

## AS Level Maths: Circles

**1** The circle  $C$  has the equation  $x^2 + y^2 - 2x + 6y = 26$

Find:

- (i) The coordinates of the centre of  $C$
- (ii) the radius of  $C$

**(Total for question 1 is 3 marks)**

---

**2** The circle  $C$  has centre  $(2, 5)$  and passes through point  $(4, 9)$ .

Find an equation for  $C$ .

**(Total for question 2 is 3 marks)**

---

**3** The circle  $C$  has centre  $(-2, 3)$  and passes through point  $(1, 8)$ .

- (a) Find an equation for  $C$ . **(4)**
- (b) Show that the point  $(3, 6)$  lies on  $C$ . **(1)**
- (c) Find the equation of the tangent to  $C$  at  $(3, 6)$ . **(5)**  
Give your answer in the form  $ax + by + c = 0$ , where  $a$ ,  $b$  and  $c$  are integers.

**(Total for question 3 is 10 marks)**

---

**4** The circle  $C$  has centre  $(2, 5)$  and radius 7.

- (a) Find an equation for  $C$ . **(2)**

The line  $y = 3x - 1$  intersects  $C$  at the points  $A$  and  $B$ .

- (b) Find the exact coordinates of  $A$  and  $B$ . **(6)**

**(Total for question 4 is 8 marks)**

---

**5** The circle  $C$  has the equation  $x^2 + y^2 + 8x - 4y + k = 0$   
Where  $k$  is a constant.

Given that the point  $(1, 5)$  lies on  $C$ .

- (a) Find the value of  $k$  **(2)**
- (b) Find the coordinates of the centre and the radius of  $C$  **(3)**

A straight line that passes through the point  $A(3, 7)$  is a tangent to the circle  $C$  at the point  $B$

- (c) Find the exact length of the line  $AB$  **(5)**

**(Total for question 5 is 10 marks)**

---

6 The points  $D$ ,  $E$  and  $F$  have coordinates  $(-3, 2)$ ,  $(4, -1)$  and  $(1, -8)$  respectively.

(a) Show that angle  $DEF$  is a right angle. (4)

Given that  $D$ ,  $E$  and  $F$  all lie on the circle  $C$ .

(b) Find the coordinates of the centre of  $C$ . (3)

(c) Find the equation of the circle  $C$ . (3)

---

(Total for question 6 is 10 marks)

7 The circle  $C$  has the equation  $x^2 + y^2 - 6x + 2y = 6$

(a) Find the coordinates of the centre and the radius of  $C$  (3)

$C$  crosses the  $y$  axis at the points  $A$  and  $B$

(b) Find the coordinates of the points  $A$  and  $B$  (3)

---

(Total for question 7 is 6 marks)

8 The points  $A$  and  $B$  have coordinates  $(-3, 5)$  and  $(13, -4)$  respectively.

Given that  $AB$  is a diameter of the circle  $C$ .

Find an equation for  $C$ .

---

(Total for question 8 is 6 marks)

9 The circle  $C$  has centre  $(1, 5)$  and passes through the point  $A(-4, 3)$ .

(a) Find an equation for  $C$ . (4)

(b) Find an equation for the tangent to  $C$  at  $A$ , giving your answer in the form  $ax + by + c = 0$ , where  $a$ ,  $b$  and  $c$  are integers (4)

---

(Total for question 9 is 8 marks)

10 The circle  $C$  has centre  $(5, k)$ , where  $k$  is a constant.

The line  $y = 2x + 1$  is a tangent to the circle  $C$ , touching  $C$  at the point  $A(3, 7)$ .

Find an equation for  $C$ .

---

(Total for question 10 is 5 marks)

**11** The circle  $C$  has the equation  $x^2 + y^2 - 14x + 2y + 40 = 0$

(a) Find:

(i) The coordinates of the centre of  $C$

(ii) the exact radius of  $C$

(3)

The line with equation  $y = kx$  where  $k$  is a constant, meets  $C$  at two distinct points.

(b) Find the range of possible values of  $k$ .

(6)

**(Total for question 11 is 9 marks)**

---

**12** The circle  $C$  has the equation  $x^2 + y^2 - 8x + 6y + 5 = 0$

(a) Find:

(i) The coordinates of the centre of  $C$

(ii) the exact radius of  $C$

(3)

The line with equation  $y = k$  where  $k$  is a constant, is a tangent to  $C$ .

(b) Find the possible values of  $k$ .

(2)

**(Total for question 12 is 5 marks)**

---

**13** The circle  $C$  has the equation  $x^2 + y^2 - 6x - 12y = 0$

The line  $l$  is a tangent to  $C$  at the point  $(8, 3)$ .

Find the equation of  $l$  in the form  $ax + by + c = 0$

**(Total for question 13 is 5 marks)**

---

**14** The circle  $C$  has the equation  $x^2 + y^2 - 8x - 10y + k = 0$ , where  $k$  is a constant.

Given the  $C$  lies entirely in the first quadrant, find the range of possible values of  $k$ .

**(Total for question 14 is 4 marks)**

---

**15** The line  $y = mx - 2$  is a tangent to the circle  $x^2 + 6x + y^2 - 8y + 5 = 0$

Find the two possible values of  $m$ , giving your answers in exact form.

**(Total for question 15 is 7 marks)**

---

**16** The line  $y = mx + 2$  is a tangent to the circle  $(x - 5)^2 + (y + 1)^2 = 15$

Find the two possible values of  $m$ , giving your answers in exact form.

**(Total for question 16 is 7 marks)**

---

**17** The circle  $C$  has the equation  $x^2 + y^2 - 4x - 6y = 48$

(a) Find the coordinates of the centre of  $C$

(2)

(b) Find equation of the tangent to the circle at the point  $(7, 9)$

(4)

**(Total for question 17 is 6 marks)**

---

**18** A circle  $C$  has centre  $(10, 6)$  and radius  $2\sqrt{17}$

A line  $L$  has equation  $y = mx$

(a) Show that the  $x$ -coordinate of any points of intersection of  $C$  and  $L$  satisfies the equation.

$$(m^2 + 1)x^2 - (12m + 20)x + 68 = 0 \quad (3)$$

(b) Find values of  $m$  for which the equation in (a) has equal roots. (3)

(c) Two lines drawn from the origin which are tangents to  $C$ .

Find the coordinates of the points of contact between the tangents and  $C$ . (4)

**(Total for question 18 is 10 marks)**

---

**19** The lines with equations  $y = \frac{2}{3}x$  and  $y = -\frac{2}{3}x$  are tangents to a circle at  $(3, 2)$  and  $(3, -2)$  respectively.

Find an equation for the circle.

**(Total for question 19 is 6 marks)**

---

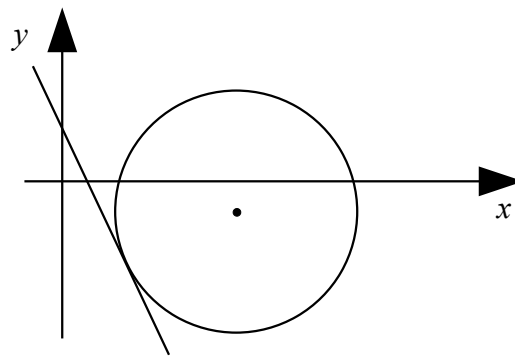
**20** The circle with equation  $x^2 + y^2 - 10x + ky + 20 = 0$  has radius 3.

Find the two possible values of the constant  $k$ .

**(Total for question 20 is 4 marks)**

---

**21** The diagram shows line with equation  $y + 3x = 2$  which is a tangent to a circle with centre  $(11, -1)$ .



Find an equation for the circle.

**(Total for question 21 is 4 marks)**

---

**22** Find the centre of the circle with equation  $x^2 + y^2 - 8x + 10y = 15$

**(Total for question 22 is 1 mark)**

---

**23** The points  $A(4, a)$  and  $B(13, 6)$  lie on a circle.

$AB$  is a diameter of the circle and has a gradient of  $\frac{1}{3}$

The circle has equation  $(x - c)^2 + (y - d)^2 = e$  where  $c$ ,  $d$  and  $e$  are rational numbers.

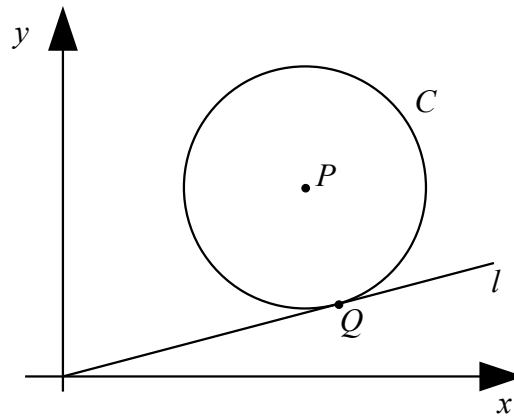
Find the values of  $a$ ,  $c$ ,  $d$  and  $e$ .

**(Total for question 23 is 6 marks)**

---

24 The sketch shows circle  $C$  with centre  $P(6.5, 7)$

The line  $l$  with equation  $y = \frac{1}{2}x$  is a tangent to  $C$  at point  $Q(8, 4)$



(a) Find an equation for  $C$  (4)

The line with equation  $y = \frac{1}{2}x + k$ , where  $k$  is a non-zero constant, is also a tangent to  $C$ .

(b) Find the value of  $k$ . (3)

(Total for question 24 is 7 marks)

25 A circle has equation  $(x + 3)^2 + (y - 4)^2 = 25$

Find the gradient of the tangent to the circle at the origin.

(Total for question 25 is 2 marks)

26 A circle  $C$  with radius 5 passes through  $(0, 0)$  and  $(0, 8)$ .

(a) Sketch the two possible positions of  $C$ . (1)

(b) Find the equations of the two circles. (3)

(Total for question 26 is 4 marks)

27 A circle  $C$  with centre  $(2, 1)$  passes through the point  $(-1, 3)$ .

A line  $L$  passes through the points  $(1, -4)$  and  $(10, 2)$ .

Show that  $L$  is a tangent to  $C$ .

(Total for question 27 is 7 marks)

28 A circle  $C$  with centre  $(4, -1)$  and radius 10.

A line  $L$  passes through the points  $(-2, 1)$  and  $(14, 9)$ .

Find the points of intersection of  $L$  and  $C$ .

(Total for question 28 is 8 marks)