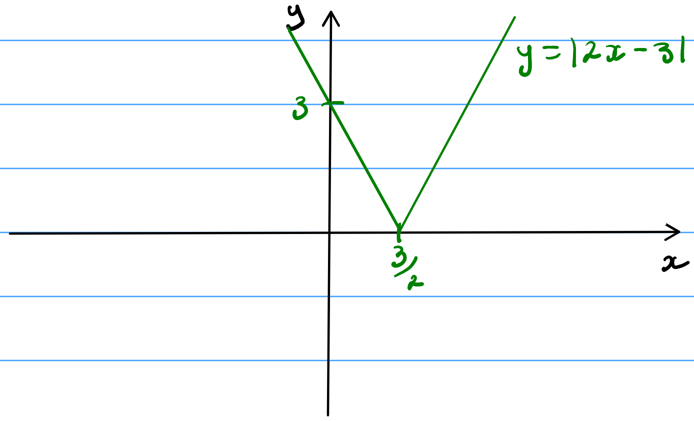
$$y = |2x - 3|$$

stating the coordinates where the graph meets the coordinate axis. **(2)**

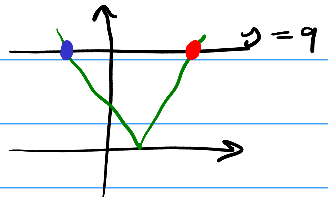
(b) Find the values of $x$ which satisfy $|2x - 3| < 9$ **(2)**

(c) Find the values of $x$ which satisfy $|2x - 3| < x + 1$ **(2)**

a)




b)




$-(2x - 3) = 9$

$2x - 3 = -9$

$2x = -6$

$x = -3$

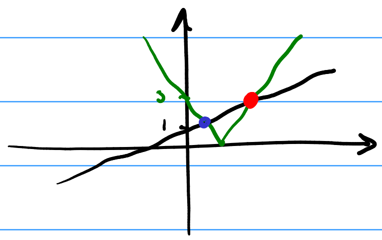
$2x - 3 = 9$

$2x = 12$

$x = 6$

$$\underline{-3 < x < 6}$$

c)




$-(2x - 3) = x + 1$

$-2x + 3 = x + 1$

$2 = 3x$

$x = \frac{2}{3}$

$2x - 3 = x + 1$

$x = 4$

$$\underline{\frac{2}{3} < x < 4}$$

**8** The functions f and g are defined by

$$f : x \rightarrow \ln(3x-2), \quad x \in \mathbb{R},\ x > \frac{2}{3}$$

$$g : x \rightarrow \frac{3}{x-2}, \quad x \in \mathbb{R},\ x \neq 2$$

(a) Find the exact value of fg(3) **(2)**

(b) Find an expression for $f^{-1}(x)$ and state its domain **(4)**

(c) Sketch the graphs of f($x$) and $f^{-1}(x)$ on the same diagram **(3)**

(d) Sketch the graph of $y = |g(x)|$ **(3)**

(e) Find the exact values of $\left|\frac{3}{x-2}\right| = 4$ **(3)**

a/ $g(3) = \frac{3}{3-2} = 3$

$f(3) = \ln(3(3)-2)$

$= \underline{\underline{\ln 7}}$

b/ $y = \ln(3x-2)$

$x = \ln(3y-2)$

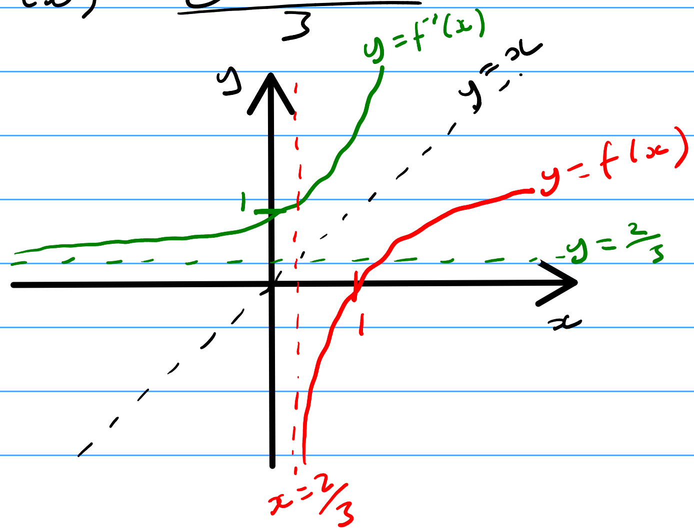
$e^x = 3y-2$

$e^x + 2 = 3y$

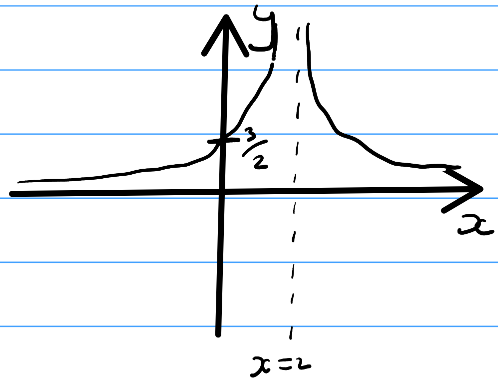
$y = \frac{e^x+2}{3}$

$f^{-1}(x) = \frac{e^x+2}{3}$

c/



d/




e/

$$\frac{3}{x-2} = 4 \qquad -\frac{3}{x-2} = 4$$

$$3 = 4(x-2) \qquad \frac{3}{x-2} = -4$$

$$3 = 4x - 8 \qquad 3 = -4(x-2)$$

$$11 = 4x \qquad 3 = -4x + 8$$

$$x = \frac{11}{4} \qquad -5 = -4x$$

$$x = \frac{5}{4}$$

9

$$f(x) = \frac{3x+1}{x-2} \qquad x \geq 4$$

(a) Find ff(4) **(2)**

(b) State the range of f. **(1)**

(c) Find $f^{-1}(x)$ and state its domain **(3)**

a) $f(4) = \frac{3(4)+1}{4-2} = \frac{13}{2}$

$f\left(\frac{13}{2}\right) = \frac{3\left(\frac{13}{2}\right)+1}{\left(\frac{13}{2}\right)-2}$

$= \frac{41}{9}$

b) when $x=4$ $f(x) = \frac{13}{2}$

As $x$ gets bigger $f(x)$ gets closer to 3.

$3 < f(x) \leq \frac{13}{2}$

c) $y = \frac{3x+1}{x-2}$

$x = \frac{3y+1}{y-2}$

$x(y-2) = 3y+1$

$xy - 2x = 3y+1$

$xy - 3y = 1 + 2x$

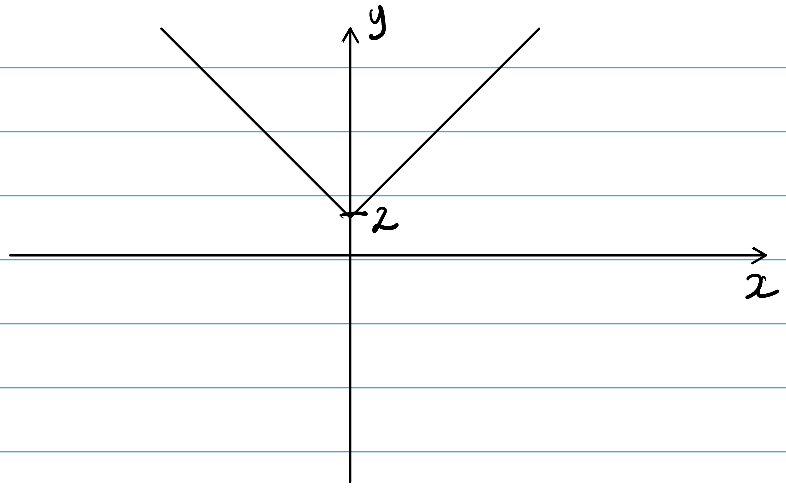
$y(x-3) = 1 + 2x$

$y = \frac{1+2x}{x-3}$

$f^{-1}(x) = \frac{1+2x}{x-3}$ $3 < x \leq \frac{13}{2}$

10 (a) (i) Sketch the graph of $y = |x| + 2$

(ii) Explain why $|x| + 2 \geq |x + 2|$ for all real values of $x$.



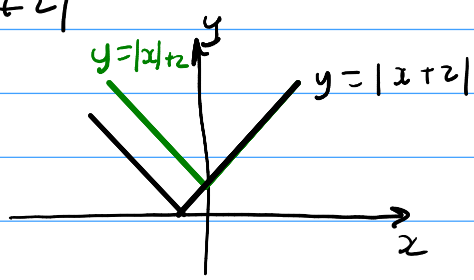

ii/ when $x \geqslant 0$ $|x| + 2 = |x + 2|$

but when $x < 0$ eg $x = -2$

$|-2| + 2 = 4$

$|-2 + 2| = 0$

$|x| + 2 > |x + 2|$ for all negative values of $x$



**11** $\mathrm{f}(x) = 2x^2 + 8x + 11 \qquad x \in \mathbb{R}$

(a) Write f(x) in the form $a(x + b)^2 + c$, where $a$, $b$ and $c$ are integers to be found. **(3)**

(b) Sketch the curve with equation $y = \mathrm{f}(x)$ showing any points of intersection with the coordinate axes and the coordinates of any turning point. **(3)**

(c) Describe fully the transformation that maps the curve with equation $y = \mathrm{f}(x)$ onto the curve with equation $y = \mathrm{g}(x)$ where **(2)**

$$\mathrm{g}(x) = 2(x-2)^2 + 8x - 5 \qquad x \in \mathbb{R}$$

(d) Find the range of the function

$$\mathrm{h}(x) = \frac{18}{2x^2 + 8x + 11} \qquad x \in \mathbb{R}$$ **(2)**

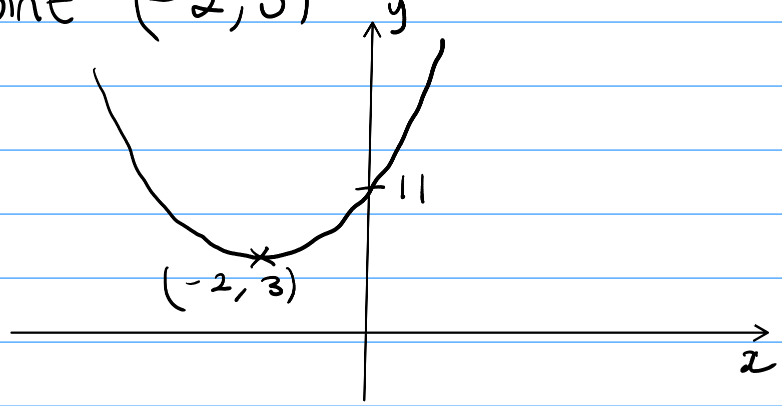
a) $2(x^2 + 4x) + 11$

$2[(x+2)^2 - 4] + 11$

$2(x+2)^2 - 8 + 11$

$\underline{2(x+2)^2 + 3}$

b) min point $(-2, 3)$




c) $f(x-2) = 2(x-2)^2 + 8(x-2) + 11$

$= 2(x-2)^2 + 8x - 16 + 11$

$= \underline{2(x-2)^2 + 8x - 5}$

Translation 2 to the right $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$

d) Range of $f(x)$ : $f(x) \geq 3$ $\qquad \frac{18}{3} = 6$

As $x$ gets bigger $h(x)$ gets closer to zero

$\therefore \quad \underline{0 < h(x) \leq 6}$

**12** The function f is defined by

$$f(x) = \frac{3x-5}{x-2}, \qquad x \in \mathbb{R},\ x \neq 2$$

(a) Find $f^{-1}(4)$ **(2)**

(b) Show that $ff(x) = \dfrac{ax+b}{x-1}$ where $a$ and $b$ are integers to be found. **(3)**

a/ $4 = \dfrac{3x-5}{x-2}$

$4(x-2) = 3x-5$

$4x-8 = 3x-5$

$\underline{\underline{x = 3}}$

b/ $f\left(\dfrac{3x-5}{x-2}\right) = \dfrac{3\left(\dfrac{3x-5}{x-2}\right) - 5}{\dfrac{3x-5}{x-2} - 2}$

$= \dfrac{3(3x-5) - 5(x-2)}{3x-5-2(x-2)}$

$= \dfrac{9x-15-5x+10}{3x-5-2x+4}$

$= \underline{\underline{\dfrac{4x-5}{x-1}}}$

**13** The function f is defined by $f(x) = e^{x+1}$, $x \in \mathbb{R}$

Find $f^{-1}(x)$ and state its domain.

range: $f(x) > 0$

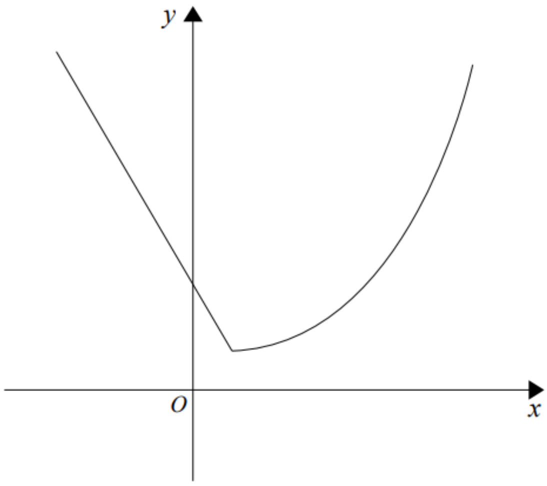
$y = e^{x+1}$

$x = e^{y+1}$

$\ln x = y+1$

$y = \ln x - 1$

$\underline{\underline{f^{-1}(x) = \ln x - 1}}$ $\underline{\underline{f^{-1}(x) > 0}}$




The diagram shows a sketch of $y = g(x)$, where

$$g(x) = \begin{cases} 5 - 3x & x < 1 \\ (x-1)^2 + 2 & x \geq 1 \end{cases}$$

(a) Find the value of gg(0) **(2)**

(b) Find all the values of $x$ for which $g(x) > 18$ **(4)**

The function h is defined as

$$h(x) = (x-1)^2 + 2 \qquad x \geq 1$$

(c) Explain why h has an inverse but g does not. **(1)**

(d) Solve the equation **(3)**

$$h^{-1}(x) = 2$$

a) $g(0) = 5 - 3(0) = \underline{\underline{2}}$

$g(2) = (2-1)^2 + 2 = \underline{\underline{3}}$

b) $5 - 3x = 18$

$-13 = 3x$

$x = \underline{-\frac{13}{3}}$

$(x-1)^2 + 2 = 18$

$(x-1)^2 = 16$

$x - 1 = \pm 4$

$x = \cancel{-3}$ or $\underline{5}$

$\underline{x < -\frac{13}{3}}$ or $\underline{x > 5}$

c) h is one to one function but g is a many to one function.

d) $(2-1)^2 + 2 = \underline{\underline{3}}$

**15**

$$f(x) = \frac{2x+1}{x-1} \qquad x \neq 1$$

(a) Find $f^{-1}(x)$ **(3)**

(b) Find an expression for $ff(x)$ **(1)**

a/ $y = \dfrac{2x+1}{x-1}$

$x = \dfrac{2y+1}{y-1}$

$x(y-1) = 2y+1$

$xy - x = 2y + 1$

$xy - 2y = 1 + x$

$y(x-2) = 1 + x$

$y = \dfrac{1+x}{x-2}$

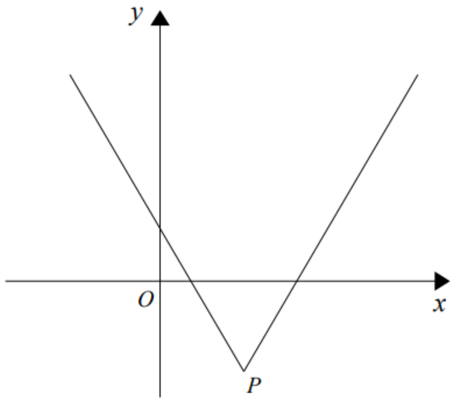
$f^{-1}(x) = \dfrac{x+1}{x-2}$

b/ $f\left(\dfrac{2x+1}{x-1}\right) = \dfrac{2\left(\dfrac{2x+1}{x-1}\right)+1}{\left(\dfrac{2x+1}{x-1}\right)-1}$ $\quad \times(x-1)$, $\times(x-1)$

$= \dfrac{2(2x+1) + (x-1)}{2x+1-(x-1)}$

$= \dfrac{4x+2+x-1}{2x+1-x+1}$

$= \dfrac{5x+1}{x+2}$




The diagram shows a sketch of $y = 2|x-3|-5$

The vertex of the graph is at point $P$.

(a) Find the coordinates of point $P$. **(2)**

(b) Solve the equation $10 - x = 2|x-3|-5$ **(2)**

A line $l$ has equation $y = ax$, where $a$ is a constant.

Given that $l$ intersects $y = 2|x-3|-5$ at least once,

(c) find the range of possible values of $a$, writing your answer in set notation. **(3)**

a/ $y = 2|x-3| - 5$

vertex when $x = 3$ $\quad y = 2(0) - 5$
$= -5$

$\underline{(3, -5)}$

b/ 2 intersections

$10 - x = 2(x-3) - 5$
$10 - x = 2x - 6 - 5$
$10 - x = 2x - 11$
$21 = 3x$
$\underline{x = 7}$

$10 - x = -2(x-3) - 5$
$10 - x = -2x + 6 - 5$
$10 - x = -2x + 1$
$\underline{x = -9}$

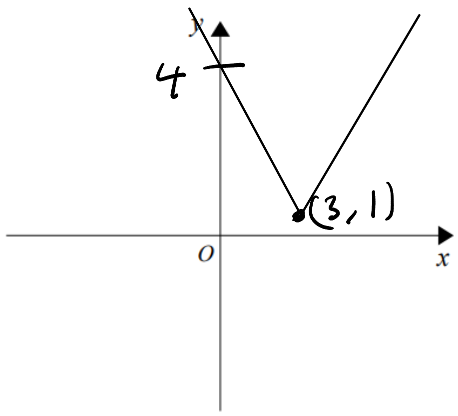
c/ if $a = -2$ no intersection
the line $(0,0)$ $(3,-5)$ intersects once
$\quad\quad x_1\ y_1 \quad x_2\ y_2$

$m = \frac{-5}{3}$

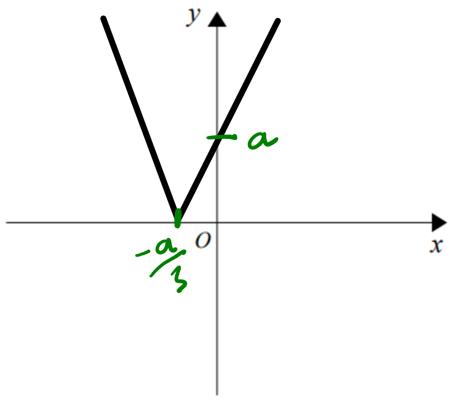
$\underline{\left\{a : -2 < a < -\frac{5}{3}\right\}}$

**17** Sketch the graph of $y = 1 + |x - 3|$

Show clearly where the graph intersects the axes.




**18**




Sketch the graph of $y = |3x + a|$, where a is a positive constant.

Show clearly where the graph intersects the axes.

**19** The function f is defined by $f(x) = 2x^2 + 5, \; x \geq 0$

(a) Find the range of f. **(1)**

(b) Find $f^{-1}(x)$ **(3)**

(c) State the range of $f^{-1}(x)$ **(1)**

a) $f(x) \geq 5$

b) $y = 2x^2 + 5$

$y - 5 = 2x^2$

$\frac{y-5}{2} = x^2$

$x = \sqrt{\frac{y-5}{2}}$ $\quad$ $f^{-1}(x) = \sqrt{\frac{x-5}{2}}$ $\quad$ $x \geq 5$

c) $f^{-1}(x) \geq 0$

**20** $f(x) = \arcsin x$

State the maximum possible domain of f.

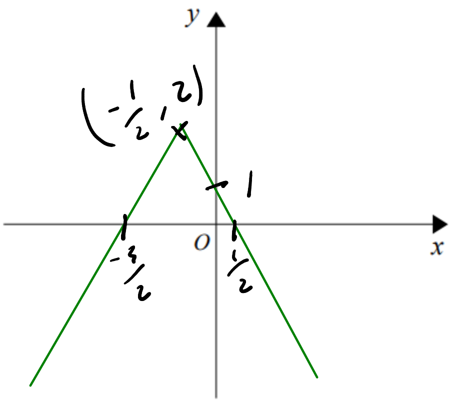
$-1 \leq x \leq 1$

**21** $f(x) = \dfrac{1}{\sqrt{x+2}}$

State the maximum possible domain of f.

$x > -2$

**22** (a) Sketch the graph of $y = 2 - |2x + 1|$




**(3)**

(b) Solve the inequality $2 - |2x + 1| > 0$ **(2)**

b/ $-\frac{3}{2} < x < \frac{1}{2}$

**23** The function f is defined by $f(x) = 2x^2 - 3x \qquad x \in \mathbb{R},\ 0 \leq x \leq 4$

(a) Find the range of f. **(3)**

(b) Determine whether f has an inverse.
Fully justify your answer. **(2)**

a/ $f(x) = 2\left(x^2 - \frac{3}{2}x\right)$

$= 2\left(\left(x - \frac{3}{4}\right)^2 - \frac{9}{16}\right)$

$= 2\left(x - \frac{3}{4}\right)^2 - \frac{9}{8}$

min value of $f(x) = -\frac{9}{8}$ when $x=4$ $f(x) = 2(4)^2 - 3(4)$
$= 20$

$-\frac{9}{8} \leq f(x) \leq 20$

b/ No. $f(x)$ is a many to one function $\therefore$ no inverse.

**24** The function f is defined for all real values of $x$ as $f(x) = x^2 - 6x + c$, where $c$ is a constant.

(i) Given that the range of f is $f(x) \geq 2$, find the value of $c$. **(3)**

(ii) Given instead that $ff(2) = 12$, find the possible values of $c$. **(4)**

i/ $f(x) = (x-3)^2 - 9 + c$

$\underline{\underline{c = 11}}$

ii/ $f(2) = (2)^2 - 6(2) + c$

$= c - 8$

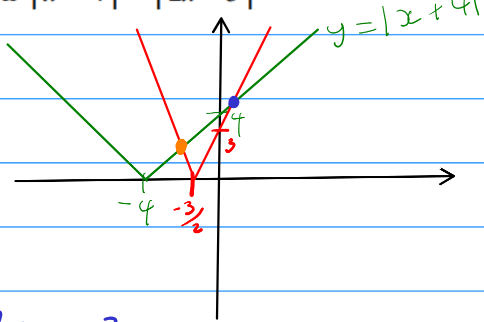
$(c-8)^2 - 6(c-8) + c = 12$

$c^2 - 16c + 64 - 6c + 48 + c = 12$

$c^2 - 21c + 98 = 0$

$\underline{\underline{c = 14}}$ or $\underline{\underline{c = 7}}$

**25** Solve the equation $|x + 4| = |2x + 3|$




$x + 4 = 2x + 3$

$\underline{\underline{1 = x}}$

$x + 4 = -(2x + 3)$

$x + 4 = -2x - 3$

$3x = -7$

$x = -\frac{7}{3}$

$\underline{\underline{x = -\frac{7}{3}}}$ and $\underline{\underline{x = 1}}$

**26** The functions f and g are defined for all real values of $x$ by

$$f(x) = x^3 \quad \text{and} \quad g(x) = x^2 - 1$$

(a) Write down expressions for fg($x$) and gf($x$) **(2)**

(b) Hence find the values of $x$ for which gf($x$) – fg($x$) = 18 **(6)**

a/ $fg(x) = (x^2 - 1)^3$

$gf(x) = (x^3)^2 - 1$

$= x^6 - 1$

b/ $x^6 - 1 - (x^2 - 1)^3 = 18$

$x^6 - 1 - (x^4 - 2x^2 + 1)(x^2 - 1) = 18$

$x^6 - 1 - (x^6 - x^4 - 2x^4 + 2x^2 + x^2 - 1) = 18$

$x^6 - 1 - x^6 + x^4 + 2x^4 - 2x^2 - x^2 + 1 = 18$

$3x^4 - 3x^2 = 18$

$3x^4 - 3x^2 - 18 = 0$

$x^4 - x^2 - 6 = 0$

$(x^2 - 3)(x^2 + 2) = 0$

$x^2 = 3$ ×

$x = \pm\sqrt{3}$

**27** The function~~s~~ f is defined by $f(x) = (x-2)^2 - 9$ for $x \geq k$, where $k$ is a constant.

(a) Given that $f^{-1}(x)$ exists, state the least possible value of $k$. **(1)**

(b) Evaluate ff(5) **(2)**

(c) Solve the equation $f(x) = x$ **(3)**

(d) Explain why your answer to part (c) is also the solution to the equation $f(x) = f^{-1}(x)$ **(1)**

a/ Min point at $(2, -9)$

$\underline{\underline{2}}$

b/ $f(5) = 0$

$f(0) = \underline{\underline{-5}}$

c/ $(x-2)^2 - 9 = x$

$x^2 - 4x + 4 - 9 = x$

$x^2 - 5x - 5 = 0$

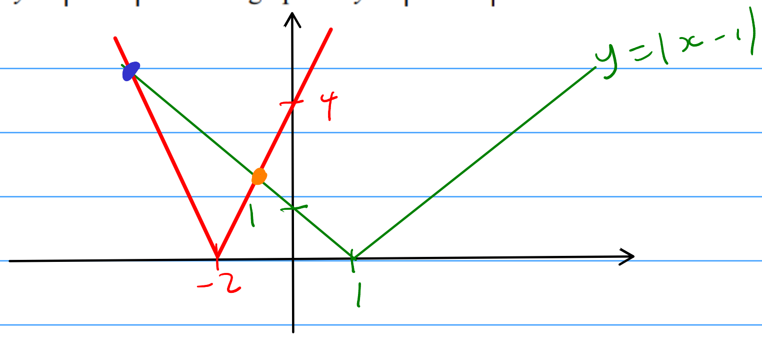
$x = \underline{\underline{\dfrac{5+3\sqrt{5}}{2}}}$ $\quad x = \dfrac{5-3\sqrt{5}}{2}$

$\times$ as $x \geqslant 2$

d/ The graphs of $f(x)$ and $f^{-1}(x)$ reflect in the line $y = x$. $\therefore$ Any point where $f(x)$ crosses $y = x$ $f^{-1}(x)$ will also cross $y = x$.

**28** (a) Solve the inequality $|x-1| \leq |2x+4|$ **(4)**

(b) Give full details of a sequence of two transformations needed to transform the graph of $y = |x-1|$ onto the graph of $y = |2x+4|$ **(3)**




Intersections where:

$-(x-1) = -(2x+4)$

$-x+1 = -2x-4$

$\underline{x = -5}$

$2x+4 = -(x-1)$

$2x+4 = -x+1$

$3x = -3$

$\underline{x = -1}$

$\underline{\underline{x \leq -5}}$ or $\underline{\underline{x \geq -1}}$

b) 1) Stretch with scale factor $\frac{1}{2}$ in the $x$ direction

2) Translation by vector $\begin{pmatrix} -2.5 \\ 0 \end{pmatrix}$

OR 1) translation by vector $\begin{pmatrix} -5 \\ 0 \end{pmatrix}$

2) stretch with s.f $\frac{1}{2}$ in $x$ direction

**29** The functions f and g are defined for all real values of by

$f(x) = 2x^2 - 3x$ and $g(x) = 4x + 6$

(a) Find the range of f. **(3)**

(b) Give a reason why f has no inverse. **(1)**

(c) Given that $gf(2) = g^{-1}(a)$, where $a$ is a constant, determine the value of $a$. **(3)**

(d) Determine the set of values for which $f(x) > g(x)$. Give your answer in set notation. **(3)**

a/ $f(x) = 2(x^2 - \frac{3}{2}x)$

$= 2[(x - \frac{3}{4})^2 - \frac{9}{16}]$

$= 2(x - \frac{3}{4})^2 - \frac{9}{8}$

$f(x) \geqslant -\frac{9}{8}$

b/ It is a many to one function (many to one functions do not have an inverse)

c/ $f(2) = 2(2)^2 - 3(2)$
$= 2$

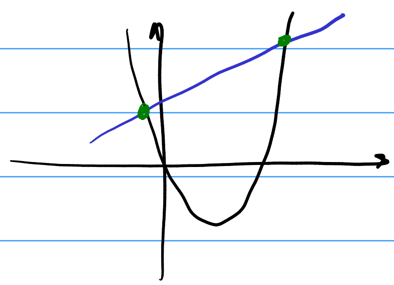
$g(2) = 4(2) + 6$
$= 14$

$g^{-1}(a) = 14$ $\quad g(14) = 4(14) + 6$
$= \underline{\underline{62}}$

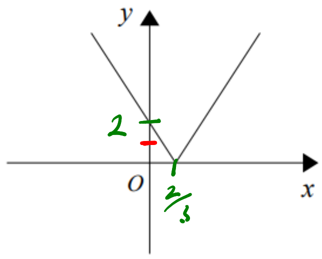
d/ $2x^2 - 3x = 4x + 6$

$2x^2 - 7x - 6 = 0$

$x = \frac{7 + \sqrt{97}}{4}$ $\quad x = \frac{7 - \sqrt{97}}{4}$



$\left\{x : x < \frac{7 - \sqrt{97}}{4}\right\} \cap \left\{x : x > \frac{7 + \sqrt{97}}{4}\right\}$




The graph shows a sketch of $y = |3x - 2|$

(a) State the coordinates of the points of intersection with the axes. **(2)**

(b) Given that the graphs of $y = |3x - 2|$ and $y = ax + 1$ have two distinct points of intersection, determine

(i) the set of possible values of $a$, **(4)**

(ii) the $x$ coordinates of the points of intersection of these graphs, giving your answers in terms of $a$. **(3)**

a/ $(0, 2)$ and $\left(\frac{2}{3}, 0\right)$

b i/ a must be less than 3

one intersection if $y = ax + 1$ goes through $\left(\frac{2}{3}, 0\right)$

$$0 = \frac{2}{3}a + 1$$
$$-1 = \frac{2}{3}a$$
$$a = -\frac{3}{2}$$

$$\therefore \quad -\frac{2}{3} < a < 3$$

ii/

$$3x - 2 = ax + 1$$
$$3x = ax + 3$$
$$3x - ax = 3$$
$$x(3 - a) = 3$$
$$x = \frac{3}{3 - a}$$

$$-(3x - 2) = ax + 1$$
$$-3x + 2 = ax + 1$$
$$1 = ax + 3x$$
$$1 = x(a + 3)$$
$$x = \frac{1}{a + 3}$$

**31** The function $f(x) = \dfrac{e^x}{2 - e^x}$ is defined on the domain $x \in \mathbb{R}, x \neq \ln 2$

(a) Find $f^{-1}(x)$ **(3)**

(b) Write down the range of $f^{-1}(x)$ **(1)**

a/ $$y = \frac{e^x}{2-e^x}$$

$$x = \frac{e^y}{2-e^y}$$

$$x(2-e^y) = e^y$$

$$2x - xe^y = e^y$$

$$2x = e^y + xe^y$$

$$2x = e^y(1+x)$$

$$e^y = \frac{2x}{1+x}$$

$$y = \ln\left(\frac{2x}{1+x}\right)$$

$$f^{-1}(x) = \ln\left(\frac{2x}{1+x}\right)$$

b/ $f^{-1}(x) \in R$ , $f^{-1}(x) \neq \ln 2$

**32** You are given that

$f(x) = x^2 - 4x - 5$ for $x > 2$ and
$g(x) = 2x + 1$

(a) Find $gf(x)$ and state its domain **(3)**

(b) State the range of $gf(x)$. **(1)**

(c) Find $gf^{-1}(x)$. **(5)**

a/ $gf(x) = 2(x^2 - 4x - 5) + 1$
$= 2x^2 - 8x - 10 + 1$
$= 2x^2 - 8x - 9$ $\qquad x > 2$

b/ $2(2)^2 - 8(2) - 9 = -17$

$gf(x) > -17$

c/ $f^{-1}(x)$ $\qquad y = (x-2)^2 - 4 - 5$

$y = (x-2)^2 - 9$

$x = (y-2)^2 - 9$

$x + 9 = (y-2)^2$

$\pm\sqrt{x+9} = y - 2$

$y = 2 \pm \sqrt{x+9}$

we only want the positive root $\quad f^{-1}(x) = 2 + \sqrt{x+9}$
$x > -9$

$gf^{-1}(x) = 2(2 + \sqrt{x+9}) + 1$
$= 4 + 2\sqrt{x+9} + 1$
$= 5 + 2\sqrt{x+9}$ $\qquad x > -9$

**33** The function f is defined for all real values of by

f($x$) = 2$x$ – 5

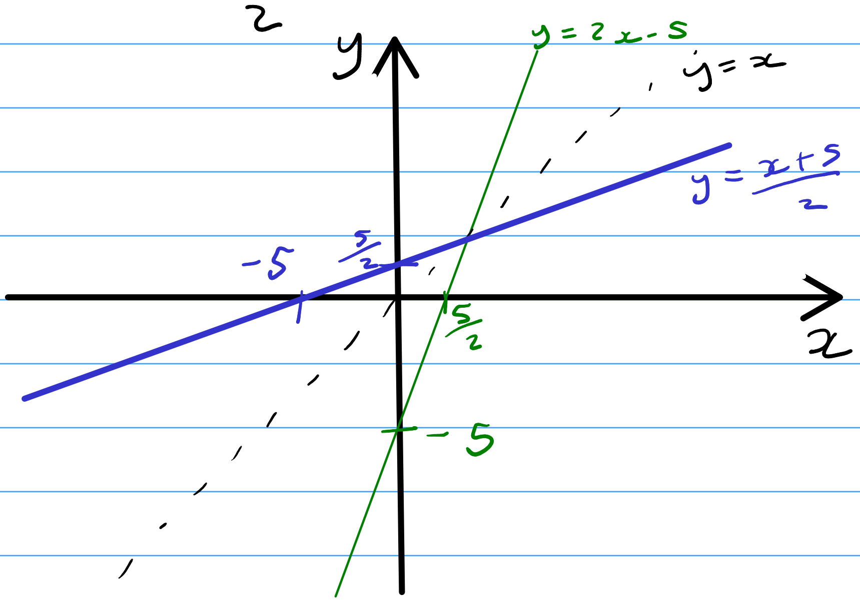
(a) Find an expression for $f^{-1}(x)$ **(2)**

(b) Sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ on the same diagram. **(2)**

(c) Find the set of values for which $f(x) > f^{-1}(x)$. **(2)**

a/ $f^{-1}(x) = \dfrac{x+5}{2}$

b/

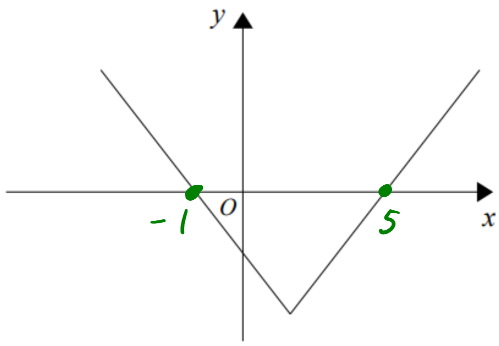



c/ Intersection where $2x - 5 = x$

$x = 5$

$\underline{\underline{x > 5}}$

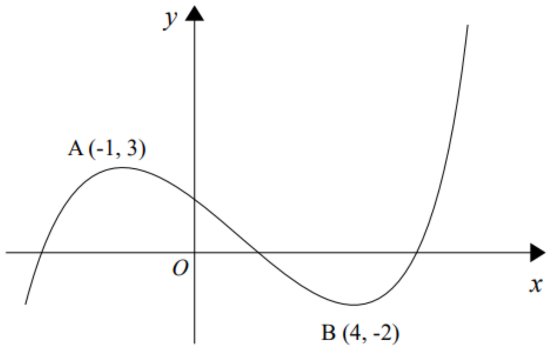
**34** The graph shows a sketch of $y = |x-2| - 3$




Find the set of values for which $|x-2| < 3$

$\{x : x > -1\} \cup \{x : x < 5\}$

**35** The diagram shows the graph of $y = \mathrm{f}(x)$




The curve has a maximum point A (-1, 3) and a minimum point B (4, -2).

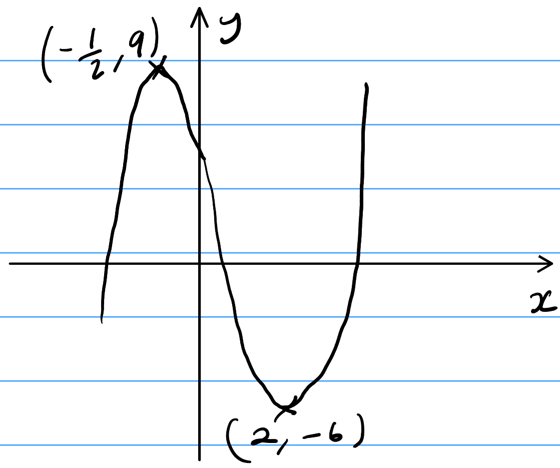
Showing the coordinates of any stationary points, on separate diagrams sketch the graphs of

(a) $y = 3\mathrm{f}(2x)$ **(3)**

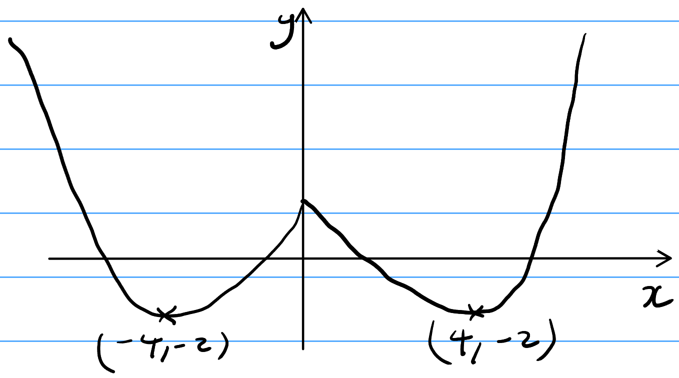
(b) $y = \mathrm{f}(|x|)$ **(3)**

(c) Write down the constants $a$ and $b$ such that the curve with equation $y = \mathrm{f}(x + a) + b$ has a minimum point at the origin $O$. **(2)**

a/



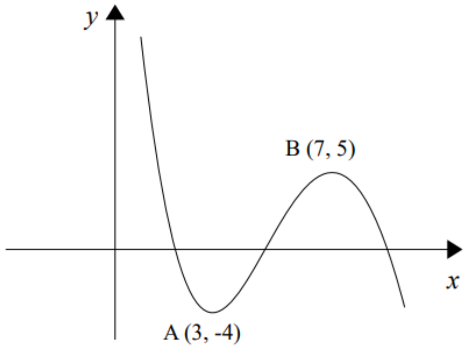

b/




c/ $a = 4 \quad b = 2$

$f(x + 4) + 2$

**36** The diagram shows the graph of $y = \mathrm{f}(x)$



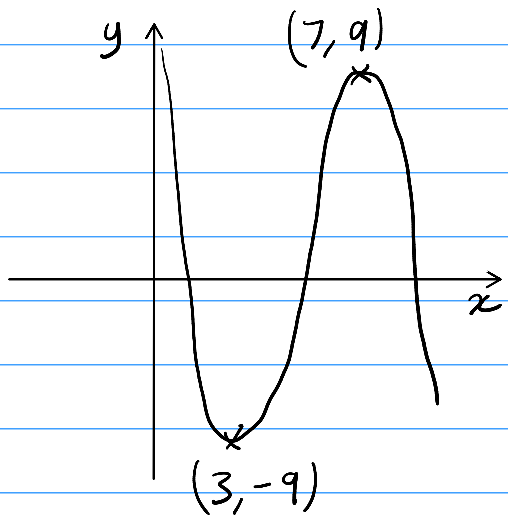

The points A (3, -4) and B (7, 5) are the turning points of the graph.

On separate diagrams sketch the graphs of

(a) $y = 2\mathrm{f}(x) - 1$ **(3)**

(b) $y = |\mathrm{f}(x)|$ **(3)**

a/




b/