

1 (a) Solve the inequality

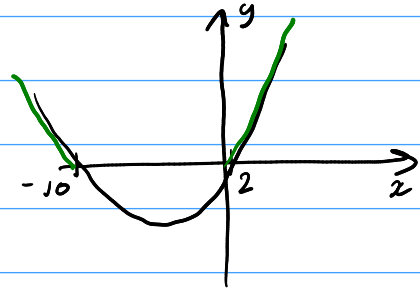
$$x^2 + 8x > 20$$

(b) Find the set of values for  $x$  which satisfy both of the inequalities

$$x^2 + 8x > 20$$

$$18 + 3x < 23 + x$$

$$\begin{aligned} a/ \quad x^2 + 8x - 20 &> 0 \\ (x+10)(x-2) &> 0 \\ x = -10 \quad x = 2 \end{aligned}$$



$$\underline{\underline{x < -10 \quad \text{or} \quad x > 2}}$$

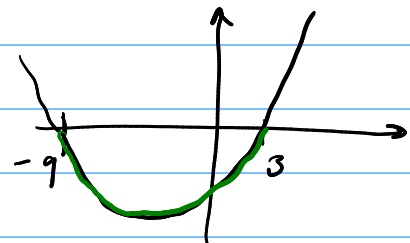
$$\begin{aligned} b/ \quad 18 + 3x &< 23 + x \\ 18 + 2x &< 23 \\ 2x &< 5 \\ x &< \frac{5}{2} \end{aligned}$$

$$\underline{\underline{x < -10 \quad \text{or} \quad 2 < x < \frac{5}{2}}}$$

2 Find the set of values of  $x$  for which

$$(x+5)(x+1) < 32$$

$$\begin{aligned} x^2 + x + 5x + 5 &< 32 \\ x^2 + 6x - 27 &< 0 \\ (x+9)(x-3) &< 0 \\ x = -9 \quad x = 3 \end{aligned}$$



$$\underline{\underline{-9 < x < 3}}$$

3 Solve the simultaneous equations

$$\begin{aligned}x + y &= 3 \\ x^2 + 2y^2 - 8x &= 6\end{aligned}$$

$$y = 3 - x$$

$$x^2 + 2(3 - x)^2 - 8x = 6$$

$$x^2 + 2(9 - 3x - 3x + x^2) - 8x = 6$$

$$x^2 + 18 - 6x - 6x + 2x^2 - 8x = 6$$

$$3x^2 - 20x + 18 = 6$$

$$3x^2 - 20x + 12 = 0$$

$$3x^2 - 2x - 18x + 12 = 0$$

$$(x - 6)(3x - 2) = 0$$

$$x = 6 \quad x = \frac{2}{3}$$

$$\begin{array}{r} 36 \quad 36 \\ 1 \quad 18 \\ \hline 2 \quad 18 \\ 3 \quad 12 \\ 4 \quad 9 \\ 6 \quad 6 \end{array}$$

$$y = 3 - x$$

$$y = -3 \quad y = \frac{7}{3}$$

$$x = 6, y = -3 \quad \text{or} \quad x = \frac{2}{3}, y = \frac{7}{3}$$

4 Solve the inequality

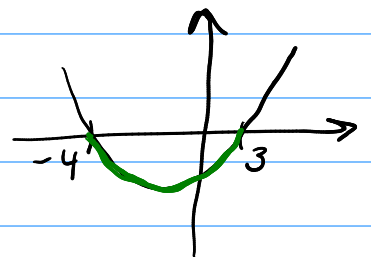
$$x(x + 1) \leq 12$$

$$x^2 + x \leq 12$$

$$x^2 + x - 12 \leq 0$$

$$(x + 4)(x - 3) \leq 0$$

$$x = -4 \quad x = 3$$



$$\underline{-4 \leq x \leq 3}$$

- 5 Find the coordinates of the points where the circle C with equation  $x^2 + y^2 - 2x = 19$  meets the line L with equation  $y = 3x - 1$

$$x^2 + (3x - 1)^2 - 2x = 19$$

$$x^2 + (3x - 1)(3x - 1) - 2x = 19$$

$$x^2 + 9x^2 - 3x - 3x + 1 - 2x = 19$$

$$10x^2 - 8x + 1 = 19$$

$$10x^2 - 8x - 18 = 0$$

$$5x^2 - 4x - 9 = 0$$

$$5x^2 + 5x - 9x - 9 = 0$$

$$(5x - 9)(x + 1) = 0$$

$$x = \frac{9}{5} \quad x = -1$$

$$y = 3\left(\frac{9}{5}\right) - 1 \quad y = 3(-1) - 1$$

$$= \frac{22}{5} \quad = -4$$

$$\left(\frac{9}{5}, \frac{22}{5}\right) \text{ and } (-1, -4)$$

$$\begin{array}{r} 45 \\ 1 \ 45 \\ \underline{3 \ 15} \\ 5 \ 9 \end{array}$$

- 6 The curve C has the equation  $y = x^2 - 2x + 7$   
The line L has the equation  $x + y = 7$   
Find the coordinates of the points where L and C intersect.

$$y = 7 - x$$

$$7 - x = x^2 - 2x + 7$$

$$0 = x^2 - x$$

$$0 = x(x - 1)$$

$$x = 0 \quad x = 1$$

$$y = 7 \quad y = 6$$

$$[y = 7 - x]$$

$$\underline{(0, 7)} \text{ and } \underline{(1, 6)}$$

7 Solve the simultaneous equations

$$\begin{aligned}x + 2y &= 3 \\x^2 + y^2 - 2xy &= 6\end{aligned}$$

$$x = 3 - 2y$$

$$\begin{aligned}(3 - 2y)^2 + y^2 - 2y(3 - 2y) &= 6 \\(3 - 2y)(3 - 2y) + y^2 - 6y + 4y^2 &= 6\end{aligned}$$

$$9 - 6y - 6y + 4y^2 + y^2 - 6y + 4y^2 = 6$$

$$9y^2 - 18y + 3 = 0$$

$$3y^2 - 6y + 1 = 0$$

$$y = \frac{3 \pm \sqrt{6}}{3}$$

$$\underline{x = \frac{3 - 2\sqrt{6}}{3}, y = \frac{3 + \sqrt{6}}{3}} \quad \text{or} \quad \underline{x = \frac{3 + 2\sqrt{6}}{3}, y = \frac{3 - \sqrt{6}}{3}}$$

8 (a) Solve the inequality

$$x^2 + 3x - 10 < 0$$

(b) Find the set of values for  $x$  which satisfy both of the inequalities

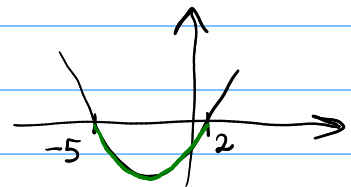
$$x^2 + 3x - 10 < 0$$

$$9 + 3x \leq 12 + x$$

$$a/ \quad (x + 5)(x - 2) < 0$$

$$x = -5 \quad x = 2$$

$$\underline{-5 < x < 2}$$



$$b/ \quad 9 + 3x \leq 12 + x$$

$$9 + 2x \leq 12$$

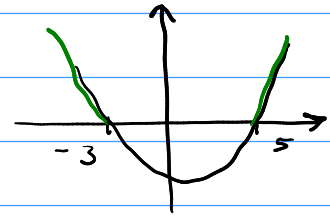
$$2x \leq 3$$

$$x \leq \frac{3}{2}$$

$$\underline{-5 < x \leq \frac{3}{2}}$$

- 9 Using algebra, solve the inequality  $x^2 - 2x > 15$  writing your answer in set notation.

$$\begin{aligned}x^2 - 2x - 15 &> 0 \\(x - 5)(x + 3) &> 0 \\x = 5 \quad x = -3\end{aligned}$$

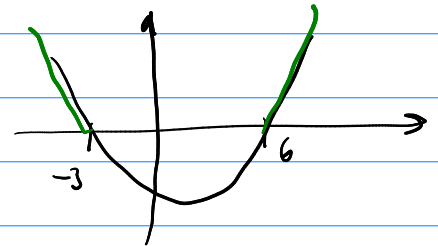


$$\underline{\{x: x < -3\} \cup \{x: x > 5\}}$$

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- 10 Solve the inequality  $18 + x^2 - 3x > 0$

$$\begin{aligned}x^2 - 3x + 18 &> 0 \\(x - 6)(x + 3) &> 0 \\x = 6 \quad x = -3\end{aligned}$$

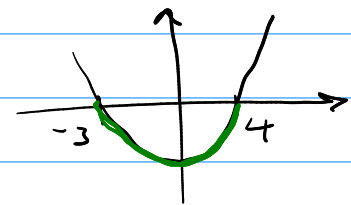


$$\underline{x < -3} \quad \text{or} \quad \underline{x > 6}$$

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- 11 Using algebra, solve the inequality  $x^2 - x + 12 < 0$  writing your answer in set notation.

$$\begin{aligned}(x - 4)(x + 3) &< 0 \\x = 4 \quad x = -3\end{aligned}$$



$$\{x: -3 < x < 4\}$$

- 12 Determine the points of intersection of the curve  $2xy + x^2 - 32 = 0$  and the line  $x + 3y = 2$

$$y = \frac{2-x}{3}$$

$$2x \left( \frac{2-x}{3} \right) + x^2 - 32 = 0$$

$$\frac{4x - 2x^2}{3} + x^2 - 32 = 0$$

$$4x - 2x^2 + 3x^2 - 96 = 0$$

$$x^2 + 4x - 96 = 0$$

$$(x+12)(x-8) = 0$$

$$x = -12 \quad x = 8$$

$$y = \frac{14}{3} \quad y = -2$$

$$\left(-12, \frac{14}{3}\right) \text{ and } (8, -2)$$

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- 13 Solve the inequality  $18 - x < 5x - 2$

$$18 < 6x - 2$$

$$20 < 6x$$

$$\frac{10}{3} < x$$

$$\underline{x > \frac{10}{3}}$$

14 The curve with equation  $y = px^2 - 4px - 5p$ , where  $p$  is a constant does not intersect the line with equation  $y = 2x - 12$ .

(a) Show that  $9p^2 - 8p + 1 < 0$

(b) Find the set of possible values for  $p$ .

a/  $px^2 - 4px - 5p = 2x - 12$  (has no real roots)  
 $px^2 - 4px - 2x - 5p + 12 = 0$   
 $px^2 + (-4p - 2)x - 5p + 12 = 0$

$$b^2 - 4ac < 0$$

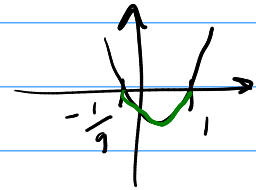
$$(-4p - 2)^2 - 4p(-5p + 12) < 0$$

$$16p^2 + 8p + 8p + 4 + 20p^2 - 48p < 0$$

$$36p^2 - 32p + 4 < 0$$

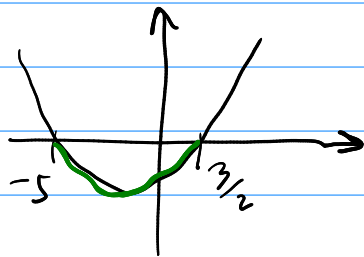
$$9p^2 - 8p + 1 < 0$$

b/  $(9p + 1)(p - 1)$   
 $p = -\frac{1}{9} \quad p = 1$



$$\underline{\underline{-\frac{1}{9} < p < 1}}$$

15 Using algebra, solve the inequality  $15 - 2x^2 > 7x$  writing your answer in set notation.



$$0 > 2x^2 + 7x - 15$$

$$0 > 2x^2 + 10x - 3x - 15$$

$$0 > (2x - 3)(x + 5)$$

$$x = \frac{3}{2} \quad x = -5$$

$$\underline{\underline{\left\{ x : -5 < x < \frac{3}{2} \right\}}}$$