AS Level Maths: Exponentials and Logs

1 Solve $3^x = 13$, giving your answer to 3 significant figures.

(Total for question 1 is 3 marks)

2 Solve $2^x = 32$

(Total for question 2 is 2 marks)

3 Solve the equation

$$2\log_2(x) - \log_2(5) = 1$$

(Total for question 3 is 4 marks)

4 Solve the equation

$$\log_3(x) + \log_3(4) = 2$$

(Total for question 4 is 4 marks)

5 Express as a single logarithm to base a

$$2\log_a(x + 1) - \log_a(4)$$

(Total for question 5 is 4 marks)

6 Giving your answers to 2 decimal places, solve the simultaneous equations

$$e^{2y} = x + 1$$
$$\ln (x - 2) = 2y - 1$$

(Total for question 6 is 7 marks)

7 Solve the equation

$$\ln(2x + 5) = 1$$

(Total for question 7 is 3 marks)

8 Given that y = $\log_2 x$, find expressions in terms of y for

(a) $\log_2 x^2$ **(2)**

(b) $\log_2 2x$ **(2)**

(c) $\log_8 x$ **(2)**

(Total for question 8 is 6 marks)

9 Solve the equation, giving your answers in exact form.

$$2e^y + 15e^{-y} = 11$$

(Total for question 9 is 5 marks)

10 The population of a species of plant in a field is modelled using the formula $P = 50e^{0.1t}$
Where t is the number of weeks since the population was first recorded.

(a) Write down the number of the plants when the population was first recorded. **(1)**

(b) Find the rate of increase in the population 10 weeks after the population was first recorded. **(2)**

(c) Find how many weeks it takes for the number of plants to exceed 300. **(4)**

(Total for question 10 is 7 marks)

11 The decay of a radioactive substance is modelled using the formula $N = 1000e^{-kt}$
Where N is the number of atoms after t years and k is a positive constant.

(a) Write down the number of atoms when the substance started to decay. **(1)**

Given it takes 14.4 years for half of the substance to decay.

(b) Find the value of k to three significant figures. **(4)**

(c) Calculate the number of atoms left when t=30. **(1)**

(Total for question 11 is 6 marks)

12 The temperature of water in a kettle is modelled using the formula $T = 75e^{-kt} + 22$

Where T is the temperature t minutes after the kettle is turned off and k is a positive constant.

(a) Find the rate of change of the temperature in terms of k **(2)**

After 5 minutes the temperature of the water is 70°C

(b) Find the value of k **(3)**

(c) Find how many minutes it takes for the water to cool to 55°C **(4)**

(Total for question 12 is 9 marks)

13 $$f(x) = e^{2x+1} - 3$$

(a) State the range of f **(1)**

The curve y = f(x) meets the *y*-axis at *A* and the *x*-axis at *B*.

(b) Find the exact coordinates of *A* and *B*. **(4)**

(c) Find the equation of the tangent to the curve at *A*. **(4)**

(Total for question 13 is 9 marks)

14 The population of bacteria is being measured.

The equation

$$\log_{10}P = 0.5t + 1.398$$

is used to model the population of bacteria, P, t hours after it was first recorded.

(a) Show that $P = ab^t$, where a and b are constants to be found.

(4)

Give the value of a to the nearest whole number and give the value of b to 3 significant figures.

(b) Interpret the meaning of the constant a in this model.

(1)

(c) Find the population of the bacteria after 10 hours. Give your answer to 2 significant figures.

(2)

(Total for question 14 is 7 marks)

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15 The growth in the population of worms, W, is modelled by the equation:

$$W = 95 - 75e^{kt}$$

Where k is a constant and t is the the number of days since the first measurement.

(a) Use the model to find the number of worms when measurements began.

(1)

After 50 days there were 35 worms

(b) Use this information to find a complete equation for the model, giving your value of k to 3 significant figures.

(4)

(c) Use the model to predict the number of worms after one year.

(1)

(d) Sketch the graph of W against t.

(3)

(Total for question 15 is 9 marks)

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16 Two experiments measuring the population of insects are started at the same time.

The number of insects in experiment A is modelled by the formula $P_A = ae^{0.2t}$

The number of insects in experiment B is modelled by the formula $P_B = be^{0.15t}$

Where t is the number of days since the experiments began.

At the start of the experiments the **total** number of insects recorded is 120
After 8 days the **total** number of insects recorded is 480

(a) Show that $a = 50$, to the nearest whole number, and find the value of b.

(3)

(b) Estimate the total number of insects after 10 days.

(1)

(c) Find the day that the number of insects in experiment A exceed the number of insects in experiment B.

(3)

(Total for question 16 is 7 marks)

17 The temperature, $\theta°C$, of a cup of tea t minutes after measurements began, is modelled using the formula

$$\theta = 68e^{-0.15t} + 21 \qquad t \geq 0$$

(a) Sketch the graph of θ against t. **(2)**

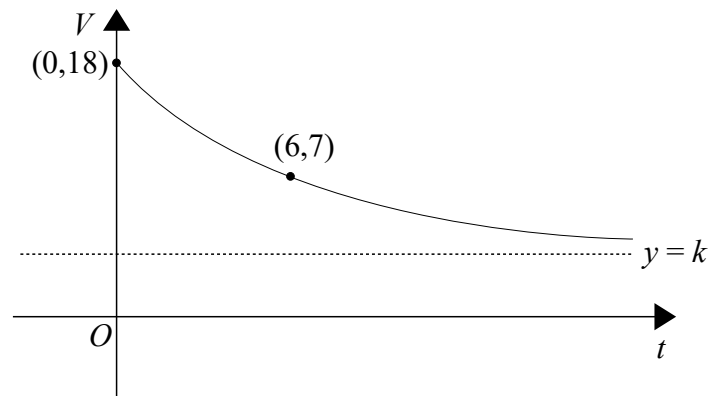Find according to the model:

(b) the initial temperature of the tea. **(1)**

(c) Find the value of t when the cup of tea reaches $40°C$. **(3)**

(Total for question 17 is 6 marks)

18 The value of a car t years after it was purchased, where V is the value of the car in thousands of pounds, is modelled by the formula

$$V = A + Be^{\frac{-1}{6}t}$$

where A and B are constants.

The graph shows a sketch of V against t. The graphs passes through $(0, 18)$ and $(6, 7)$.
$y = k$ is an asymptote to the curve, where k is a constant.

Find the values of A, B and k.

(Total for question 18 is 5 marks)

19 Show that

$$\log_2\left(\frac{x}{8}\right) + 2\log_2 3 \equiv \log_2(9x) - 3$$

(Total for question19 is 3 marks)

20 (a) Using $y = 2^x$ as a substitution, show that

$$4^x - 2^{x+2} - 5 = 0$$

can be written as

$$y^2 - 4y - 5 = 0$$

(2)

(b) Hence, show that the equation

$$4^x - 2^{x+2} - 5 = 0$$

has $x = \log_2 5$ as its only solution.

(4)

(Total for question 20 is 6 marks)

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21 A curve has the equation $y = e^{2x}$

At point P on the curve the tangent is parallel to the line $9x - 2y + 3 = 0$

Find the coordinates of P stating your answer in the form $(\ln p, q)$, where p and q are rational.

(Total for question 21 is 7 marks)

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22 The population of people in a town is modelled using the formula

$$P = a(10^{bt})$$

where t is the time in years since 2001, and a and b are constants.

(a) Explain what the value of a represents

(1)

In 2008 the population of the town was 54 000
In 2013 the population of the town was 59 000

(b) Use the data to calculate the value of a and the value of b.

(4)

(Total for question 22 is 5 marks)

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23 Find the solution to

$$5^{2x} = 9$$

giving your answer in the form $\log_5 a$, where a in an integer.

(Total for question 23 is 2 marks)

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24 A curve has the equation $y = e^{2x}$

(a) Find, in terms of a, the equation of the tangent to the curve at the point (a, e^{2a}) **(3)**

(b) Find the value of a for which this tangent passes through the origin. **(2)**

(c) Hence, find the set of values of m for which the equation

$$e^{2x} = mx$$

has no real solutions. **(3)**

(Total for question 24 is 8 marks)

25 Jonathan is investigating the spread of a virus measured by the number of daily recorded cases N.

He believes that V and t are connected by a formula:
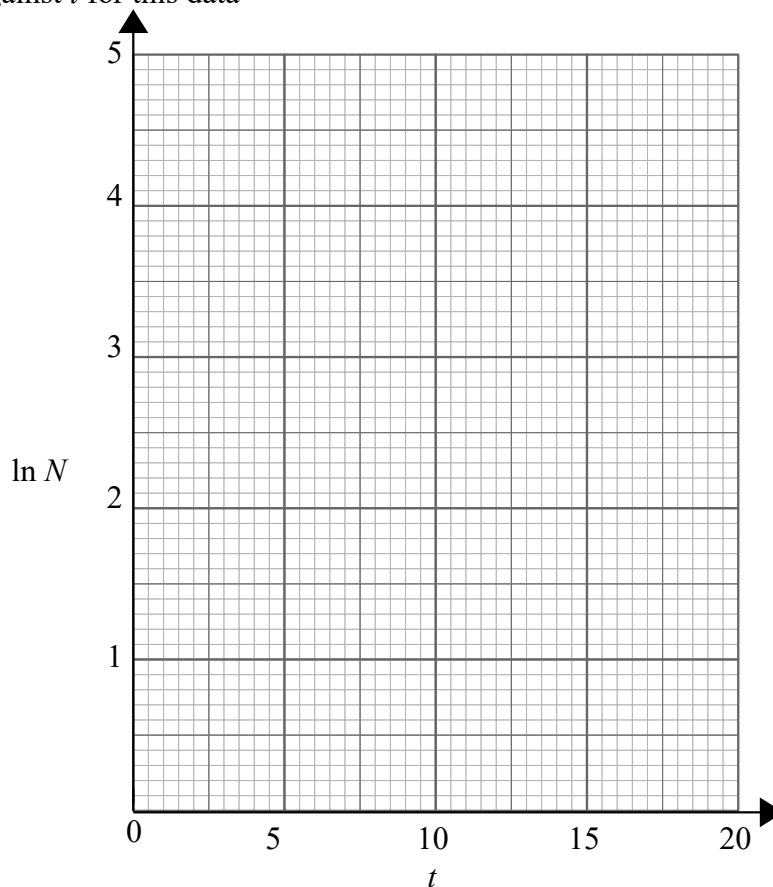
$$N = Ae^{kt}$$

where t is the number of days since the virus was first recorded and where A and k are constants.

(a) Express $\ln N$ in terms of t

(2)

Jonathan collects the following data

t	8	11	18
N	12	24	90

(b) Plot $\ln N$ against t for this data

(2)

(4)

(c) Using the graph, estimate the value of A and the value of k.

(Total for question 25 is 8 marks)

26 The temperature of water in a kettle is modelled using the equation $T = 75e^{-kt} + 25$

Where T is the temperature t minutes after the kettle is turned off and k is a positive constant.

(a) Explain what the 25 represents in the equation $T = 75e^{-kt} + 25$ **(1)**

When the kettle is turned off the rate of decrease of the water temperature is 20°C per minute.

(b) Find the value of k **(3)**

(c) Find how many minutes it takes for the water to cool to 55°C **(3)**

(Total for question 26 is 7 marks)

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27 The line L is a tangent to the curve $y = e^{\frac{1}{3}x}$ at the point where $x = 3$

Show that L passes through the origin.

(Total for question 27 is 4 marks)

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28 Find the coordinates of the point of intersection of the curves $y = e^x$ and $y = 3 - 2e^{\frac{1}{2}x}$

(Total for question 28 is 6 marks)

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29 Find the exact solution to the equation $(2^x)^2 = 2(3^x)$

(Total for question 29 is 5 marks)

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30 On the same axes sketch the curves $y = e^x$ and $y = \ln x$

(Total for question 30 is 4 marks)

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31 (a) Write down the value of $\log_a a$
(b) Write down the value of $\log_a a^2$

(Total for question 31 is 2 marks)

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32 (a) Show that the equation $2\log_2 x = \log_2(x + a) + 3$, can be expressed in the form $x^2 - 8x - 8a = 0$ **(3)**

(b) Given the equation $2\log_2 x = \log_2(x + a) + 3$ has only one real root find the value of a. **(3)**

(Total for question 32 is 6 marks)

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33 Solve the equation $2\log_2(x + 6) = \log_2(x + 4) + 3$,
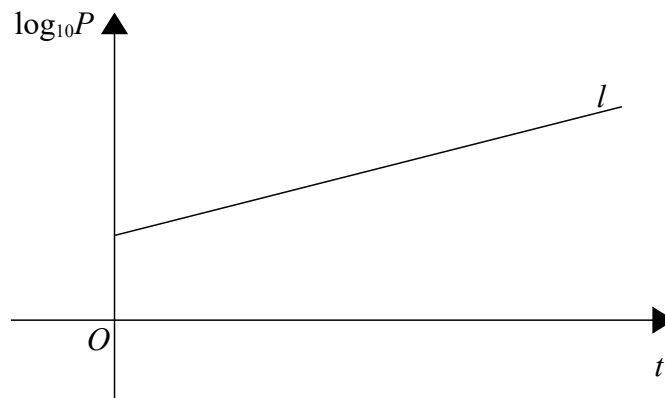
(Total for question 33 is 5 marks)

34   Solve the equation
$$2\log_2(x) + \log_2(\sqrt{x}) = 10$$

(Total for question 34 is 4 marks)

35

The world population, P (billion), is modelled by the equation $P = ab^t$, where a and b are constants and t is the number of years since 1 April 1974.

The line l illustrates the linear relationship between t and $\log_{10}P$ since 1 April 1974.
The equation of line l is $\log_{10}P = 0.008t + 0.6$

(a) Find, to 4 significant figures, the value of a and the value of b

(4)

(b) With reference to the model interpret
      (i) the value of the constant a,
      (ii) the value of the constant b.

(2)

(c) Find the world population, as predicted by the model, on 1 April 2020.

(2)

(d) Given the world population on April 2020 was 7.8 billion use your answer in part (c) to comment on the suitability of the model.

(1)

(Total for question 35 is 9 marks)

36   The value of a car is modelled using the formula $V = 17100e^{-0.2t} + 2000$

Where V is the value of the car and the car's age is t years.

(a) Find the initial value of the car. **(1)**

Given the model predicts that the value of the car is decreasing at a rate of £1000 per year at the instant when $t = T$

(b)   (i) Show that $3420e^{-0.2T} = 1000$

      (ii) Hence find the age of the car at this instant, giving your answer in years and
         months to the nearest month. **(6)**

(Total for question 36 is 7 marks)

37
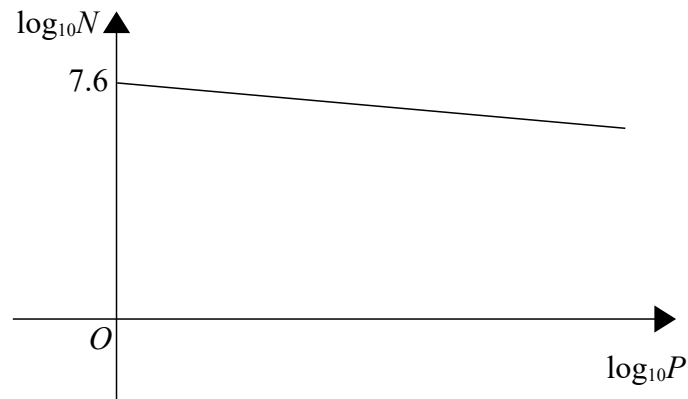
A company model the number of products they will sell *N,* and the price of the products, *P,* by the equation

$$N = AP^B$$

where *A* and *B* are constants.

The graph shows the linear relationship between $\log_{10}N$ and $\log_{10}P$

The line meets the vertical axis at 7.6 and has a gradient of -1.32

(a) Find, to the 3 significant figures, the value of *A* and the value of *B*. **(3)**

The company sets a selling price at £74.95.

(b) Find, according to the model, an estimate for the number of products the company will sell. **(2)**

The company estimate their production costs to be £20 per product.

(c) Estimate the profit the company will make from the product. **(2)**

(Total for question 37 is 7 marks)

38 Express as a single logarithm
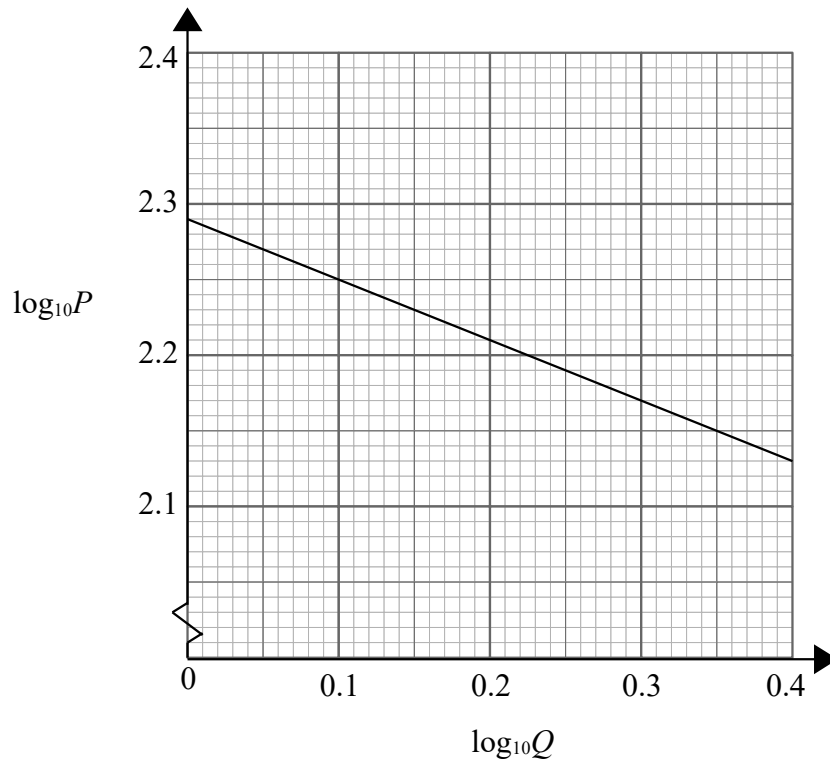
$$3 \log_a 4 - \log_a 2$$

(Total for question 38 is 2 marks)

39 The value of an investment grew exponentially from £3.2 million in 2001 to £6.1 million in 2021.

Estimate the value of the investment in 2031 if the exponential growth continued.

(Total for question 39 is 2 marks)

40

The graph shows the relationship between $\log_{10}P$ and $\log_{10}Q$.

Show that P and Q can be connected by the equation $P = aQ^b$ where a and b are constants.

(Total for question 40 is 5 marks)

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41 Michael's car is valued after 2 years and after 5 years. The valuations are shown in the table below.

Time (years)	2	5
Value (£)	47200	29200

Michael models the relationship between V, the value of the car and t, the time in years since Michael bought the car, by

$$V = Ae^{-kt}$$

where A and k are constants.

(a) Explain what the value of A represents.   **(1)**

(b) Show that

$$\ln V = \ln A - kt$$ **(1)**

(c) Calculate the value of A and the value of k **(4)**

(d) Use the model to predict the value of the car after 10 years.   **(2)**

(Total for question 41 is 8 marks)

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