advancing learning, changing lives

Mark Scheme (Results)

Summer 2014

Pearson Edexcel GCE in Core Mathematics 4 (6666/01)

ALWAYS LEARNING

Edexcel and BTEC Qualifications

Edexcel and BTEC qualifications come from Pearson, the world's leading learning company. We provide a wide range of qualifications including academic, vocational, occupational and specific programmes for employers. For further information, please visit our website at www.edexcel.com.

Our website subject pages hold useful resources, support material and live feeds from our subject advisors giving you access to a portal of information. If you have any subject specific questions about this specification that require the help of a subject specialist, you may find our Ask The Expert email service helpful.

www.edexcel.com/contactus

Pearson: helping people progress, everywhere

Our aim is to help everyone progress in their lives through education. We believe in every kind of learning, for all kinds of people, wherever they are in the world. We've been involved in education for over 150 years, and by working across 70 countries, in 100 languages, we have built an international reputation for our commitment to high standards and raising achievement through innovation in education. Find out more about how we can help you and your students at: www.pearson.com/uk

Summer 2014
Publications Code UA038467
All the material in this publication is copyright
© Pearson Education Ltd 2014

General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate’s response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 75.

2. The Edexcel Mathematics mark schemes use the following types of marks:

- **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⎣ The second mark is dependent on gaining the first mark
- dM1 denotes a method mark which is dependent upon the award of the previous method mark.

4. All A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but manifestly absurd answers should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. If a candidate makes more than one attempt at any question:
   - If all but one attempt is crossed out, mark the attempt which is NOT crossed out.
   - If either all attempts are crossed out or none are crossed out, mark all the attempts and score the highest single attempt.

7. Ignore wrong working or incorrect statements following a correct answer.

General Principles for Core Mathematics Marking

(But note that specific mark schemes may sometimes override these general principles).

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2 + bx + c) = (x + p)(x + q)$, where $|pq| = |c|$, leading to x = ...

$(ax^2 + bx + c) = (mx + p)(nx + q)$, where $|pq| = |c|$ and $|mn| = |a|$, leading to x = ...

2. Formula

Attempt to use the correct formula (with values for a, b and c).

3. Completing the square

Solving $x^2 + bx + c = 0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0, \quad q \neq 0$, leading to x = ...

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. ($x^n \to x^{n-1}$)

2. Integration

Power of at least one term increased by 1. ($x^n \to x^{n+1}$)

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values, but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question Number	Scheme	Marks
1.	$x^3 + 2xy - x - y^3 - 20 = 0$	
(a)	$\left\{\dfrac{\cancel{dy}}{\cancel{dx}} \times\right\}$ $\underline{3x^2} + \underline{\underline{\left(2y + 2x\dfrac{dy}{dx}\right)}} \underline{- 1 - 3y^2\dfrac{dy}{dx} = 0}$	M1 A1 B1
	$3x^2 + 2y - 1 + (2x - 3y^2)\dfrac{dy}{dx} = 0$	dM1
	$\dfrac{dy}{dx} = \dfrac{3x^2 + 2y - 1}{3y^2 - 2x}$ or $\dfrac{1 - 3x^2 - 2y}{2x - 3y^2}$	A1 **cso**
		[5]
(b)	At $P(3, -2)$, $m(\mathbf{T}) = \dfrac{dy}{dx} = \dfrac{3(3)^2 + 2(-2) - 1}{3(-2)^2 - 2(3)}; = \dfrac{22}{6}$ or $\dfrac{11}{3}$	
	and either T: $y - -2 = \dfrac{"11"}{3}(x - 3)$ **see notes**	M1
	or $(-2) = \left(\dfrac{11}{3}\right)(3) + c \Rightarrow c = \ldots,$	
	T: $11x - 3y - 39 = 0$ or $K(11x - 3y - 39) = 0$	A1 **cso**
		[2] **7**
(a)	**<u>Alternative method for part (a)</u>**	
	$\left\{\dfrac{\cancel{dx}}{\cancel{dy}} \times\right\}$ $\underline{3x^2\dfrac{dx}{dy}} + \underline{\underline{\left(2y\dfrac{dx}{dy} + 2x\right)}} \underline{- \dfrac{dx}{dy} - 3y^2 = 0}$	M1 A1 B1
	$2x - 3y^2 + (3x^2 + 2y - 1)\dfrac{dx}{dy} = 0$	dM1
	$\dfrac{dy}{dx} = \dfrac{3x^2 + 2y - 1}{3y^2 - 2x}$ or $\dfrac{1 - 3x^2 - 2y}{2x - 3y^2}$	A1 **cso**
		[5]

Question 1 Notes

(a) **General**	**Note**	Writing down $\dfrac{dy}{dx} = \dfrac{3x^2 + 2y - 1}{3y^2 - 2x}$ or $\dfrac{1 - 3x^2 - 2y}{2x - 3y^2}$ from **no working** is full marks.
	Note	Writing down $\dfrac{dy}{dx} = \dfrac{3x^2 + 2y - 1}{2x - 3y^2}$ or $\dfrac{1 - 3x^2 - 2y}{3y^2 - 2x}$ from **no working is** M1A0B0M1A0
	Note	Few candidates will write $3x^2 + 2y + 2x\,dy - 1 - 3y^2 dy = 0$ leading to $\dfrac{dy}{dx} = \dfrac{3x^2 + 2y - 1}{3y^2 - 2x}$, o.e. This should get full marks.
1. (a)	**M1**	Differentiates implicitly to include either $2x\dfrac{dy}{dx}$ or $-y^3 \to \pm k y^2\dfrac{dy}{dx}$. (Ignore $\left(\dfrac{dy}{dx} = \right)$).
	A1	$x^3 \to 3x^2$ **and** $-x - y^3 - 20 = 0 \to -1 - 3y^2\dfrac{dy}{dx} = 0$
	B1	$2xy \to 2y + 2x\dfrac{dy}{dx}$
	Note	If an extra term appears then award 1st A0.

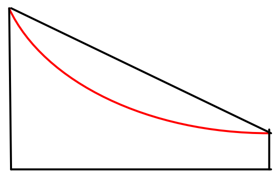
1. (a) ctd	**Note**	$3x^2 + 2y + 2x\frac{\mathrm{d}y}{\mathrm{d}x} - 1 - 3y^2\frac{\mathrm{d}y}{\mathrm{d}x} \rightarrow 3x^2 + 2y - 1 = 3y^2\frac{\mathrm{d}y}{\mathrm{d}x} - 2x\frac{\mathrm{d}y}{\mathrm{d}x}$ will get 1st A1 (implied) as the $"= 0"$ can be implied by rearrangement of their equation.
	dM1	**dependent on the first method mark being awarded.** An attempt to factorise out **all the terms in** $\frac{\mathrm{d}y}{\mathrm{d}x}$ as long as there are ***at least two terms*** in $\frac{\mathrm{d}y}{\mathrm{d}x}$. ie. $... + \left(2x - 3y^2\right)\frac{\mathrm{d}y}{\mathrm{d}x} = ...$
	Note	Placing an extra $\frac{\mathrm{d}y}{\mathrm{d}x}$ at the beginning and then including it in their factorisation is fine for dM1.
	A1	For $\frac{1 - 2y - 3x^2}{2x - 3y^2}$ or equivalent. Eg: $\frac{3x^2 + 2y - 1}{3y^2 - 2x}$ **cso:** If the candidate's solution is not completely correct, then do not give this mark. **isw:** You can, however, ignore subsequent working following on from correct solution.
1. (b)	**M1**	**Some** attempt to substitute **both** $x = 3$ **and** $y = -2$ into their $\frac{\mathrm{d}y}{\mathrm{d}x}$ which contains both x and y to find m_T **and** • **either** applies $y - -2 = \left(\text{their } m_T\right)\left(x - 3\right)$, where m_T is a numerical value. • **or** finds c by solving $(-2) = \left(\text{their } m_T\right)(3) + c$, where m_T is a numerical value.
	Note	Using a changed gradient (i.e. applying $\frac{-1}{\text{their } \frac{\mathrm{d}y}{\mathrm{d}x}}$ or $\frac{1}{\text{their } \frac{\mathrm{d}y}{\mathrm{d}x}}$ is M0).
	A1	Accept any integer multiple of $11x - 3y - 39 = 0$ or $11x - 39 - 3y = 0$ or $-11x + 3y + 39 = 0$, where their tangent equation is equal to 0.
	cso	A correct solution is required from a correct $\frac{\mathrm{d}y}{\mathrm{d}x}$.
	isw	You can ignore subsequent working following a correct solution.
	<u>Alternative method for part (a): Differentiating with respect to *y*</u>	
1. (a)	**M1**	Differentiates implicitly to include either $2y\frac{\mathrm{d}x}{\mathrm{d}y}$ or $x^3 \rightarrow \pm kx^2\frac{\mathrm{d}x}{\mathrm{d}y}$ or $-x \rightarrow -\frac{\mathrm{d}x}{\mathrm{d}y}$ (Ignore $\left(\frac{\mathrm{d}x}{\mathrm{d}y} = \right)$).
	A1	$x^3 \rightarrow 3x^2\frac{\mathrm{d}x}{\mathrm{d}y}$ **and** $-x - y^3 - 20 = 0 \rightarrow -\frac{\mathrm{d}x}{\mathrm{d}y} - 3y^2 = 0$
	B1	$2xy \rightarrow 2y\frac{\mathrm{d}x}{\mathrm{d}y} + 2x$
	dM1	**dependent on the first method mark being awarded.** An attempt to factorise out **all the terms in** $\frac{\mathrm{d}x}{\mathrm{d}y}$ as long as there are ***at least two terms*** in $\frac{\mathrm{d}x}{\mathrm{d}y}$.
	A1	For $\frac{1 - 2y - 3x^2}{2x - 3y^2}$ or equivalent. Eg: $\frac{3x^2 + 2y - 1}{3y^2 - 2x}$ **cso:** If the candidate's solution is not completely correct, then do not give this mark.

Question Number	Scheme		Marks
2.	$\left\{(1+kx)^{-4} = 1 + (-4)(kx) + \dfrac{(-4)(-4-1)}{2!}(kx)^2 + ...\right\}$		
(a)	**Either** $(-4)k = -6$ **or** $(1+kx)^{-4} = 1 + (-4)(kx)$	**see notes**	M1
	leading to $k = \dfrac{3}{2}$	$k = \dfrac{3}{2}$ or 1.5 or $\dfrac{6}{4}$	A1
			[2]
(b)	$\dfrac{(-4)(-5)}{2}(k)^2$	**Either** $\dfrac{(-4)(-5)}{2!}$ **or** $(k)^2$ **or** $(kx)^2$	M1
		Either $\dfrac{(-4)(-5)}{2!}(k)^2$ **or** $\dfrac{(-4)(-5)}{2!}(kx)^2$	M1
	$\left\{A = \dfrac{(-4)(-5)}{2!}\left(\dfrac{3}{2}\right)^2\right\} \Rightarrow A = \dfrac{45}{2}$	$\dfrac{45}{2}$ or 22.5	A1
			[3] **5**

	Question 2 Notes		
Note	**In this question ignore part labelling and mark part (a) and part (b) together.**		
	Note	Writing down $\left\{(1+kx)^{-4}\right\} = 1 + (-4)(kx) + \dfrac{(-4)(-4-1)}{2!}(kx)^2 + ...$ gets all the method marks in Q2. i.e. (a) M1 and (b) M1M1	
(a)	**M1**	Award M1 for • **either** writing down $(-4)k = -6$ or $4k = 6$ • **or** expanding $(1+kx)^{-4}$ to give $1 + (-4)(kx)$ • **or** writing down $(-4)kx = -6$ or $(-4k) = -6x$ or $-4kx = -6x$	
	A1	$k = \dfrac{3}{2}$ or 1.5 or $\dfrac{6}{4}$ **from no incorrect sign errors.**	
	Note	The M1 mark can be implied by a candidate writing down the correct value of k.	
	Note	Award M1 for writing down $4k = 6$ and then A1 for $k = 1.5$ (or equivalent).	
	Note	Award M0 for $4k = -6$ (if there is no evidence that $(1+kx)^{-4}$ expands to give $1 + (-4)(kx) + ...$)	
	Note	$1 + (-4)(kx)$ leading to $(-4)k = 6$ leading to $k = \dfrac{3}{2}$ is M1A0.	
(b)	**M1**	For **either** $\dfrac{(-4)(-4-1)}{2!}$ **or** $\dfrac{(-4)(-5)}{2!}$ **or** 10 **or** $(k)^2$ **or** $(kx)^2$	
	M1	**Either** $\dfrac{(-4)(-4-1)}{2!}(k)^2$ **or** $\dfrac{(-4)(-5)}{2!}(k)^2$ **or** $\dfrac{(-4)(-5)}{2!}(kx)^2$ **or** $\dfrac{(-4)(-5)}{2!}(\text{their } k)^2$ **or** $10k^2$	
	Note	Candidates are allowed to use 2 instead of 2!	
	A1	Uses $k = 1.5$ to give $A = \dfrac{45}{2}$ or 22.5	
	Note	$A = \dfrac{90}{4}$ which has not been simplified is A0.	
	Note	Award A0 for $A = \dfrac{45}{2}x^2$.	
	Note	Allow A1 for $A = \dfrac{45}{2}x^2$ followed by $A = \dfrac{45}{2}$	
	Note	$k = -1.5$ leading to $A = \dfrac{45}{2}$ or 22.5 is A0.	

Question Number	Scheme	Marks
3.	x: 1, 2, 3, 4; y: 1.42857, 0.90326, ***0.682116...***, 0.55556 $y = \dfrac{10}{2x + 5\sqrt{x}}$	
(a)	$\{\text{At } x = 3,\}\ y = 0.68212$ (5 dp) 0.68212	B1 **cao**
		[1]
(b)	$\frac{1}{2}\times 1\times\left[1.42857 + 0.55556 + 2\left(0.90326 + \text{their } 0.68212\right)\right]$ Outside brackets $\frac{1}{2}\times 1$ or $\frac{1}{2}$	B1 aef
	For structure of $[\ldots\ldots\ldots\ldots]$	M1
	$\{= \frac{1}{2}(5.15489)\} = 2.577445 = 2.5774$ (4 dp) anything that rounds to 2.5774	A1
		[3]
(c)	• Overestimate **and** a reason such as • {top of} trapezia lie above the curve • a diagram which gives reference to the extra area • concave or convex • $\dfrac{d^2y}{dx^2} > 0$ (can be implied) • bends inwards • curves downwards	B1
		[1]
(d)	$\left\{u = \sqrt{x} \Rightarrow\right\} \dfrac{du}{dx} = \dfrac{1}{2}x^{-\frac{1}{2}}$ or $\dfrac{dx}{du} = 2u$	B1
	$\displaystyle\int \frac{10}{2u^2 + 5u}.\ 2u\ du$ Either $\left\{\int\right\}\dfrac{\pm ku}{\alpha u^2 \pm \beta u}\ \{du\}$ or $\left\{\int\right\}\dfrac{\pm k}{u\left(\alpha u^2 \pm \beta u\right)}\ \{du\}$	M1
	$\left\{= \displaystyle\int \frac{20}{2u + 5}\ du\right\} = \dfrac{20}{2}\ln(2u + 5)$ $\pm\lambda\ln(2u + 5)$ or $\pm\lambda\ln\left(u + \dfrac{5}{2}\right),\ \lambda \neq 0$ **with no other terms.**	M1
	$\dfrac{20}{2u + 5} \rightarrow \dfrac{20}{2}\ln(2u + 5)$ or $10\ln\left(u + \dfrac{5}{2}\right)$	A1 **cso**
	$\left\{\left[\dfrac{20}{2}\ln(2u + 5)\right]_1^2\right\} = 10\ln\left(2(2) + 5\right) - 10\ln\left(2(1) + 5\right)$ Substitutes limits of 2 and 1 in u (or 4 and 1 in x) and subtracts the correct way round.	M1
	$10\ln 9 - 10\ln 7$ or $10\ln\left(\dfrac{9}{7}\right)$ or $20\ln 3 - 10\ln 7$	A1 oe **cso**
		[6] **11**

	Question 3 Notes	
3. (a)	**B1**	0.68212 correct answer only. Look for this on the table or in the candidate's working.
(b)	**B1**	Outside brackets $\dfrac{1}{2}\times 1$ or $\dfrac{1}{2}$ or equivalent.
	M1	For structure of trapezium rule $[\ \ldots\ldots\ldots\ldots\ ]$
	Note	No errors are allowed [eg. an omission of a y-ordinate or an extra y-ordinate or a repeated y ordinate].
	A1	anything that rounds to 2.5774
	Note	Working must be seen to demonstrate the use of the trapezium rule. (Actual area is 2.51314428…)

3. (b) contd	**Note**	Award B1M1A1 for $\frac{1}{2}(1.42857+0.55556)+\left(0.90326+\text{their } 0.68212\right)=2.577445$
		<u>Bracketing mistake</u>: Unless the final answer implies that the calculation has been done correctly award B1M0A0 for $\frac{1}{2}\times 1\ +1.42857+2\left(0.90326+\text{their } 0.68212\right)+0.55556$ (nb: answer of 5.65489). award B1M0A0 for $\frac{1}{2}\times 1\ (1.42857+0.55556)+2\left(0.90326+\text{their } 0.68212\right)$ (nb: answer of 4.162825).
		<u>Alternative method: Adding individual trapezia</u> $\text{Area} \approx 1\times\left[\frac{1.42857+0.90326}{2}+\frac{0.90326+\text{"}0.68212\text{"}}{2}+\frac{\text{"}0.68212\text{"}+0.55556}{2}\right] = 2.577445$
	B1 **M1** **A1**	**B1:** 1 and a divisor of 2 on all terms inside brackets. **M1:** First and last ordinates once and two of the middle ordinates twice inside brackets ignoring the 2. **A1:** anything that rounds to 2.5774
(c)	**B1**	Overestimate **and** either trapezia lie above curve **or** a diagram that gives reference to the extra area **eg.** This diagram is sufficient. It must show the top of a trapezium lying above the curve.  **or** concave or convex **or** $\frac{d^2y}{dx^2}>0$ (can be implied) **or** bends inwards **or** curves downwards.
	Note	Reason of "gradient is negative" by itself is B0.
(d)	**B1**	$\frac{du}{dx}=\frac{1}{2}x^{-\frac{1}{2}}$ or $du=\frac{1}{2\sqrt{x}}\,dx$ or $2\sqrt{x}\,du=dx$ or $dx=2u\,du$ or $\frac{dx}{du}=2u$ o.e.
	M1	Applying the substitution and achieving $\left\{\int\right\}\frac{\pm ku}{\alpha u^2\pm\beta u}\{du\}$ or $\left\{\int\right\}\frac{\pm k}{u\left(\alpha u^2\pm\beta u\right)}\{du\}$, $k,\alpha,\beta\neq 0$. Integral sign and du not required for this mark.
	M1	Cancelling u and integrates to achieve $\pm\lambda\ln(2u+5)$ or $\pm\lambda\ln\left(u+\frac{5}{2}\right)$, $\lambda\neq 0$ **with no other terms.**
	A1	**cso**. Integrates $\frac{20}{2u+5}$ to give $\frac{20}{2}\ln(2u+5)$ or $10\ln\left(u+\frac{5}{2}\right)$, un-simplified or simplified.
	Note	BE CAREFUL! Candidates must be integrating $\frac{20}{2u+5}$ or equivalent. So $\int\frac{10}{2u+5}\,du = 10\ln(2u+5)$ WOULD BE A0 and final A0.
	M1	Applies limits of 2 and 1 in u or 4 and 1 in x in their (i.e. any) changed function and subtracts the correct way round.
	A1	Exact answers of either $10\ln 9-10\ln 7$ or $10\ln\left(\frac{9}{7}\right)$ or $20\ln 3-10\ln 7$ or $20\ln\left(\frac{3}{\sqrt{7}}\right)$ or $\ln\left(\frac{9^{10}}{7^{10}}\right)$ or equivalent. **Correct solution only.**
	Note	You can ignore subsequent working which follows from a correct answer.
	Note	A decimal answer of 2.513144283... (without a correct **exact** answer) is A0.

<table>
<tr><th>Question Number</th><th>Scheme</th><th>Marks</th></tr>
<tr><td>4.</td><td>$\frac{dV}{dt} = 80\pi,\ V = 4\pi h(h+4) = 4\pi h^2 + 16\pi h,$
$\frac{dV}{dh} = 8\pi h + 16\pi$ $\pm\alpha h \pm \beta,\ \alpha \neq 0,\ \beta \neq 0$
$8\pi h + 16\pi$</td><td>M1
A1</td></tr>
<tr><td></td><td>$\left\{\frac{dV}{dh}\times\frac{dh}{dt} = \frac{dV}{dt} \Rightarrow\right\}\ (8\pi h + 16\pi)\frac{dh}{dt} = 80\pi$ $\left(\text{Candidate's } \frac{dV}{dh}\right)\times\frac{dh}{dt} = 80\pi$
$\left\{\frac{dh}{dt} = \frac{dV}{dt} \div \frac{dV}{dh} \Rightarrow\right\}\ \frac{dh}{dt} = 80\pi \times \frac{1}{8\pi h + 16\pi}$ or $80\pi \div \text{Candidate's } \frac{dV}{dh}$</td><td>M1 oe</td></tr>
<tr><td></td><td>When $h = 6$, $\left\{\frac{dh}{dt} =\right\} \frac{1}{8\pi(6) + 16\pi} \times 80\pi\ \left\{= \frac{80\pi}{64\pi}\right\}$ dependent on the previous M1 see notes
$\frac{dh}{dt} = \underline{1.25}\ (\text{cm s}^{-1})$ 1.25 or $\frac{5}{4}$ or $\frac{10}{8}$ or $\frac{80}{64}$</td><td>dM1

A1 oe
[5]
5</td></tr>
<tr><td></td><td><u>Alternative Method for the first M1A1</u>
Product rule: $\left\{\begin{array}{ll} u = 4\pi h & v = h + 4 \\ \frac{du}{dh} = 4\pi & \frac{dv}{dh} = 1 \end{array}\right\}$
$\frac{dV}{dh} = 4\pi(h+4) + 4\pi h$ $\pm\alpha h \pm \beta,\ \alpha \neq 0,\ \beta \neq 0$
$4\pi(h+4) + 4\pi h$</td><td>M1
A1</td></tr>
<tr><td></td><td></td><td></td></tr>
</table>

<table>
<tr><th></th><th colspan="2">Question 4 Notes</th></tr>
<tr><td></td><td>M1</td><td>An expression of the form $\pm\alpha h \pm \beta,\ \alpha \neq 0,\ \beta \neq 0$. Can be simplified or un-simplified.</td></tr>
<tr><td></td><td>A1

Note
Note</td><td>Correct simplified or un-simplified differentiation of V.
eg. $8\pi h + 16\pi$ or $4\pi(h+4) + 4\pi h$ or $8\pi(h+2)$ or equivalent.
Some candidates will use the product rule to differentiate V with respect to h. (See Alt Method 1).
$\frac{dV}{dh}$ does not have to be explicitly stated, but it should be clear that they are differentiating their V.</td></tr>
<tr><td></td><td>M1

Note

Note

dM1

A1

Note

Note</td><td>$\left(\text{Candidate's } \frac{dV}{dh}\right)\times\frac{dh}{dt} = 80\pi$ or $80\pi \div \text{Candidate's } \frac{dV}{dh}$
Also allow 2nd M1 for $\left(\text{Candidate's } \frac{dV}{dh}\right)\times\frac{dh}{dt} = \mathbf{80}$ or $\mathbf{80} \div \text{Candidate's } \frac{dV}{dh}$
Give 2nd M0 for $\left(\text{Candidate's } \frac{dV}{dh}\right)\times\frac{dh}{dt} = \mathbf{80\pi t}$ or $\mathbf{80k}$ or $\mathbf{80\pi t}$ or $\mathbf{80k} \div \text{Candidate's } \frac{dV}{dh}$
which is dependent on the previous M1 mark.
Substitutes $h = 6$ into an expression which is a result of a quotient of their $\frac{dV}{dh}$ and 80π (or 80)
1.25 or $\frac{5}{4}$ or $\frac{10}{8}$ or $\frac{80}{64}$ (units are not required).
$\frac{80\pi}{64\pi}$ as a final answer is A0.
Substituting $h = 6$ into a correct $\frac{dV}{dh}$ gives 64π but the final M1 mark can only be awarded if this is used as a quotient with 80π (or 80)</td></tr>
</table>

Question Number	Scheme		Marks
5.	$x = 4\cos\left(t + \frac{\pi}{6}\right), \quad y = 2\sin t$		
(a)	**<u>Main Scheme</u>**		
	$x = 4\left(\cos t\cos\left(\frac{\pi}{6}\right) - \sin t\sin\left(\frac{\pi}{6}\right)\right)$	$\cos\left(t + \frac{\pi}{6}\right) \to \cos t\cos\left(\frac{\pi}{6}\right) \pm \sin t\sin\left(\frac{\pi}{6}\right)$	M1 oe
	So, $\{x + y\} = 4\left(\cos t\cos\left(\frac{\pi}{6}\right) - \sin t\sin\left(\frac{\pi}{6}\right)\right) + 2\sin t$	Adds their expanded x (which is in terms of t) to $2\sin t$	dM1
	$= 4\left(\left(\frac{\sqrt{3}}{2}\right)\cos t - \left(\frac{1}{2}\right)\sin t\right) + 2\sin t$		
	$= 2\sqrt{3}\cos t$ *	Correct proof	A1 *
			[3]
(a)	**<u>Alternative Method 1</u>**		
	$x = 4\left(\cos t\cos\left(\frac{\pi}{6}\right) - \sin t\sin\left(\frac{\pi}{6}\right)\right)$	$\cos\left(t + \frac{\pi}{6}\right) \to \cos t\cos\left(\frac{\pi}{6}\right) \pm \sin t\sin\left(\frac{\pi}{6}\right)$	M1 oe
	$= 4\left(\left(\frac{\sqrt{3}}{2}\right)\cos t - \left(\frac{1}{2}\right)\sin t\right) = 2\sqrt{3}\cos t - 2\sin t$		
	So, $x = 2\sqrt{3}\cos t - y$	Forms an equation in x, y and t.	dM1
	$x + y = 2\sqrt{3}\cos t$ *	Correct proof	A1 *
			[3]
(b)	**<u>Main Scheme</u>**		
	$\left(\frac{x + y}{2\sqrt{3}}\right)^2 + \left(\frac{y}{2}\right)^2 = 1$	Applies $\cos^2 t + \sin^2 t = 1$ to achieve an equation containing **only** x's and y's.	M1
	$\Rightarrow \frac{(x + y)^2}{12} + \frac{y^2}{4} = 1$		
	$\Rightarrow (x + y)^2 + 3y^2 = 12$	$(x + y)^2 + 3y^2 = 12$	A1
		$\{a = 3, b = 12\}$	**[2]**
(b)	**<u>Alternative Method 1</u>**		
	$(x + y)^2 = 12\cos^2 t = 12(1 - \sin^2 t) = 12 - 12\sin^2 t$		
	So, $(x + y)^2 = 12 - 3y^2$	Applies $\cos^2 t + \sin^2 t = 1$ to achieve an equation containing **only** x's and y's.	M1
	$\Rightarrow (x + y)^2 + 3y^2 = 12$	$(x + y)^2 + 3y^2 = 12$	A1
			[2]
(b)	**<u>Alternative Method 2</u>**		
	$(x + y)^2 = 12\cos^2 t$		
	As $12\cos^2 t + 12\sin^2 t = 12$		
	then $(x + y)^2 + 3y^2 = 12$		M1, A1
			[2]
			5

	Question 5 Notes	
5. (a)	M1	$\cos\left(t+\frac{\pi}{6}\right) \to \cos t\cos\left(\frac{\pi}{6}\right) \pm \sin t\sin\left(\frac{\pi}{6}\right)$ or $\cos\left(t+\frac{\pi}{6}\right) \to \left(\frac{\sqrt{3}}{2}\right)\cos t \pm \left(\frac{1}{2}\right)\sin t$
	Note	**If a candidate states** $\cos(A+B) = \cos A\cos B \pm \sin A\sin B$, but there is an error ***in its application*** then give M1. **Awarding the dM1 mark which is dependent on the first method mark**
Main	dM1 **Note**	Adds their expanded x (which is in terms of t) to $2\sin t$ Writing $x + y = ...$ is not needed in the **Main Scheme** method.
Alt 1	dM1	Forms an equation in x, y and t.
	A1*	Evidence of $\cos\left(\frac{\pi}{6}\right)$ and $\sin\left(\frac{\pi}{6}\right)$ evaluated and the proof is correct with no errors.
	Note	$\{x+y\} = 4\cos\left(t+\frac{\pi}{6}\right) + 2\sin t$, by itself is M0M0A0.
(b)	M1 A1	Applies $\cos^2 t + \sin^2 t = 1$ to achieve an equation containing **only** x's and y's. leading $(x+y)^2 + 3y^2 = 12$
	SC	Award **Special Case B1B0** for a candidate who writes down **either** • $(x+y)^2 + 3y^2 = 12$ from no working • $a = 3,\ b = 12$, but **<u>does not provide a correct proof</u>**.
	Note **Note**	Alternative method 2 is fine for M1 A1 Writing $(x+y)^2 = 12\cos^2 t$ followed by $12\cos^2 t + a(4\sin^2 t) = b \Rightarrow a = 3,\ b = 12$ is SC: B1B0
	Note	Writing $(x+y)^2 = 12\cos^2 t$ followed by $12\cos^2 t + a(4\sin^2 t) = b$ • states $a = 3,\ b = 12$ • and refers to either $\cos^2 t + \sin^2 t = 1$ or $12\cos^2 t + 12\sin^2 t = 12$ • and there is no incorrect working would get M1A1

Question Number	Scheme		Marks
6. (i)	$\int x\mathrm{e}^{4x}\,\mathrm{d}x = \frac{1}{4}x\mathrm{e}^{4x} - \int \frac{1}{4}\mathrm{e}^{4x}\,\{\mathrm{d}x\}$	$\pm\alpha x\mathrm{e}^{4x} - \int \beta\mathrm{e}^{4x}\{\mathrm{d}x\},\quad \alpha \neq 0,\ \beta > 0$	M1
		$\frac{1}{4}x\mathrm{e}^{4x} - \int \frac{1}{4}\mathrm{e}^{4x}\,\{\mathrm{d}x\}$	A1
	$= \frac{1}{4}x\mathrm{e}^{4x} - \frac{1}{16}\mathrm{e}^{4x}\,\{+c\}$	$\frac{1}{4}x\mathrm{e}^{4x} - \frac{1}{16}\mathrm{e}^{4x}$	A1
			[3]
(ii)	$\int \frac{8}{(2x-1)^3}\,\mathrm{d}x = \frac{8(2x-1)^{-2}}{(2)(-2)}\,\{+c\}$	$\pm\lambda(2x-1)^{-2}$	M1
		$\frac{8(2x-1)^{-2}}{(2)(-2)}$ or equivalent.	A1
	$\{= -2(2x-1)^{-2}\,\{+c\}\}$	*{Ignore subsequent working}.*	**[2]**
(iii)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{e}^x \operatorname{cosec} 2y \operatorname{cosec} y \qquad y = \frac{\pi}{6}$ at $x = 0$		
	Main Scheme		
	$\int \frac{1}{\operatorname{cosec} 2y \operatorname{cosec} y}\,\mathrm{d}y = \int \mathrm{e}^x\,\mathrm{d}x$ **or** $\int \sin 2y \sin y\,\mathrm{d}y = \int \mathrm{e}^x\,\mathrm{d}x$		B1 oe
	$\int 2\sin y\cos y\sin y\,\mathrm{d}y = \int \mathrm{e}^x\,\mathrm{d}x$	Applying $\frac{1}{\operatorname{cosec}2y}$ or $\sin 2y \to 2\sin y\cos y$	M1
		Integrates to give $\pm\mu\sin^3 y$	M1
	$\frac{2}{3}\sin^3 y = \mathrm{e}^x\,\{+c\}$	$2\sin^2 y\cos y \to \frac{2}{3}\sin^3 y$	A1
		$\mathrm{e}^x \to \mathrm{e}^x$	B1
	$\frac{2}{3}\sin^3\left(\frac{\pi}{6}\right) = \mathrm{e}^0 + c$ **or** $\frac{2}{3}\left(\frac{1}{8}\right) - 1 = c$	Use of $y = \frac{\pi}{6}$ and $x = 0$ in an integrated equation containing c	M1
	$\left\{\Rightarrow c = -\frac{11}{12}\right\}$ giving $\frac{2}{3}\sin^3 y = \mathrm{e}^x - \frac{11}{12}$	$\frac{2}{3}\sin^3 y = \mathrm{e}^x - \frac{11}{12}$	A1
			[7]
	Alternative Method 1		
	$\int \frac{1}{\operatorname{cosec} 2y \operatorname{cosec} y}\,\mathrm{d}y = \int \mathrm{e}^x\,\mathrm{d}x$ **or** $\int \sin 2y \sin y\,\mathrm{d}y = \int \mathrm{e}^x\,\mathrm{d}x$		B1 oe
	$\int -\frac{1}{2}(\cos 3y - \cos y)\,\mathrm{d}y = \int \mathrm{e}^x\,\mathrm{d}x$	$\sin 2y\sin y \to \pm\lambda\cos 3y \pm \lambda\cos y$	M1
		Integrates to give $\pm\alpha\sin 3y \pm \beta\sin y$	M1
	$-\frac{1}{2}\left(\frac{1}{3}\sin 3y - \sin y\right) = \mathrm{e}^x\,\{+c\}$	$-\frac{1}{2}\left(\frac{1}{3}\sin 3y - \sin y\right)$	A1
		$\mathrm{e}^x \to \mathrm{e}^x$ ***as part of solving their DE.***	B1
	$-\frac{1}{2}\left(\frac{1}{3}\sin\left(\frac{3\pi}{6}\right) - \sin\left(\frac{\pi}{6}\right)\right) = \mathrm{e}^0 + c$ **or** $-\frac{1}{2}\left(\frac{1}{3} - \frac{1}{2}\right) - 1 = c$	Use of $y = \frac{\pi}{6}$ and $x = 0$ in an integrated equation containing c	M1
	$\left\{\Rightarrow c = -\frac{11}{12}\right\}$ giving $-\frac{1}{6}\sin 3y + \frac{1}{2}\sin y = \mathrm{e}^x - \frac{11}{12}$	$-\frac{1}{6}\sin 3y + \frac{1}{2}\sin y = \mathrm{e}^x - \frac{11}{12}$	A1
			[7]
			12

	Question 6 Notes	
6. (i)	**M1**	Integration by parts is applied in the form $\pm\alpha x e^{4x} - \int \beta e^{4x}\{dx\}$, where $\alpha \neq 0,\ \beta > 0$. (must be in this form).
	A1	$\frac{1}{4}xe^{4x} - \int \frac{1}{4}e^{4x}\{dx\}$ or equivalent.
	A1	$\frac{1}{4}xe^{4x} - \frac{1}{16}e^{4x}$ with/without + c. Can be un-simplified.
	isw	You can ignore subsequent working following on from a correct solution.
	SC	**SPECIAL CASE:** A candidate who uses $u = x,\ \frac{dv}{dx} = e^{4x}$, writes down the correct "by parts" formula, but makes only one error when applying it can be awarded Special Case M1.
(ii)	**M1**	$\pm\lambda(2x-1)^{-2},\ \lambda \neq 0$. **Note** that λ can be 1.
	A1	$\frac{8(2x-1)^{-2}}{(2)(-2)}$ or $-2(2x-1)^{-2}$ or $\frac{-2}{(2x-1)^2}$ with/without + c. Can be un-simplified.
	Note	You can ignore subsequent working which follows from a correct answer.
(iii)	**B1**	Separates variables as shown. dy and dx should be in the correct positions, though this mark can be implied by later working. Ignore the integral signs.
	Note	**Allow B1 for** $\int \frac{1}{\operatorname{cosec} 2y \operatorname{cosec} y} = \int e^x$ **or** $\int \sin 2y \sin y = \int e^x$
	M1	$\frac{1}{\operatorname{cosec} 2y} \to 2\sin y\cos y$ **or** $\sin 2y \to 2\sin y \cos y$ **or** $\sin 2y \sin y \to \pm\lambda\cos 3y \pm \lambda\cos y$ seen anywhere in the candidate's working to (iii).
	M1	Integrates to give $\pm\mu\sin^3 y,\ \mu \neq 0$ **or** $\pm\alpha\sin 3y \pm \beta\sin y,\ \alpha \neq 0,\ \beta \neq 0$
	A1	$2\sin^2 y\cos y \to \frac{2}{3}\sin^3 y$ (with no extra terms) **or** integrates to give $-\frac{1}{2}\left(\frac{1}{3}\sin 3y - \sin y\right)$
	B1	Evidence that e^x has been integrated to give e^x ***as part of solving their DE.***
	M1	Some evidence of using both $y = \frac{\pi}{6}$ and $x = 0$ in an integrated or changed equation containing c.
	Note	that is mark can be implied by the correct value of c.
	A1	$\frac{2}{3}\sin^3 y = e^x - \frac{11}{12}$ **or** $-\frac{1}{6}\sin 3y + \frac{1}{2}\sin y = e^x - \frac{11}{12}$ **or any equivalent correct answer.**
	Note	You can ignore subsequent working which follows from a correct answer.

Alternative Method 2 (Using integration by parts twice)

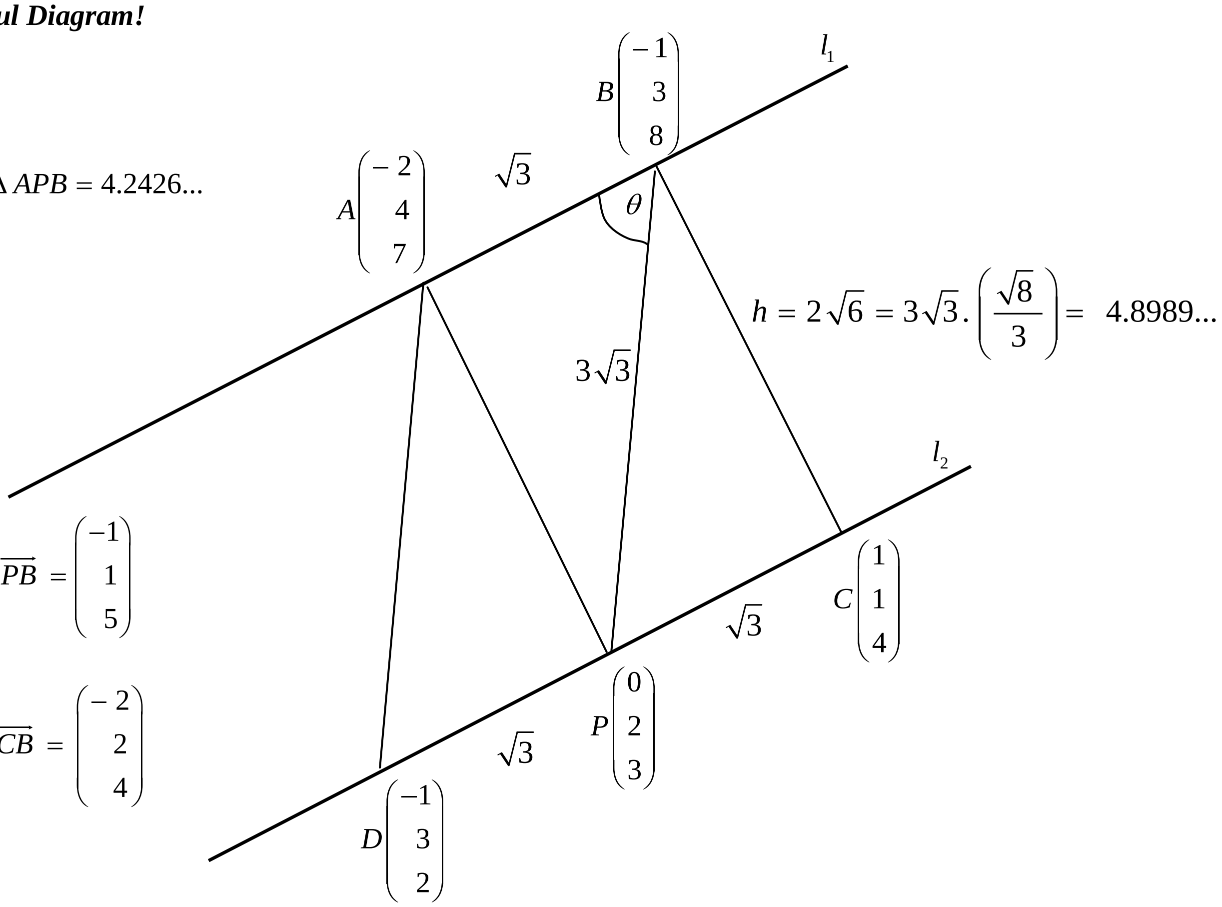
$\int \sin 2y \sin y\, dy = \int e^x\, dx$		B1 oe
	Applies integration by parts **twice** to give $\pm\alpha\cos y\sin 2y \pm \beta\sin y\cos 2y$	**M2**
$\frac{1}{3}\cos y \sin 2y - \frac{2}{3}\sin y\cos 2y = e^x\ \{+c\}$	$\frac{1}{3}\cos y\sin 2y - \frac{2}{3}\sin y\cos 2y$ (simplified or un-simplified)	A1
	$e^x \to e^x$ ***as part of solving their DE.***	B1
	as in the main scheme	M1
$\frac{1}{3}\cos y \sin 2y - \frac{2}{3}\sin y\cos 2y = e^x - \frac{11}{12}$	$-\frac{1}{6}\sin 3y + \frac{1}{2}\sin y = e^x - \frac{11}{12}$	A1
		[7]

Question Number	Scheme		Marks
7.	$x = 3\tan\theta, \quad y = 4\cos^2\theta$ or $y = 2 + 2\cos 2\theta, \quad 0 \leqslant \theta < \frac{\pi}{2}.$		
(a)	$\frac{dx}{d\theta} = 3\sec^2\theta, \quad \frac{dy}{d\theta} = -8\cos\theta\sin\theta$ **or** $\frac{dy}{d\theta} = -4\sin 2\theta$		
	$\frac{dy}{dx} = \frac{-8\cos\theta\sin\theta}{3\sec^2\theta} \quad \left\{= -\frac{8}{3}\cos^3\theta\sin\theta = -\frac{4}{3}\sin 2\theta\cos^2\theta\right\}$	their $\frac{dy}{d\theta}$ divided by their $\frac{dx}{d\theta}$ Correct $\frac{dy}{dx}$	M1 A1 oe
	At $P(3, 2)$, $\theta = \frac{\pi}{4}$, $\frac{dy}{dx} = -\frac{8}{3}\cos^3\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{4}\right) \quad \left\{= -\frac{2}{3}\right\}$ So, $m(\mathbf{N}) = \frac{3}{2}$	Some evidence of substituting $\theta = \frac{\pi}{4}$ into their $\frac{dy}{dx}$ applies $m(\mathbf{N}) = \frac{-1}{m(\mathbf{T})}$	M1 M1
	Either N: $y - 2 = "\frac{3}{2}"(x - 3)$ **or** $2 = \left("\frac{3}{2}"\right)(3) + c$	**see notes**	M1
	$\{$At Q, $y = 0$, so, $-2 = \frac{3}{2}(x - 3)\}$ **giving** $\underline{x = \frac{5}{3}}$	$x = \frac{5}{3}$ or $1\frac{2}{3}$ or awrt 1.67	A1 **cso** **[6]**
(b)	$\left\{\int y^2 dx = \int y^2 \frac{dx}{d\theta} d\theta\right\} = \left\{\int\right\}(4\cos^2\theta)^2 3\sec^2\theta \quad \{d\theta\}$	**see notes**	M1
	So, $\pi\int y^2 dx = \pi\int(4\cos^2\theta)^2 3\sec^2\theta\,\{d\theta\}$	**see notes**	A1
	$\int y^2 dx = \int 48\cos^2\theta\, d\theta$	$\int 48\cos^2\theta\{d\theta\}$	A1
	$= \{48\}\int\left(\frac{1+\cos 2\theta}{2}\right)d\theta \quad \left\{= \int(24 + 24\cos 2\theta)d\theta\right\}$	Applies $\cos 2\theta = 2\cos^2\theta - 1$	M1
	$= \{48\}\left(\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta\right) \quad \{= 24\theta + 12\sin 2\theta\}$	**Dependent on the first method mark.** For $\pm\alpha\theta \pm \beta\sin 2\theta$ $\cos^2\theta \to \left(\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta\right)$	dM1 A1
	$\int_0^{\frac{\pi}{4}} y^2 dx \left\{= 48\left[\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta\right]_0^{\frac{\pi}{4}}\right\} = \{48\}\left(\left(\frac{\pi}{8} + \frac{1}{4}\right) - (0 + 0)\right) \quad \{= 6\pi + 12\}$	**Dependent on the third method mark.**	dM1
	$\{$So $V = \pi\int_0^{\frac{\pi}{4}} y^2 dx = 6\pi^2 + 12\pi\}$		
	$V_{cone} = \frac{1}{3}\pi(2)^2\left(3 - \frac{5}{3}\right) \left\{= \frac{16\pi}{9}\right\}$	$V_{cone} = \frac{1}{3}\pi(2)^2(3 - \text{their }(a))$	M1
	$\left\{\text{Vol}(S) = 6\pi^2 + 12\pi - \frac{16\pi}{9}\right\} \Rightarrow \text{Vol}(S) = \underline{\frac{92}{9}\pi + 6\pi^2}$	$\frac{92}{9}\pi + 6\pi^2$ $\left\{p = \frac{92}{9},\ q = 6\right\}$	A1 **[9]**
			15

		Question 7 Notes
7. (a)	**1st M1**	Applies their $\frac{dy}{d\theta}$ divided by their $\frac{dx}{d\theta}$ **or** applies $\frac{dy}{d\theta}$ multiplied by their $\frac{d\theta}{dx}$
	SC	Award **Special Case 1st M1** if **both** $\frac{dx}{d\theta}$ **and** $\frac{dy}{d\theta}$ are both correct.
	1st A1	Correct $\frac{dy}{dx}$ i.e. $\frac{-8\cos\theta\sin\theta}{3\sec^2\theta}$ or $-\frac{8}{3}\cos^3\theta\sin\theta$ or $-\frac{4}{3}\sin 2\theta\cos^2\theta$ or any equivalent form.
	2nd M1	***Some evidence*** of substituting $\theta = \frac{\pi}{4}$ or $\theta = 45^\circ$ into their $\frac{dy}{dx}$
	Note	For 3rd M1 and 4th M1, $m(\mathbf{T})$ must be found by using $\frac{dy}{dx}$.
	3rd M1	applies $m(\mathbf{N}) = \frac{-1}{m(\mathbf{T})}$. Numerical value for $m(\mathbf{N})$ is required here.
	4th M1	• Applies $y - 2 = (\text{their } m_N)(x-3)$, where $\text{m}(\mathbf{N})$ is a numerical value, • or ***finds c*** by solving $2 = (\text{their } m_N)3 + c$, where $\text{m}(\mathbf{N})$ is a numerical value, and $m_N = -\frac{1}{\text{their m}(\mathbf{T})}$ or $m_N = \frac{1}{\text{their m}(\mathbf{T})}$ or $m_N = -\text{their m}(\mathbf{T})$.
	Note	This mark can be implied by subsequent working.
	2nd A1	$x = \frac{5}{3}$ or $1\frac{2}{3}$ or awrt 1.67 **from a correct solution only.**
(b)	**1st M1**	Applying $\int y^2 dx$ as $y^2 \frac{dx}{d\theta}$ with their $\frac{dx}{d\theta}$. Ignore π or $\frac{1}{3}\pi$ outside integral.
	Note	You can ignore the omission of an integral sign and/or $d\theta$ for the 1st M1.
	Note	Allow 1st M1 for $\int (\cos^2\theta)^2 \times$ "their $3\sec^2\theta$" $d\theta$ or $\int 4(\cos^2\theta)^2 \times$ "their $3\sec^2\theta$" $d\theta$
	1st A1	Correct expression $\left\{\pi\int y^2 dx\right\} = \pi\int (4\cos^2\theta)^2 3\sec^2\theta \{d\theta\}$ (Allow the omission of $d\theta$)
	Note	**IMPORTANT:** The π can be recovered later, **but as a correct statement only**.
	2nd A1	$\left\{\int y^2 dx\right\} = \int 48\cos^2\theta\{d\theta\}$. (Ignore $d\theta$). **Note**: 48 can be written as 24(2) for example.
	2nd M1	Applies $\cos 2\theta = 2\cos^2\theta - 1$ to their integral. (Seen or **implied**.)
	3rd dM1*	**which is dependent on the 1st M1 mark.** Integrating $\cos^2\theta$ to give $\pm\alpha\theta \pm \beta\sin 2\theta$, $\alpha \neq 0, \beta \neq 0$, un-simplified or simplified.
	3rd A1	**which is dependent on the 3rd M1 mark and the 1st M1 mark.** Integrating $\cos^2\theta$ to give $\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta$, un-simplified or simplified. This can be implied by $k\cos^2\theta$ giving $\frac{k}{2}\theta + \frac{k}{4}\sin 2\theta$**,** un-simplified or simplified.
	4th dM1	**which is dependent on the 3rd M1 mark and the 1st M1 mark.** Some evidence of applying limits of $\frac{\pi}{4}$ and 0 (0 can be implied) to an integrated function in θ
	5th M1	Applies $V_{\text{cone}} = \frac{1}{3}\pi(2)^2(3 - \text{their part}(a) \text{ answer})$.
	Note	Also allow the 5th M1 for $V_{\text{cone}} = \pi\int_{\text{their } \frac{5}{3}}^{3}\left(\frac{3}{2}x - \frac{5}{2}\right)^2\{dx\}$, which includes the correct limits.
	4th A1	$\frac{92}{9}\pi + 6\pi^2$ or $10\frac{2}{9}\pi + 6\pi^2$
	Note	A decimal answer of 91.33168464... (without a correct **exact** answer) is A0.
	Note	The π in the volume formula is only needed for the 1st A1 mark and the final accuracy mark.

7.		**<u>Working with a Cartesian Equation</u>** A cartesian equation for C is $y = \dfrac{36}{x^2 + 9}$
(a)	**1st M1** **1st A1** **2nd dM1**	$\dfrac{dy}{dx} = \pm\lambda x\left(\pm\alpha x^2 \pm \beta\right)^{-2}$ **or** $\dfrac{dy}{dx} = \dfrac{\pm\lambda x}{\left(\pm\alpha x^2 \pm \beta\right)^2}$ $\dfrac{dy}{dx} = -36(x^2+9)^{-2}(2x)$ **or** $\dfrac{dy}{dx} = \dfrac{-72x}{(x^2+9)^2}$ un-simplified or simplified. **Dependent on the 1st M1 mark if a candidate uses this method** For substituting $x = 3$ into their $\dfrac{dy}{dx}$ i.e. at $P(3, 2)$, $\dfrac{dy}{dx} = \dfrac{-72(3)}{(3^2+9)^2}\left\{= -\dfrac{2}{3}\right\}$ From this point onwards the original scheme can be applied.
(b)	**1st M1** **A1**	For $\displaystyle\int\left(\frac{\pm\lambda}{\pm\alpha x^2 \pm \beta}\right)^2 \{dx\}$ (π not required for this mark) For $\pi\displaystyle\int\left(\frac{36}{x^2+9}\right)^2 \{dx\}$ (π required for this mark) To integrate, a substitution of $x = 3\tan\theta$ is required which will lead to $\int 48\cos^2\theta\, d\theta$ and so from this point onwards the original scheme can be applied.
(a)	**1st M1** **1st A1** **2nd dM1**	Another cartesian equation for C is $x^2 = \dfrac{36}{y} - 9$ $\pm\alpha x = \pm\dfrac{\beta}{y^2}\dfrac{dy}{dx}$ or $\pm\alpha x\dfrac{dx}{dy} = \pm\dfrac{\beta}{y^2}$ $2x = -\dfrac{36}{y^2}\dfrac{dy}{dx}$ or $2x\dfrac{dx}{dy} = -\dfrac{36}{y^2}$ **Dependent on the 1st M1 mark if a candidate uses this method** For substituting $x = 3$ to find $\dfrac{dy}{dx}$ i.e. at $P(3, 2)$, $2(3) = -\dfrac{36}{4}\dfrac{dy}{dx} \Rightarrow \dfrac{dy}{dx} = ...$ From this point onwards the original scheme can be applied.

Question Number	Scheme	Marks
8.	$\overrightarrow{OA} = -2\mathbf{i} + 4\mathbf{j} + 7\mathbf{k}$, $\overrightarrow{OB} = -\mathbf{i} + 3\mathbf{j} + 8\mathbf{k}$ & $\overrightarrow{OP} = 0\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$	
(a)	$\overrightarrow{AB} = = \pm\left((-\mathbf{i} + 3\mathbf{j} + 8\mathbf{k}) - (-2\mathbf{i} + 4\mathbf{j} + 7\mathbf{k})\right); = \mathbf{i} - \mathbf{j} + \mathbf{k}$	M1; A1
		[2]
(b)	$\{l_1 : \mathbf{r}\ \} = \begin{pmatrix} -2 \\ 4 \\ 7 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ or $\{\mathbf{r}\} = \begin{pmatrix} -1 \\ 3 \\ 8 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$	B1ft
		[1]
(c)	$\overrightarrow{PB} = \overrightarrow{OB} - \overrightarrow{OP} = \begin{pmatrix} -1 \\ 3 \\ 8 \end{pmatrix} - \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 5 \end{pmatrix}$ or $\overrightarrow{BP} = \begin{pmatrix} 1 \\ -1 \\ -5 \end{pmatrix}$	M1
	$\{\cos\theta =\}\ \dfrac{\overrightarrow{AB} \bullet \overrightarrow{PB}}{\left\|\overrightarrow{AB}\right\| . \left\|\overrightarrow{PB}\right\|} = \dfrac{\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \bullet \begin{pmatrix} -1 \\ 1 \\ 5 \end{pmatrix}}{\sqrt{(1)^2 + (-1)^2 + (1)^2} . \sqrt{(-1)^2 + (1)^2 + (5)^2}}$ — Applies dot product formula between their $\left(\overrightarrow{AB}\text{ or }\overrightarrow{BA}\right)$ and their $\left(\overrightarrow{PB}\text{ or }\overrightarrow{BP}\right)$.	M1
	$\{\cos\theta\} = \dfrac{-1-1+5}{\sqrt{3}.\sqrt{27}} = \dfrac{3}{9} = \underline{\dfrac{1}{3}}$ — Correct proof	A1 cso
		[3]
(d)	$\{l_2 : \mathbf{r} =\} \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ — $\mathbf{p} + \lambda\mathbf{d}$ or $\mathbf{p} + \mu\mathbf{d}$, $\mathbf{p} \neq 0$, $\mathbf{d} \neq 0$ with either $\mathbf{p} = 0\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$ or $\mathbf{d} =$ their $\overrightarrow{AB}$, or a multiple of their $\overrightarrow{AB}$.	M1
	Correct vector equation.	A1 ft
		[2]
(e)	$\overrightarrow{OC} = \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 4 \end{pmatrix}$ or $\overrightarrow{OD} = \begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ — Either $\overrightarrow{OP}$ + their $\overrightarrow{AB}$ or $\overrightarrow{OP}$ − their $\overrightarrow{AB}$	M1
	$\{C(1, 1, 4),\ D(-1, 3, 2)\}$ — At least one set of coordinates are correct.	A1 ft
	Both sets of coordinates are correct.	A1 ft
		[3]
(f) **Way 1**	$\dfrac{h}{\sqrt{(-1)^2 + (1)^2 + (5)^2}} = \sin\theta$ — $\dfrac{h}{\text{their } \left\|\overrightarrow{PB}\right\|} = \sin\theta$	M1
	$h = \sqrt{27}\sin(70.5...) \left\{ = \sqrt{27}\dfrac{\sqrt{8}}{3} = 2\sqrt{6} = \text{awrt } 4.9 \right\}$ — $\sqrt{27}\sin(70.5...)$ or $\sqrt{27}.\dfrac{\sqrt{8}}{3}$ or $2\sqrt{6}$ or awrt 4.9 or equivalent	A1 oe
	Area $ABCD = \dfrac{1}{2}2\sqrt{6}\left(\sqrt{3} + 2\sqrt{3}\right)$ — $\dfrac{1}{2}(\text{their } h)(\text{their } AB + \text{their } CD)$	dM1
	$\left\{ = \dfrac{1}{2}2\sqrt{6}\left(3\sqrt{3}\right) = 3\sqrt{18} \right\} = \underline{9\sqrt{2}}$ — $9\sqrt{2}$	A1 **cao**
		[4]
		15

8. (f)	***Helpful Diagram!*** Area $\Delta APB = 4.2426...$  $\overrightarrow{DA} = \overrightarrow{PB} = \begin{pmatrix}-1\\1\\5\end{pmatrix}$ $\overrightarrow{PA} = \overrightarrow{CB} = \begin{pmatrix}-2\\2\\4\end{pmatrix}$		
	$\overrightarrow{PA} = \overrightarrow{CB} = \begin{pmatrix}-2\\2\\4\end{pmatrix}$ and $\overrightarrow{AB} = \begin{pmatrix}1\\-1\\1\end{pmatrix}$, so $BC \perp AB$	Candidates do not need to prove this result for part (f)	
8. (f) **Way 2**	$h = \left\lvert\overrightarrow{CB}\right\rvert = \sqrt{(-2)^2 + (2)^2 + (4)^2} = \sqrt{24} = 2\sqrt{6} = 4.8989...$	Attempts $\left\lvert\overrightarrow{PA}\right\rvert$ or $\left\lvert\overrightarrow{CB}\right\rvert$	M1
		$\left\lvert\overrightarrow{PA}\right\rvert = \left\lvert\overrightarrow{CB}\right\rvert = \sqrt{24}$	A1 oe
	Area $ABCD = \frac{1}{2}\sqrt{24}\left(\sqrt{3} + 2\sqrt{3}\right)$ or $\frac{1}{2}\sqrt{24}\sqrt{3} + \sqrt{24}\sqrt{3}$	$\frac{1}{2}h(\text{their } AB + \text{their } CD)$	dM1 oe
	$= \underline{9\sqrt{2}}$	$9\sqrt{2}$	A1 **cso**
			[4]
Way3	**<u>Finds the area of either triangle *APB* or *APD* or *BCP* and triples the result.</u>**		
8. (f)	Area $\Delta APB = \frac{1}{2}\sqrt{3}\left(3\sqrt{3}\right)\sin\theta$	Attempts $\frac{1}{2}$ (their AB)(their PB)$\sin\theta$	M1
	$= \frac{1}{2}\sqrt{3}\left(3\sqrt{3}\right)\sin(70.5...)$	$\frac{1}{2}\sqrt{3}\left(3\sqrt{3}\right)\sin(70.5...)$ or $3\sqrt{2}$ or awrt 4.24 or equivalent	A1
	Area $ABCD = 3\left(3\sqrt{2}\right)$	$3 \times$ Area of ΔAPB	dM1
	$= \underline{9\sqrt{2}}$	$9\sqrt{2}$	A1 **cso**
			[4]

	Question 8 Notes	
8. (a)	**M1**	Finding the difference (either way) between $\overrightarrow{OB}$ and $\overrightarrow{OA}$. If no "subtraction" seen, you can award M1 for 2 out of 3 correct components of the difference.
	A1	$\mathbf{i} - \mathbf{j} + \mathbf{k}$ or $\begin{pmatrix}1\\-1\\1\end{pmatrix}$ or $(1, -1, 1)$ or benefit of the doubt $\begin{matrix}1\\-1\\1\end{matrix}$
(b)	**B1ft**	$\{\mathbf{r}\} = \begin{pmatrix}-2\\4\\7\end{pmatrix} + \lambda\begin{pmatrix}1\\-1\\1\end{pmatrix}$ **or** $\{\mathbf{r}\} = \begin{pmatrix}-1\\3\\8\end{pmatrix} + \lambda\begin{pmatrix}1\\-1\\1\end{pmatrix}$, with $\overrightarrow{AB}$ or $\overrightarrow{BA}$ correctly followed through from (a).
	Note	$\mathbf{r} =$ is not needed.
(c)	**M1**	An attempt to find either the vector $\overrightarrow{PB}$ or $\overrightarrow{BP}$. If no "subtraction" seen, you can award M1 for 2 out of 3 correct components of the difference.
	M1	Applies dot product formula between their $\left(\overrightarrow{AB} \text{ or } \overrightarrow{BA}\right)$ and their $\left(\overrightarrow{PB} \text{ or } \overrightarrow{BP}\right)$.
	A1	Obtains $\{\cos\theta\} = \frac{1}{3}$ ***by correct solution only.***
	Note	If candidate starts by applying $\dfrac{\overrightarrow{AB} \bullet \overrightarrow{PB}}{\lvert\overrightarrow{AB}\rvert . \lvert\overrightarrow{PB}\rvert}$ correctly (without reference to $\cos\theta = ...$) they can gain both 2nd M1 and A1 mark.
	Note	Award the final A1 mark if candidate achieves $\{\cos\theta\} = \frac{1}{3}$ by either taking the dot product between (i) $\begin{pmatrix}1\\-1\\1\end{pmatrix}$ and $\begin{pmatrix}-1\\1\\5\end{pmatrix}$ **or** (ii) $\begin{pmatrix}-1\\1\\-1\end{pmatrix}$ and $\begin{pmatrix}1\\-1\\-5\end{pmatrix}$. Ignore if any of these vectors are labelled incorrectly.
	Note	Award final A0, cso for those candidates who take the dot product between (iii) $\begin{pmatrix}1\\-1\\1\end{pmatrix}$ and $\begin{pmatrix}1\\-1\\-5\end{pmatrix}$ or (iv) $\begin{pmatrix}-1\\1\\-1\end{pmatrix}$ and $\begin{pmatrix}-1\\1\\5\end{pmatrix}$ They will usually find $\{\cos\theta\} = -\frac{1}{3}$ or may fudge $\{\cos\theta\} = \frac{1}{3}$. If these candidates give a convincing detailed explanation which must include reference to the direction of their vectors then this can be given A1 cso

(c)	**<u>Alternative Method 1: The Cosine Rule</u>**		
	$\overrightarrow{PB} = \overrightarrow{OB} - \overrightarrow{OP} = \begin{pmatrix}-1\\3\\8\end{pmatrix} - \begin{pmatrix}0\\2\\3\end{pmatrix} = \begin{pmatrix}-1\\1\\5\end{pmatrix}$ or $\overrightarrow{BP} = \begin{pmatrix}1\\-1\\-5\end{pmatrix}$	Mark in the same way as the main scheme.	M1
	Note $\lvert\overrightarrow{PB}\rvert = \sqrt{27}$, $\lvert\overrightarrow{AB}\rvert = \sqrt{3}$ and $\lvert\overrightarrow{PA}\rvert = \sqrt{24}$		
	$\left(\sqrt{24}\right)^2 = \left(\sqrt{27}\right)^2 + \left(\sqrt{3}\right)^2 - 2\left(\sqrt{27}\right)\left(\sqrt{3}\right)\cos\theta$	Applies the cosine rule the correct way round	M1 oe
	$\cos\theta = \dfrac{27 + 3 - 24}{18} = \dfrac{1}{3}$	Correct proof	A1 cso
			[3]

8. (c)	**Alternative Method 2: Right-Angled Trigonometry**		
	$\overrightarrow{PB} = \overrightarrow{OB} - \overrightarrow{OP} = \begin{pmatrix}-1\\3\\8\end{pmatrix} - \begin{pmatrix}0\\2\\3\end{pmatrix} = \begin{pmatrix}-1\\1\\5\end{pmatrix}$ or $\overrightarrow{BP} = \begin{pmatrix}1\\-1\\-5\end{pmatrix}$	Mark in the same way as the main scheme.	M1
	Either $\left(\sqrt{24}\right)^2 + \left(\sqrt{3}\right)^2 = \left(\sqrt{27}\right)^2$ or $\overrightarrow{AB} \bullet \overrightarrow{PA} = \begin{pmatrix}1\\-1\\1\end{pmatrix} \bullet \begin{pmatrix}-2\\2\\4\end{pmatrix} = -2-2+4=0$	Confirms ΔPAB is right-angled	M1
	So, $\left\{\cos\theta = \frac{AB}{PB} \Rightarrow\right\} \cos\theta = \frac{\sqrt{3}}{\sqrt{27}} = \frac{1}{3}$	Correct proof	A1 cso
			[3]

(d)	**M1**	Writing down a line in the form $\mathbf{p} + \lambda\mathbf{d}$ or $\mathbf{p} + \mu\mathbf{d}$ with either $\mathbf{a} = \begin{pmatrix}0\\2\\3\end{pmatrix}$ or $\mathbf{d}$ = their $\overrightarrow{AB}$ $\mathbf{d}$ = their $\overrightarrow{AB}$, or a multiple of their $\overrightarrow{AB}$ found in part (a).
	A1ft	Writing $\begin{pmatrix}0\\2\\3\end{pmatrix} + \mu\begin{pmatrix}1\\-1\\1\end{pmatrix}$ or $\begin{pmatrix}0\\2\\3\end{pmatrix} + \mu\mathbf{d}$, where $\mathbf{d}$ = their $\overrightarrow{AB}$ or a multiple of their $\overrightarrow{AB}$ found in part (a).
	Note	$\mathbf{r} =$ is not needed.
	Note	Using the same scalar parameter as in part (b) is fine for A1.
(e)	**M1**	Either $\overrightarrow{OP}$ + their $\overrightarrow{AB}$ or $\overrightarrow{OP}$ − their $\overrightarrow{AB}$.
	Note	This can be implied at least two out of three correct components for either their *C* or their *D*.
	A1ft	At least one set of coordinates are correct. Ignore labelling of C, D
	A1ft	Both sets of coordinates are correct. Ignore labelling of C, D
	Note	You can follow through either or both accuracy marks in this part using their $\overrightarrow{AB}$ from part (a).
(f)	**M1**	Way 1: $\frac{h}{\text{their } \lvert\overrightarrow{PB}\rvert} = \sin\theta$ Way 2: Attempts $\lvert\overrightarrow{PA}\rvert$ or $\lvert\overrightarrow{CB}\rvert$ Way 3: Attempts $\frac{1}{2}$ (their PB)(their AB)$\sin\theta$
	Note	Finding *AD* by itself is M0.
	A1	Either • $h = \sqrt{27}\sin(70.5...)$ or $\lvert\overrightarrow{PA}\rvert = \lvert\overrightarrow{CB}\rvert = \sqrt{24}$ or equivalent. (See Way 1 and Way 2) or • the area of either triangle *APB* or *APD* or *BDP* $= \frac{1}{2}\sqrt{3}\left(3\sqrt{3}\right)\sin\left(70.5...\right)$ o.e. (See Way 3).
	dM1	**which is dependent on the 1st M1 mark.** A full method to find the area of trapezium *ABCD*. (See Way 1, Way 2 and Way 3).
	A1	$9\sqrt{2}$ from a correct solution only.
	Note	A decimal answer of 12.7279... (without a correct **exact** answer) is A0.

Pearson Education Limited. Registered company number 872828
with its registered office at Edinburgh Gate, Harlow, Essex CM20 2JE