Centre No.						Paper Reference							Surname	Initial(s)
Candidate No.						6	6	6	6	/	0	1	Signature	

Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4

Advanced

Thursday 21 June 2012 – Afternoon

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 32 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

Printer's Log. No.
P41484A
W850/R6666/57570 5/5/5/5/

Turn over

PEARSON

Leave blank

1. $$\mathrm{f}(x) = \frac{1}{x(3x-1)^2} = \frac{A}{x} + \frac{B}{(3x-1)} + \frac{C}{(3x-1)^2}$$

(a) Find the values of the constants A, B and C.

(4)

(b) (i) Hence find $\int \mathrm{f}(x)\,\mathrm{d}x$.

(ii) Find $\int_1^2 \mathrm{f}(x)\,\mathrm{d}x$, leaving your answer in the form $a + \ln b$, where a and b are constants.

(6)

a)

$1 = A(3x-1)^2 + B(x)(3x-1) + C(x)$

Let $x = 0$

$1 = A$

Let $x = 1/3$

$1 = 1/3\, C$

$C = 3$

Let ~~$x=0$~~ $x = 1$

~~$1 = A$~~ $1 = 4A + 2B + C$

$1 = 4 + 2B + 3$

$-6 = 2B$

$B = -3$

b/ $\int \frac{1}{x} - \frac{3}{3x-1} + 3(3x-1)^{-2}\,dx$

$\ln x - \ln(3x-1) - (3x-1)^{-1} + c$

ii $\left[\ln x - \ln(3x-1) - (3x-1)^{-1}\right]_1^2$

$\left[\ln 2 - \ln 5 - \frac{1}{5}\right] - \left[\ln 1 - \ln 2 - \frac{1}{2}\right]$

$\ln\left(\frac{2\times 2}{5}\right) - \frac{1}{5} + \frac{1}{2}$

$= \frac{3}{10} + \ln\left(\frac{4}{5}\right)$

Leave blank

2.

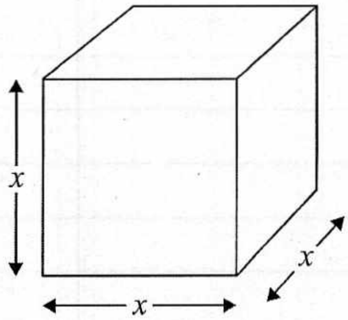

Figure 1

Figure 1 shows a metal cube which is expanding uniformly as it is heated.
At time t seconds, the length of each edge of the cube is x cm,
and the volume of the cube is V cm^3.

(a) Show that $\dfrac{dV}{dx} = 3x^2$ **(1)**

Given that the volume, V cm^3, increases at a constant rate of 0.048 cm^{3}s^{-1},

(b) find $\dfrac{dx}{dt}$, when $x = 8$ **(2)**

(c) find the rate of increase of the total surface area of the cube, in cm^{2}s^{-1}, when $x = 8$ **(3)**

a) $V = x \times x \times x$

$V = x^3$

$\dfrac{dV}{dx} = 3x^2$

b/ $\dfrac{dV}{dt} = 0.048$

$\dfrac{dx}{dt} = \dfrac{dx}{dV} \times \dfrac{dV}{dt}$

$= \dfrac{1}{3x^2} \times 0.048$

$x = 8$ $\quad \dfrac{dx}{dt} = \dfrac{1}{3(8)^2} \times 0.048$

$= \dfrac{1}{4000}$ cm^3 s^{-1}

c/ $S = 6x^2$

$\dfrac{dS}{dx} = 12x$

Leave blank

Question 2 continued

$$\frac{dS}{dt} = \frac{dS}{dx} \times \frac{dx}{dt}$$

$$= 12x \times \frac{1}{4000}$$

when $x=8$ $= 12(8) \times \frac{1}{4000}$

$$= \frac{3}{125} \text{ cm}^2 s^{-1}$$

Turn over

Leave blank

3. $$f(x) = \frac{6}{\sqrt{(9-4x)}}, \quad |x| < \frac{9}{4}$$

(a) Find the binomial expansion of $f(x)$ in ascending powers of x, up to and including the term in x^3. Give each coefficient in its simplest form.

(6)

Use your answer to part (a) to find the binomial expansion in ascending powers of x, up to and including the term in x^3, of

(b) $g(x) = \frac{6}{\sqrt{(9+4x)}}, \quad |x| < \frac{9}{4}$

(1)

(c) $h(x) = \frac{6}{\sqrt{(9-8x)}}, \quad |x| < \frac{9}{8}$

(2)

a/ $6(9-4x)^{-1/2}$

$6 \cdot 9^{-1/2}(1 - \frac{4}{9}x)^{-1/2}$

$2(1 - \frac{4}{9}x)^{-1/2}$

$2\left(1 + (-\tfrac{1}{2})(-\tfrac{4}{9}x) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2}(-\tfrac{4}{9}x)^2 + \frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{6}(-\tfrac{4}{9}x)^3\right)$

$2\left(1 + \frac{2}{9}x + \frac{2}{27}x^2 + \frac{20}{729}x^3\right)$

$2 + \frac{4}{9}x + \frac{4}{27}x^2 + \frac{40}{729}x^3$

b/ $2 - \frac{4}{9}x + \frac{4}{27}x^2 - \frac{40}{729}x^3$

c/ $2 + \frac{4}{9}(2x) + \frac{4}{27}(2x)^2 + \frac{40}{729}(2x)^3$

$2 + \frac{8}{9}x + \frac{16}{27}x^2 + \frac{320}{729}x^3$

Leave blank

4. Given that $y = 2$ at $x = \dfrac{\pi}{4}$, solve the differential equation

$$\frac{dy}{dx} = \frac{3}{y\cos^2 x}$$

(5)

$$\int y \, dy = \int 3\sec^2 x \, dx$$

$$\frac{1}{2}y^2 = 3\tan x + c$$

$\left(\frac{\pi}{4}, 2\right)$

$$\frac{1}{2}(2)^2 = 3\tan\frac{\pi}{4} + c$$

$$2 = 3 + c$$

$$c = -1$$

$$\frac{1}{2}y^2 = 3\tan x - 1$$

Leave blank

5. The curve C has equation

$$16y^3 + 9x^2y - 54x = 0$$

(a) Find $\frac{dy}{dx}$ in terms of x and y. **(5)**

(b) Find the coordinates of the points on C where $\frac{dy}{dx} = 0$. **(7)**

$u = 9x^2 \quad v = y$

$\frac{du}{dx} = 18x \quad \frac{dv}{dx} = \frac{dy}{dx}$

$48y^2 \frac{dy}{dx} + 18xy + 9x^2\frac{dy}{dx} - 54 = 0$

$48y^2\frac{dy}{dx} + 9x^2\frac{dy}{dx} = 54 \cancel{- 48y^2} - 18xy$

$\cancel{\frac{dx}{dy} \quad \frac{dy}{dx} = 48y^2 \pm a}$

$\frac{dy}{dx}\left(48y^2 + 9x^2\right) = 54 - 18xy$

$\frac{dy}{dx} = \frac{54 - 18xy}{48y^2 + 9x^2}$

b/ $0 = \frac{54 - 18xy}{48y^2 + 9x^2}$

$0 = 54 - 18xy$

$0 = 3 - xy$

$xy = 3$ 1

$16y^3 + 9x^2y - 54x = 0$

$16y^3 + 9y\left(\frac{3}{y}\right)^2 - 54\left(\frac{3}{y}\right) = 0$

$16y^3 + \frac{81}{y} - \frac{162}{y} = 0$

$16y^3 - \frac{81}{y} = 0$

$16y^4 - 81 = 0$

$y^4 = \frac{81}{16}$

$y = \pm \frac{3}{2}$

Leave blank

Question 5 continued

$x = \frac{3}{y}$

$x = \frac{3}{3/2}$ $\qquad$ $x = \frac{3}{-3/2}$

$= 2$ $\qquad$ $= -2$

$(2, 3/2)$ $\qquad$ $(-2, -3/2)$

Leave blank

6.

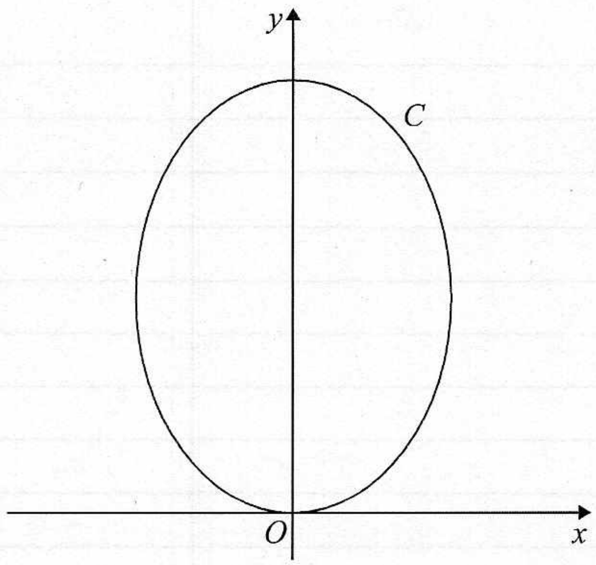

Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = (\sqrt{3})\sin 2t, \quad y = 4\cos^2 t, \quad 0 \leqslant t \leqslant \pi$$

$4(\cos t)^2$

(a) Show that $\frac{dy}{dx} = k(\sqrt{3})\tan 2t$, where k is a constant to be determined. **(5)**

(b) Find an equation of the tangent to C at the point where $t = \frac{\pi}{3}$.

Give your answer in the form $y = ax + b$, where a and b are constants. **(4)**

(c) Find a cartesian equation of C. **(3)**

a) $\frac{dx}{dt} = 2\sqrt{3}\cos 2t \qquad \frac{dy}{dt} = -8\cos t \sin t$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{-8\cos t \sin t}{2\sqrt{3}\cos 2t}$$

$$= \frac{-\cancel{8}^{4}(2\cos t \sin t)}{2\sqrt{3}\cos 2t}$$

$$= \frac{-\cancel{4}^{2}\sin 2t}{\cancel{2}\sqrt{3}\cos 2t}$$

$$= \frac{-2\tan 2t}{\sqrt{3}}$$

Leave blank

Question 6 continued

$\frac{-2}{\sqrt{3}} \tan 2t = \frac{-2\sqrt{3}}{3} \tan 2t$

$[k = -2/3]$

b/

$\frac{dy}{dx} = \frac{-2}{3}\sqrt{3} \tan\left(2\left(\frac{\pi}{3}\right)\right)$

$= 2$

$y = 2x + c$

$x = \sqrt{3} \sin\left(2\left(\frac{\pi}{3}\right)\right)$ $\quad y = 4\left(\cos\frac{\pi}{3}\right)^2$

$= \frac{3}{2}$ $\quad = 1$

$1 = 2\left(\frac{3}{2}\right) + c$

$c = -2$

$y = 2x - 2$

c/ $x^2 = 3\sin^2 2t$, $y = 4\cos^2 t$

$= 3(2\sin t\cos t)^2$

$= 12\sin^2 t\cos^2 t$

$\sin^2 t = \frac{x^2}{3(4\cos^2 t)}$

$= \frac{x^2}{3y}$ $\quad \cos^2 t = \frac{y}{4}$

$\sin^2 t + \cos^2 t = 1$

$\frac{x^2}{3y} + \frac{y}{4} = 1$

or//

$x^2 = 12\left(1 - \frac{y}{4}\right)\frac{y}{4}$

OR//

$\frac{x^2}{3} + \frac{(y-2)^2}{4} = 1$

7.

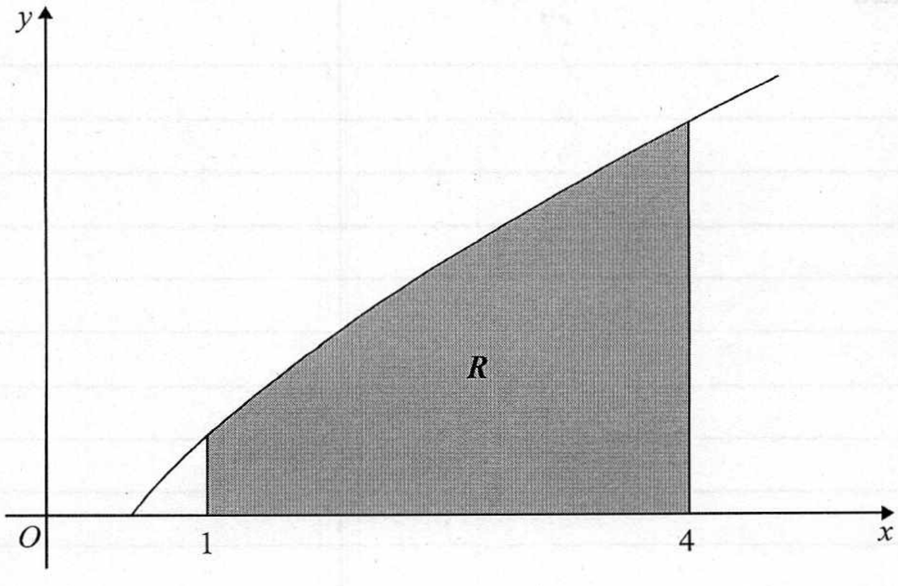

Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = x^{\frac{1}{2}} \ln 2x$.

The finite region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and the lines $x = 1$ and $x = 4$

(a) Use the trapezium rule, with 3 strips of equal width, to find an estimate for the area of R, giving your answer to 2 decimal places.
(4)

(b) Find $\int \underset{\frac{dv}{dx}}{x^{\frac{1}{2}}} \underset{u}{\ln 2x}\, dx$. $\quad \int u \frac{dv}{dx}\,dx = uv - \int v \frac{du}{dx}\,dx$
(4)

(c) Hence find the exact area of R, giving your answer in the form $a \ln 2 + b$, where a and b are exact constants.
(3)

a/

x	1	2	3	4
y	$\ln 2$	$\sqrt{2}\ln 4$	$\sqrt{3}\ln 6$	$2\ln 8$

$1\left(\frac{\ln 2}{2} + \sqrt{2}\ln 4 + \sqrt{3}\ln 6 + \frac{2\ln 8}{2}\right)$

$= 7.49$ units^2

b/ $\int \overset{\frac{dv}{dx}}{x^{\frac{1}{2}}} \overset{u}{\ln 2x}\,dx = \frac{2}{3}x^{\frac{3}{2}}\ln 2x - \int \frac{2}{3}x^{\frac{1}{2}}\,dx$

$v \rightarrow \boxed{\frac{2}{3}x^{\frac{3}{2}}}\ \boxed{\frac{1}{x}}_{\frac{du}{dx}} \qquad = \frac{2}{3}x^{\frac{3}{2}}\ln 2x - \frac{4}{9}x^{3/2} + C$

Question 7 continued

c/ $\left[\frac{2}{3}x^{\frac{3}{2}}\ln 2x - \frac{4}{9}x^{\frac{3}{2}}\right]_1^4$

$\boxed{\ln 8 = \ln 2^3 = 3\ln 2}$

$\left(\frac{16}{3}\ln 8 - \frac{32}{9}\right) - \left(\frac{2}{3}\ln 2 - \frac{4}{9}\right)$

$\left(16\ln 2 - \frac{32}{9}\right) - \left(\frac{2}{3}\ln 2 - \frac{4}{9}\right)$

$\frac{46}{3}\ln 2 - \frac{28}{9}$

Leave blank

Leave blank

8. Relative to a fixed origin O, the point A has position vector $(10\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})$, and the point B has position vector $(8\mathbf{i} + 3\mathbf{j} + 4\mathbf{k})$.

The line l passes through the points A and B.

(a) Find the vector $\overrightarrow{AB}$.

(2)

(b) Find a vector equation for the line l.

(2)

The point C has position vector $(3\mathbf{i} + 12\mathbf{j} + 3\mathbf{k})$.

The point P lies on l. Given that the vector $\overrightarrow{CP}$ is perpendicular to l,

(c) find the position vector of the point P.

(6)

a/ $(8i + 3j + 4k) - (10i + 2j + 3k)$

$-2i + j + k$

b/ $10i + 2j + 3k + t(-2i + j + k)$

c/ $P: \begin{pmatrix} 10-2t \\ 2+t \\ 3+t \end{pmatrix}$

$\overrightarrow{CP} = \begin{pmatrix} 10-2t \\ 2+t \\ 3+t \end{pmatrix} - \begin{pmatrix} 3 \\ 12 \\ 3 \end{pmatrix}$

$= \begin{pmatrix} 7-2t \\ -10+t \\ t \end{pmatrix}$

$a.b = 0 \quad (7-2t)(-2) + (-10+t)(1) + t(1) = 0$

$-14 + 4t - 10 + t + t = 0$

$6t - 24 = 0$

$t = 4$

$\cancel{P}:$

Leave blank

Question 8 continued

$$P = \begin{matrix} 10 - 2(4) \\ 2 + 4 \\ 3 + 4 \end{matrix}$$

$$= \cancel{\begin{pmatrix} 2 \\ 8 \\ 7 \end{pmatrix}} \quad \underline{\begin{pmatrix} 2 \\ 6 \\ 7 \end{pmatrix}}$$