1 The circle C has the equation $x^2 + y^2 - 2x + 6y = 26$

Find:

(i) The coordinates of the centre of C

(ii) the radius of C

$$x^2 - 2x + y^2 + 6y = 26$$

$$(x-1)^2 - 1 + (y+3)^2 - 9 = 26$$

$$(x-1)^2 + (y+3)^2 = 36$$

i) $(1, -3)$

ii) 6

---

2 The circle C has centre (2, 5) and passes through point (4, 9).

Find and equation for C.

$$(x-2)^2 + (y-5)^2 = r^2$$

$$(4-2)^2 + (9-5)^2 = r^2$$

$$20 = r^2$$

$$(x-2)^2 + (y-5)^2 = 20$$

3 The circle C has centre $(-2, 3)$ and passes through point $(1, 8)$.

(a) Find and equation for C. **(4)**

(b) Show that the point $(3, 6)$ lies on C. **(1)**

(c) Find the equation to the tangent to C at $(3,6)$. **(5)**
Give your answer in the form $ax + by + c = 0$, where a, b and c are integers.

a/ $(x+2)^2 + (y-3)^2 = r^2$

$(1+2)^2 + (8-3)^2 = r^2$

$34 = r^2$

$(x+2)^2 + (y-3)^2 = 34$

b/ $(3+2)^2 + (6-3)^2 = 34$

$34 = 34$ ✓

c/ $\underset{x_1\ y_1}{(-2, 3)}$ $\underset{x_2\ y_2}{(3, 6)}$

gradient of radius $= \dfrac{6-3}{3--2} = \dfrac{3}{5}$

tangent is perpendicular $\therefore$ $m = -\dfrac{5}{3}$ $\underset{x_1\ y_1}{(3, 6)}$

$y - 6 = -\dfrac{5}{3}(x-3)$

$3(y-6) = -5(x-3)$

$3y - 18 = -5x + 15$

$5x + 3y - 33 = 0$

4 The circle C has centre (2, 5) and radius 7.

(a) Find and equation for C. **(2)**

The line $y = 3x - 1$ intersects C at the points A and B.

(b) Find the exact coordinates of A and B. **(6)**

a/ $$(x-2)^2 + (y-5)^2 = 49$$

b/ $$(x-2)^2 + (3x-1-5)^2 = 49$$

$$(x-2)^2 + (3x-6)^2 = 49$$

$$x^2 - 2x - 2x + 4 + 9x^2 - 18x - 18x + 36 = 49$$

$$10x^2 - 40x - 9 = 0$$

$$x = \frac{20 + 7\sqrt{10}}{10} \qquad x = \frac{20 - 7\sqrt{10}}{10}$$

$$y = \frac{50 + 21\sqrt{10}}{10} \qquad y = \frac{50 - 21\sqrt{10}}{10}$$

$$\left(\frac{20 + 7\sqrt{10}}{10}, \frac{50 + 21\sqrt{10}}{10} \right) \text{ and } \left(\frac{20 - 7\sqrt{10}}{10}, \frac{50 - 21\sqrt{10}}{10} \right)$$

5 The circle C has the equation $x^2 + y^2 + 8x - 4y + k = 0$
Where k is a constant.

Given that the point (1, 5) lies on C.

(a) Find the value of k **(2)**

(b) Find the coordinates of the centre and the radius of C **(3)**

A straight line that passes through the point A (3, 7) is a tangent to the circle C at the point B

(c) Find the exact length of the line AB **(5)**

a/ $(1)^2 + (5)^2 + 8(1) - 4(5) + k = 0$

$14 + k = 0$

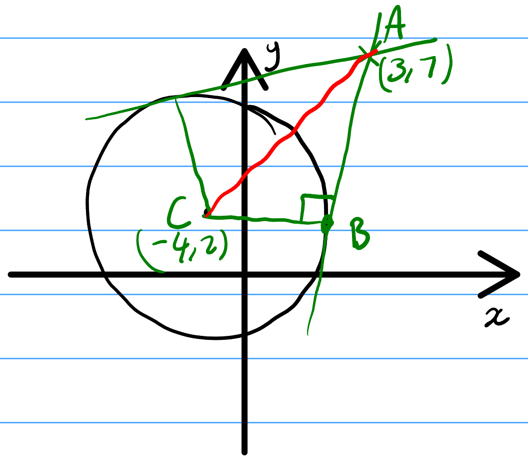
$k = -14$

b/ $(x+4)^2 - 16 + (y-2)^2 - 4 - 14 = 0$

$(x+4)^2 + (y-2)^2 = 34$

centre : $(-4, 2)$ radius $= \sqrt{34}$

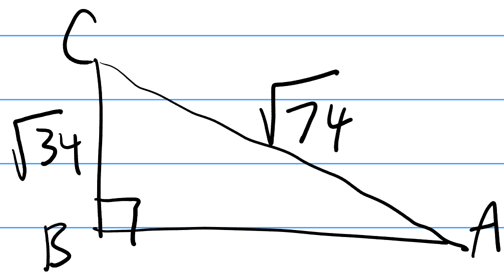
c/



$AC \text{ length}^2 = (3--4)^2 + (7-2)^2$

$AC \text{ length}^2 = 7^2 + 5^2$

$\text{length} = \sqrt{74}$



$AB^2 = AC^2 - BC^2$

$AB = \sqrt{40}$

6 The points D, E and F have coordinates $(-3, 2)$, $(4, -1)$ and $(1, -8)$ respectively.

$x_1\ y_1\ \ x_2\ y_2$

$x_1\ y_1\ x_2\ y_2$

(a) Show that angle DEF is a right angle. **(4)**

Given that D, E and F all lie on the circle C.

(b) Find the coordinates of the centre of C. **(3)**

(c) Find the equation of the circle C. **(3)**

a/ DE must be perpendicular to EF.

$$DE \quad m = \frac{-1-2}{4--3} = \frac{-3}{7}$$

$$EF \quad m = \frac{-8--1}{1-4} = \frac{-7}{-3} = \frac{7}{3}$$

$$\frac{-3}{7} \times \frac{7}{3} = -1 \quad \therefore \text{ perpendicular } \checkmark$$

b/ As DEF is a right angle DF must be a diameter.
Centre is midpoint of DF.

$$\left(\frac{-3+1}{2}, \frac{2-8}{2}\right)$$

$$\underline{\underline{(-1, -3)}}$$

c/

$$(x+1)^2 + (y+3)^2 = r^2$$

$$(-3+1)^2 + (2+3)^2 = r^2$$

$$r^2 = 29$$

$$\underline{\underline{(x+1)^2 + (y+3)^2 = 29}}$$

7 The circle C has the equation $x^2 + y^2 - 6x + 2y = 6$

(a) Find the coordinates of the centre and the radius of C **(3)**

C crosses the y axis at the points A and B

(b) Find the coordinates of the points A and B **(3)**

a) $x^2 - 6x + y^2 + 2y = 6$

$(x-3)^2 - 9 + (y+1)^2 - 1 = 6$

$(x-3)^2 + (y+1)^2 = 16$

centre $(3, -1)$ radius $= 4$

b) crosses y when $x = 0$

$y^2 + 2y = 6$

$y^2 + 2y - 6 = 0$

$y = -1 + \sqrt{7}$ or $y = -1 - \sqrt{7}$

$(0, -1+\sqrt{7})$ $(0, -1-\sqrt{7})$

8 The points A and B have coordinates (-3, 5) and (13, -4) respectively.

Given that AB is a diameter of the circle C.

Find an equation for C.

centre $\left(\frac{-3+13}{2}, \frac{5-4}{2}\right)$

$$\left(5, \frac{1}{2}\right)$$

$$(x-5)^2 + \left(y-\frac{1}{2}\right)^2 = r^2$$

$$(-3-5)^2 + \left(5-\frac{1}{2}\right)^2 = r^2$$

$$r^2 = \frac{337}{4}$$

$$(x-5)^2 + \left(y-\frac{1}{2}\right)^2 = \frac{337}{4}$$

9 The circle C has centre (1, 5) and passes through the point A (-4, 3).

(a) Find an equation for C. **(4)**

(b) Find an equation for the tangent to C at A, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers **(4)**

a) $(x-1)^2 + (y-5)^2 = r^2$

$(-4-1)^2 + (3-5)^2 = r^2$

$r^2 = 29$

$(x-1)^2 + (y-5)^2 = 29$

b) gradient of radius $= \frac{3-5}{-4-1} = \frac{-2}{-5} = \frac{2}{5}$

$\therefore \quad m = -\frac{5}{2}$

$y - 3 = -\frac{5}{2}(x+4)$

$2(y-3) = -5(x+4)$

$2y - 6 = -5x - 20$

$\underline{\underline{5x + 2y + 14 = 0}}$

10 The circle C has centre (5, k), where k is a constant.

The line $y = 2x + 1$ is a tangent to the circle C, touching C at the point A (3, 7).

Find an equation for C.

gradient of radius $= -\frac{1}{2}$ (perpendicular)

$$\frac{7-k}{3-5} = -\frac{1}{2}$$

$$2(7-k) = -1(-2)$$

$$14 - 2k = 2$$

$$12 = 2k$$

$$\underline{\underline{k = 6}}$$

$$(x-5)^2 + (y-6)^2 = r^2$$

$$(3-5)^2 + (7-6)^2 = r^2$$

$$r^2 = 5$$

$$\underline{\underline{(x-5)^2 + (y-6)^2 = 5}}$$

11 The circle C has the equation $x^2 + y^2 - 14x + 2y + 40 = 0$

(a) Find:

(i) The coordinates of the centre of C

(ii) the exact radius of C **(3)**

The line with equation $y = kx$ where k is a constant, meets C at two distinct points.

(b) Find the range of possible values of k. **(6)**

a) $(x-7)^2 - 49 + (y+1)^2 - 1 + 40 = 0$

$(x-7)^2 + (y+1)^2 = 10$

i) $(7, -1)$

ii) $\sqrt{10}$

b) $y = kx$

$x^2 + (kx)^2 - 14x + 2(kx) + 40 = 0$

$x^2 + k^2x^2 - 14x + 2kx + 40 = 0$

$(k^2+1)x^2 + (2k-14)x + 40 = 0$

2 solutions $\therefore$ $b^2 - 4ac > 0$

$(2k-14)^2 - 4(k^2+1)(40) > 0$

$4k^2 - 28k - 28k + 196 - 160k^2 - 160 > 0$

$-156k^2 - 56k + 36 > 0$

$-\frac{9}{13} < k < \frac{1}{3}$

12 The circle C has the equation $x^2 + y^2 - 8x + 6y + 5 = 0$

(a) Find:

(i) The coordinates of the centre of C

(ii) the exact radius of C **(3)**

The line with equation $y = k$ where k is a constant, is a tangent to C.

(b) Find the possible values of k. **(2)**

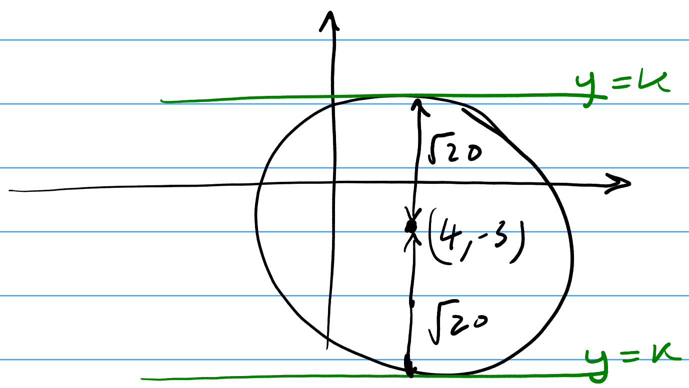
$$(x-4)^2 - 16 + (y+3)^2 - 9 + 5 = 0$$

$$(x-4)^2 + (y+3)^2 = 20$$

i) $(4, -3)$

ii) $\sqrt{20}$

b)



$k = -3 + \sqrt{20}$ or $k = -3 - \sqrt{20}$

13 The circle C has the equation $x^2 + y^2 - 6x - 12y = 0$

The line l is a tangent to C at the point $(\underset{x_1}{8}, \underset{y_1}{3})$.

Find the equation of l in the form $ax + by + c = 0$

$$(x-3)^2 - 9 + (y-6)^2 - 36 = 0$$

$$(x-3)^2 + (y-6)^2 = 45$$

centre $(\underset{x_2}{3}, \underset{y_2}{6})$

gradient of radius $= \frac{6-3}{3-8} = -\frac{3}{5}$

$\therefore$ gradient of tangent $= \frac{5}{3}$

$$y - 3 = \frac{5}{3}(x-8)$$

$$3(y-3) = 5(x-8)$$

$$3y - 9 = 5x - 40$$

$$\underline{\underline{0 = 5x - 3y - 31}}$$

14 The circle C has the equation $x^2 + y^2 - 8x - 10y + k = 0$, where k is a constant.

Given the C lies entirely in the first quadrant, find the range of possible values of k.

$$(x-4)^2 - 16 + (y-5)^2 - 25 + k = 0$$

$$(x-4)^2 + (y-5)^2 = 41 - k$$

centre $(4, 5)$ radius must be less than 4

$\therefore r^2 < 16$

$r^2 = 41 - k$ (k must be less than 41)

$$41 - k < 16$$

$$25 < k$$

$$\underline{25 < k < 41}$$

15 The line $y = mx - 2$ is a tangent to the circle $x^2 + 6x + y^2 - 8y + 5 = 0$

Find the two possible values of m, giving your answers in exact form.

$$x^2 + 6x + (mx-2)^2 - 8(mx-2) + 5 = 0$$
$$x^2 + 6x + m^2x^2 - 4mx + 4 - 8mx + 16 + 5 = 0$$
$$(1 + m^2)x^2 + (6 - 12m)x + 25 = 0$$

Tangent has one solution $\therefore b^2 - 4ac = 0$

$$(6 - 12m)^2 - 4(1 + m^2)(25) = 0$$
$$36 - 72m - 72m + 144m^2 - 100 - 100m^2 = 0$$
$$44m^2 - 144m - 64 = 0$$

$$m = \frac{18 + 10\sqrt{5}}{11} \quad \text{or} \quad m = \frac{18 - 10\sqrt{5}}{11}$$

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16 The line $y = mx + 2$ is a tangent to the circle $(x - 5)^2 + (y + 1)^2 = 15$

Find the two possible values of m, giving your answers in exact form.

$$(x-5)^2 + (mx + 2 + 1)^2 = 15$$
$$(x-5)^2 + (mx + 3)^2 = 15$$
$$x^2 - 10x + 25 + m^2x^2 + 6mx + 9 = 15$$
$$(1 + m^2)x^2 + (6m - 10)x + 19 = 0$$

tangent $\therefore b^2 - 4ac = 0$

$$(6m - 10)^2 - 4(1 + m^2)(19) = 0$$
$$36m^2 - 120m + 100 - 76 - 76m^2 = 0$$
$$-40m^2 - 120m + 24 = 0$$

$$m = \frac{-15 + \sqrt{285}}{10} \quad \text{or} \quad m = \frac{-15 - \sqrt{285}}{10}$$

17 The circle C has the equation $x^2 + y^2 - 4x - 6y = 48$

(a) Find the coordinates of the centre of C **(2)**

(b) Find equation of the tangent to the circle at the point (7, 9) **(4)**

a/ $(x-2)^2 - 4 + (y-3)^2 - 9 = 48$

$(x-2)^2 + (y-3)^2 = 61$

$(2, 3)$

b/ gradient of radius $= \dfrac{9-3}{7-2} = \dfrac{6}{5}$

gradient of tangent $= -\dfrac{5}{6}$

$y - 9 = -\dfrac{5}{6}(x-7)$

$6(y-9) = -5(x-7)$

$6y - 54 = -5x + 35$

$5x + 6y - 89 = 0$

18 A circle C has centre (10, 6) and radius $2\sqrt{17}$

A line L has equation $y = mx$

(a) Show that the x-coordinate of any points of intersection of C and L satisfies the equation.

$$(m^2 + 1)x^2 - (12m + 20)x + 68 = 0$$ **(3)**

(b) Find values of m for which the equation in (a) has equal roots. **(3)**

(c) Two lines drawn from the origin which are tangents to C.

Find the coordinates of the points of contact between the tangents and C. **(4)**

$$r^2 = (2\sqrt{17})^2$$
$$= 68$$
$$(x-10)^2 + (y-6)^2 = 68$$
$$(x-10)^2 + (mx-6)^2 = 68$$

$$x^2 - 10x - 10x + 100 + m^2x^2 - 6mx - 6mx + 36 = 68$$

$$(1 + m^2)x^2 + (-20 - 12m)x + 68 = 0$$

$$(m^2 + 1)x^2 - (12m + 20)x + 68 = 0$$

b) $b^2 - 4ac = 0$
$$(12m + 20)^2 - 4(m^2+1)(68) = 0$$
$$144m^2 + 480m + 400 - 272m^2 - 272 = 0$$
$$-128m^2 + 480m + 128 = 0$$
$$4m^2 - 15m - 4 = 0$$

$$m = 4 \quad \text{or} \quad m = -\frac{1}{4}$$

c) $y = 4x$ and $y = -\frac{1}{4}x$

when $m = 4$

$17x^2 - 68x + 68 = 0$

$x = 2$ $y = 8$

$(2, 8)$

when $m = -\frac{1}{4}$

$\frac{17}{16}x^2 - 17x + 68 = 0$

$x = 8$ $y = -2$

$(8, -2)$

19 The lines with equations $y = \frac{2}{3}x$ and $y = -\frac{2}{3}x$ are tangents to a circle at (3, 2) and (3, -2) respectively.

$x_1\ y_1 \qquad x_1\ y_1$

Find an equation for the circle.

radius at (3, 2)

$m = -\frac{3}{2}$

$y - 2 = -\frac{3}{2}(x - 3)$

$2(y - 2) = -3(x - 3)$

$2y - 4 = -3x + 9$

$\underline{3x + 2y = 13}$

radius at (3, -2)

$m = \frac{3}{2}$

$y + 2 = \frac{3}{2}(x - 3)$

$2y + 4 = 3x - 9$

$\underline{13 = 3x - 2y}$

Intersection is solution to sim. equations

$x = \frac{13}{3} \qquad y = 0$

$\left(x - \frac{13}{3}\right)^2 + y^2 = r^2$

$\left(3 - \frac{13}{3}\right)^2 + 2^2 = r^2$

$r^2 = \frac{52}{9}$

$$\underline{\underline{\left(x - \frac{13}{3}\right)^2 + y^2 = \frac{52}{9}}}$$

20 The circle with equation $x^2 + y^2 - 10x + ky + 20 = 0$ has radius 3.

Find the two possible values of the constant k.

$$(x-5)^2 - 25 + \left(y + \frac{k}{2}\right)^2 - \frac{k^2}{4} + 20 = 0$$

$$(x-5)^2 + \left(y + \frac{k}{2}\right)^2 = \frac{k^2}{4} + 5$$

$$\sqrt{\frac{k^2}{4} + 5} = 3$$

$$\frac{k^2}{4} + 5 = 9$$

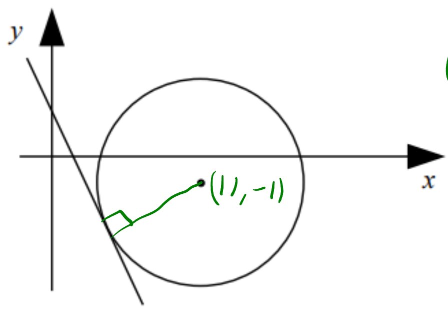
$$\frac{k^2}{4} = 4$$

$$k^2 = 16$$

$$\underline{\underline{k = \pm 4}}$$

21 The diagram shows line with equation $y + 3x = 2$ which is a tangent to a circle with centre (11, -1).

$y = -3x + 2$



$(y+1)^2 + (x-11)^2 = r^2$

Find an equation for the circle.

gradient of radius $= \frac{1}{3}$ (perpendicular)

equation of radius:

$$y + 1 = \frac{1}{3}(x - 11)$$

$$3y + 3 = x - 11$$

$$14 = x - 3y \qquad 3x + y = 2$$

intersection $(2, -4)$

$r^2 = (11-2)^2 + (-1 - -4)^2$

$r^2 = 90$

$$\underline{\underline{(x-11)^2 + (y+1)^2 = 90}}$$

22 Find the centre of the circle with equation $x^2 + y^2 - 8x + 10y = 15$

$$(x-4)^2 - 16 + (y+5)^2 - 25 = 15$$

$$\underline{(4, -5)}$$

---

23 The points $A(4, a)$ and $B(13, 6)$ lie on a circle.

AB is a diameter of the circle and has a gradient of $\frac{1}{3}$

The circle has equation $(x - c)^2 + (y - d)^2 = e$ where c, d and e are rational numbers.

Find the values of a, c, d and e.

equation of diameter: $y - 6 = \frac{1}{3}(x - 13)$

$$3y - 18 = x - 13$$

$$3y = x + 5$$

$$\underline{y = \frac{1}{3}x + \frac{5}{3}}$$

when $x = 4$ $\quad y = \frac{1}{3}(4) + \frac{5}{3}$

$$\underline{= 3}$$

centre $\left(\frac{4+13}{2}, \frac{3+6}{2}\right)$

$$\left(\frac{17}{2}, \frac{9}{2}\right)$$

$$\left(x - \frac{17}{2}\right)^2 + \left(y - \frac{9}{2}\right)^2 = r^2$$

$$\left(13 - \frac{17}{2}\right)^2 + \left(6 - \frac{9}{2}\right)^2 = r^2$$

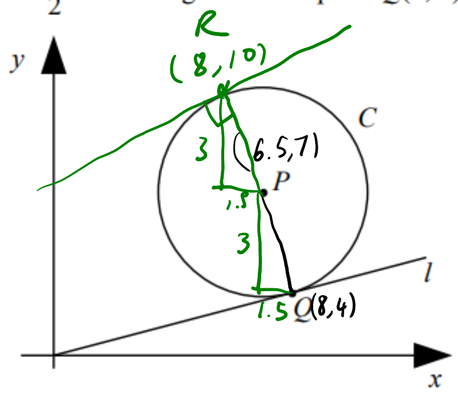
$$r^2 = \frac{45}{2}$$

$$\left(x - \frac{17}{2}\right)^2 + \left(y - \frac{9}{2}\right)^2 = \frac{45}{2}$$

$a = 3 \quad c = \frac{17}{2} \quad d = \frac{9}{2} \quad e = \frac{45}{2}$

24 The sketch shows circle C with centre $P(6.5, 7)$

The line l with equation $y = \frac{1}{2}x$ is a tangent to C at point $Q(8, 4)$



(a) Find and equation for C **(4)**

The line with equation $y = \frac{1}{2}x + k$, where k is a non-zero constant, is also a tangent to C.

(b) Find the value of k. **(3)**

a) $(x-8)^2 + (y-4)^2 = r^2$

$r^2 = 1.5^2 + 3^2$

$r^2 = \frac{45}{4}$

$(x-8)^2 + (y-4)^2 = \frac{45}{4}$

b) passes through $(8, 10)$ $QP = PR$

$y = \frac{1}{2}x + k$

$10 = \frac{1}{2}(8) + k$

$10 = 4 + k$

$k = 6$

25 A circle has equation $(x + 3)^2 + (y - 4)^2 = 25$

Find the gradient of the tangent to the circle at the origin.

$(0, 0)$ $(-3, 4)$

gradient of radius $= \frac{4}{-3} = -\frac{4}{3}$

$\therefore$ gradient of tangent $= \frac{3}{4}$

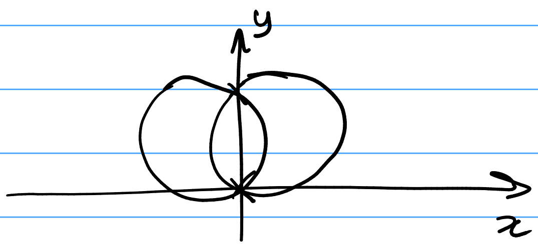
---

26 A circle C with radius 5 passes through (0, 0) and (0, 8).

(a) Sketch the two possible positions of C. **(1)**

(b) Find the equations of the two circles. **(3)**

a/



b/ centre has y coordinate 4

$$(x - a)^2 + (y - 4)^2 = 25$$

$$(-a)^2 + (-4)^2 = 25$$
$$a^2 + 16 = 25$$
$$a^2 = 9$$
$$a = \pm 3$$

$$(x - 3)^2 + (y - 4)^2 = 25$$
and $$(x + 3)^2 + (y - 4)^2 = 25$$

27 A circle C with centre (2, 1) passes through the point (-1, 3).

A line L passes through the points (1, -4) and (10, 2).

Show that L is a tangent to C.

$C:$

$$(x-2)^2 + (y-1)^2 = r^2$$
$$(-1-2)^2 + (3-1)^2 = r^2$$
$$13 = r^2$$
$$(x-2)^2 + (y-1)^2 = 13$$

$L:$

$$m = \frac{2--4}{10-1} = \frac{6}{9} = \frac{2}{3}$$
$$y-2 = \frac{2}{3}(x-10)$$
$$3(y-2) = 2(x-10)$$
$$3y-6 = 2x-20$$
$$3y = 2x-14$$
$$y = \frac{2}{3}x - \frac{14}{3}$$

L and C intersect where

$$(x-2)^2 + \left(\frac{2}{3}x - \frac{14}{3} - 1\right)^2 = 13$$
$$x^2 - 4x + 4 + \left(\frac{2}{3}x - \frac{17}{3}\right)^2 = 13$$
$$x^2 - 4x + 4 + \frac{4}{9}x^2 - \frac{68}{9}x + \frac{289}{9} = 13$$
$$9x^2 - 36x + 36 + 4x^2 - 68x + 289 = 117$$
$$13x^2 - 104x + 208 = 0$$
$$x^2 - 8x + 16 = 0$$
$$(x-4)^2 = 0$$
$$\underline{\underline{x = 4}}$$

one solution

$\therefore$ tangent at $(4, -2)$

28 A circle C with centre (4, -1) and radius 10.

A line L passes through the points (-2, 1) and (14, 9).
x_1 y_1 x_2 y_2

Find the points of intersection of L and C.

$C: \quad (x-4)^2 + (y+1)^2 = 100$

$L: \quad m = \frac{9-1}{14+2} = \frac{8}{16} = \frac{1}{2}$

$y - 1 = \frac{1}{2}(x+2)$

$y - 1 = \frac{1}{2}x + 1$

$y = \frac{1}{2}x + 2$

Intersection where:

$(x-4)^2 + (\frac{1}{2}x + 2 + 1)^2 = 100$

$(x-4)^2 + (\frac{1}{2}x + 3)^2 = 100$

$x^2 - 8x + 16 + \frac{1}{4}x^2 + 3x + 9 = 100$

$\frac{5}{4}x^2 - 5x + 25 = 100$

$\frac{5}{4}x^2 - 5x - 75 = 0$

$x^2 - 4x - 60 = 0$

$(x-10)(x+6) = 0$

$x = 10 \quad x = -6$

$y = \frac{1}{2}(10) + 2 \qquad y = \frac{1}{2}(-6) + 2$

$= 7 \qquad\qquad = -1$

$\underline{(10, 7)}$ and $\underline{(-6, -1)}$