Centre No.						Paper Reference							Surname	Initial(s)
Candidate No.						6	6	6	6	/	0	1	Signature	

Paper Reference(s)

6666/01

Edexcel GCE

Core Mathematics C4

Advanced

Monday 28 January 2013 – Morning

Time: 1 hour 30 minutes

Examiner's use only

Team Leader's use only

Question Number	Leave Blank
1	
2	
3	
4	
5	
6	
7	
8	
Total	

Materials required for examination	Items included with question papers
Mathematical Formulae (Pink)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.

Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 28 pages in this question paper. Any blank pages are indicated.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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PEARSON

1. Given

$$f(x) = (2 + 3x)^{-3}, \qquad |x| < \frac{2}{3}$$

find the binomial expansion of f(x), in ascending powers of x, up to and including the term in x^3.

Give each coefficient as a simplified fraction.

(5)

$$2^{-3}\left(1+\frac{3}{2}x\right)^{-3}$$

$$\frac{1}{8}\left(1+\frac{3}{2}x\right)^{-3}$$

$$\frac{1}{8}\left(1+(-3)\left(\frac{3}{2}x\right)+\frac{(-3)(-4)}{2}\left(\frac{3}{2}x\right)^2+\frac{(-3)(-4)(-5)}{6}\left(\frac{3}{2}x\right)^3\right)$$

$$\frac{1}{8}\left(1-\frac{9}{2}x+\frac{27}{2}x^2-\frac{135}{4}x^3\right)$$

$$\frac{1}{8}-\frac{9}{16}x+\frac{27}{16}x^2-\frac{135}{32}x^3$$

Leave blank

2. (a) Use integration to find

$$\int \frac{1}{x^3} \ln x \, dx$$

u

$\frac{dv}{dx}$

(5)

(b) Hence calculate

$$\int_1^2 \frac{1}{x^3} \ln x \, dx$$

(2)

$$\int u \frac{dv}{dx} \, dx = uv - \int v \frac{du}{dx} \, dx$$

$u = \ln x \qquad \frac{dv}{dx} = x^{-3}$

$\frac{du}{dx} = \frac{1}{x} \qquad v = \frac{-1}{2} x^{-2}$

$$= -\frac{1}{2} x^{-2} \ln x - \int -\frac{1}{2} x^{-3} \, dx$$

$$= -\frac{1}{2} x^{-2} \ln x - \frac{1}{4} x^{-2} + c$$

b/ $\left[-\frac{1}{2} x^{-2} \ln x - \frac{1}{4} x^{-2} \right]_1^2$

$\left(-\frac{1}{2}\left(\frac{1}{4}\right) \ln 2 - \frac{1}{4}\left(\frac{1}{4}\right)\right) - \left(-\frac{1}{2} \ln 1 - \frac{1}{4}\right)$

$-\frac{1}{8} \ln 2 - \frac{1}{16} + \frac{1}{4}$

$-\frac{1}{8} \ln 2 + \frac{3}{16}$

3. Express $\dfrac{9x^2 + 20x - 10}{(x+2)(3x-1)}$ in partial fractions.

(4)

0

$$\begin{array}{r} 3 \\ 3x^2 + 5x - 2 \overline{)\, 9x^2 + 20x - 10} \\ 9x^2 + 15x - 6 \\ 5x - 4 \end{array}$$

$$3 + \frac{5x - 4}{(x+2)(3x-1)}$$

$$\frac{5x - 4}{(x+2)(3x-1)} = \frac{A}{x+2} + \frac{B}{3x-1}$$

~~Let $x = 2$~~

~~$5(-2)\ 4 =$~~

$$5x - 4 = A(3x - 1) + B(x + 2)$$

Let $x = -2$

$$5(-2) - 4 = A(3(-2) - 1)$$

$$-14 = -7A$$

$$A = 2.$$

Let $x = 1/3 \quad 5(1/3) - 4 = 7/3\ B$

$$-7/3 = 7/3\ B$$

$$B = -1$$

$$3 + \frac{2}{x+2} - \frac{1}{3x-1}$$

4.

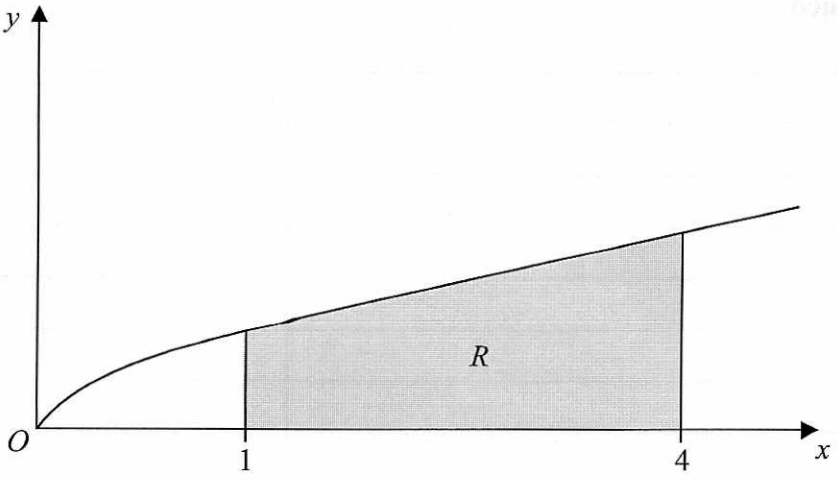

Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = \dfrac{x}{1+\sqrt{x}}$. The finite region R, shown shaded in Figure 1, is bounded by the curve, the x-axis, the line with equation $x = 1$ and the line with equation $x = 4$.

(a) Complete the table with the value of y corresponding to $x = 3$, giving your answer to 4 decimal places.

(1)

x	1	2	3	4
y	0.5	0.8284	1.0981	1.3333

(b) Use the trapezium rule, with all the values of y in the completed table, to obtain an estimate of the area of the region R, giving your answer to 3 decimal places.

(3)

(c) Use the substitution $u = 1 + \sqrt{x}$, to find, by integrating, the exact area of R.

(8)

b/ $1\left(\frac{0.5}{2} + 0.8284 + 1.0981 + \frac{1.3333}{2}\right)$

$= 2.843$ units^2 (3dp)

c/ $\int_1^4 \frac{x}{1+\sqrt{x}}\,dx$

$u = 1 + x^{1/2}$

$\frac{du}{dx} = \frac{1}{2}x^{-1/2}$

$\frac{1}{2\sqrt{x}}$

$1+\sqrt{4}$

Question 4 continued

$$\int_{2}^{3} \frac{x}{u}\,\frac{dx}{du}\,du$$

$1+\sqrt{1}$

$$\frac{dx}{du} = 2\sqrt{x}$$

$$\int_{2}^{3} \frac{2\sqrt{x}\cdot x}{u}\,du \qquad u-1=\sqrt{x}$$

$$(u-1)^2 = x$$

$$\int_{2}^{3} \frac{2(u-1)(u-1)^2}{u}\,du$$

$$\int_{2}^{3} \frac{2(u-1)^3}{u}\,du$$

$$\int_{2}^{3} \frac{2(u-1)(u^2-2u+1)}{u}\,du$$

$$2\int_{2}^{3} \frac{(u^3-2u^2+u-u^2+2u-1)}{u}\,du$$

$$2\int_{2}^{3} \frac{u^3-3u^2+3u-1}{u}\,du$$

$$2\int_{2}^{3} u^2-3u+3-\frac{1}{u}\,du$$

$$2\left[\frac{u^3}{3}-\frac{3u^2}{2}+3u-\ln u+c\right]_{2}^{3}$$

$$2\left[\left(\frac{27}{3}-\frac{27}{2}+9-\ln 3\right)-\left(\frac{8}{3}-\frac{\cancel{6}\,12}{2}+6-\ln 2\right)\right]$$

$$2\left[\left(\frac{9}{2}-\ln 3\right)-\left(\frac{8}{3}-\ln 2\right)\right]$$

$$2\left[\frac{11}{6}+\ln\frac{2}{3}\right]$$

$$\frac{11}{3}+2\ln\frac{2}{3}$$

5.

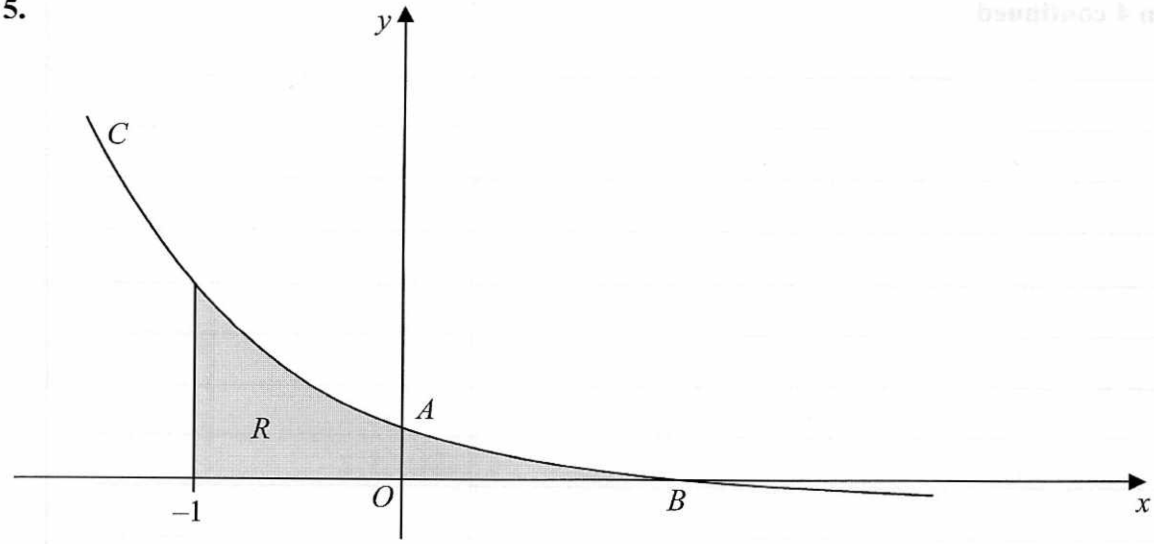

Figure 2

Figure 2 shows a sketch of part of the curve C with parametric equations

$$x = 1 - \frac{1}{2}t, \quad y = 2^t - 1$$

The curve crosses the y-axis at the point A and crosses the x-axis at the point B.

(a) Show that A has coordinates (0, 3).

(2)

(b) Find the x coordinate of the point B.

(2)

(c) Find an equation of the normal to C at the point A.

(5)

The region R, as shown shaded in Figure 2, is bounded by the curve C, the line $x = -1$ and the x-axis.

(d) Use integration to find the exact area of R.

(6)

a) A is where $x = 0$

$0 = 1 - \frac{1}{2}t$

$t = 2$

$y = 2^2 - 1$

$= 3$

$\underline{\underline{(0, 3)}}$

Question 5 continued

b/ x coordinate of B $\quad y=0$

$0 = 2^t - 1$

$1 = 2^t$

$t = 0$

$x = 1 - \frac{1}{2}(0)$

$\underline{\underline{= 1}}$

c/ gradient at A $\quad [x=0 \quad y=3 \quad t=2)$

$\frac{dx}{dt} = -\frac{1}{2} \qquad \frac{dy}{dt} = 2^t \ln 2$

$\frac{dy}{dx} = \cancel{2t} \ \frac{2^t \ln 2}{-1/2}$

$= -2(2^t \ln 2)$

$= -8 \ln 2 \qquad [t=2]$

$m\ (\text{normal}) = \frac{1}{8 \ln 2}$

$y = mx + c$

$3 = \frac{1}{8\ln 2}(0) + c$

$c = 3$

$\underline{y = \frac{1}{8 \ln 2} x + c}$

d/ $\int_{-1}^{1} \cancel{x} \ y \ dx$ $\qquad 1 = 1 - \frac{1}{2}t$

$t = 0$

$-1 = 1 - \frac{1}{2}t$

$t = 4$

$\int_{4}^{0} y \frac{dx}{dt} \, dt$

$\int_{4}^{0} (2^t - 1)\left(-\frac{1}{2}\right) dt$

Question 5 continued

$$-\frac{1}{2}\int_4^0 2^t - 1 \quad dt$$

$$\frac{1}{2}\int_0^4 2^t - 1 \quad dt$$

$$\frac{1}{2}\left[2^t\left(\frac{1}{\ln 2}\right) - t\right]_0^4$$

$$\frac{1}{2}\left[\left(2^4\left(\frac{1}{\ln 2}\right) - 4\right) - \left(2^0\left(\frac{1}{\ln 2}\right) - 0\right)\right]$$

$$\frac{1}{2}\left[16\left(\frac{1}{\ln 2}\right) - 4 - \frac{1}{\ln 2}\right]$$

$$\frac{1}{2}\left[15\left(\frac{1}{\ln 2}\right) - 4\right]$$

$$\frac{15}{2}\left(\frac{1}{\ln 2}\right) - 2$$

$$\frac{15}{2\ln 2} - 2$$

Q5

(Total 15 marks)

6.

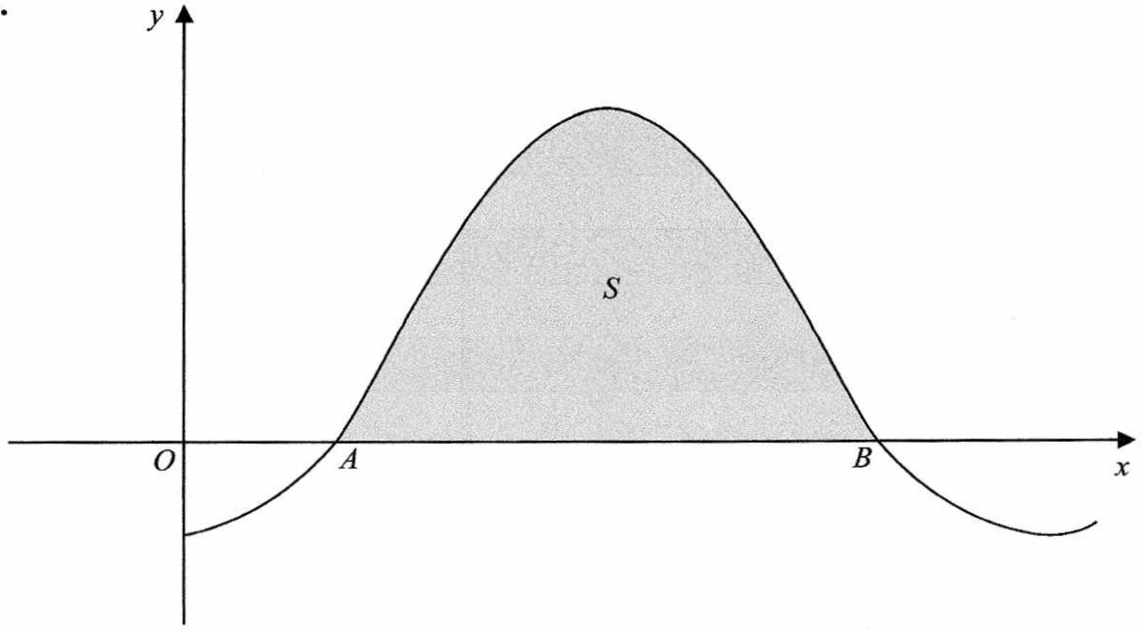

Figure 3

Figure 3 shows a sketch of part of the curve with equation $y = 1 - 2\cos x$, where x is measured in radians. The curve crosses the x-axis at the point A and at the point B.

(a) Find, in terms of π, the x coordinate of the point A and the x coordinate of the point B. **(3)**

The finite region S enclosed by the curve and the x-axis is shown shaded in Figure 3. The region S is rotated through 2π radians about the x-axis.

(b) Find, by integration, the exact value of the volume of the solid generated. **(6)**

a/ x coordinates of A and B where $y=0$

$$0 = 1 - 2\cos x$$
$$-1 = -2\cos x$$
$$\frac{1}{2} = \cos x$$
$$x = \frac{1}{3}\pi, \frac{5}{3}\pi$$

b/ $$\pi\int_{\frac{1}{3}\pi}^{\frac{5}{3}\pi} y^2 \, dx$$

$$\pi\int_{\frac{1}{3}\pi}^{\frac{5}{3}\pi} (1-2\cos x)^2 \, dx$$

Question 6 continued

$$\pi \int_{\frac{1}{3}\pi}^{\frac{5}{3}\pi} 1 - 4\cos x + 4\cos^2 x \quad dx$$

$$\cos 2x = 2\cos^2 x - 1$$

$$\cos 2x + 1 = 2\cos^2 x$$

$$\pi \int_{\frac{1}{3}\pi}^{\frac{5}{3}\pi} 1 - 4\cos x + 2\cos 2x + 2 \quad dx$$

$$\pi \int_{\frac{1}{3}\pi}^{\frac{5}{3}\pi} 3 \mp 4\cos x + 2\cos 2x \quad dx$$

$$\pi \Big[3x \mp 4\sin x + \sin 2x \Big]_{\frac{1}{3}\pi}^{\frac{5}{3}\pi}$$

$$\pi \left[\left(5\pi + 2\sqrt{3} - \frac{\sqrt{3}}{2} \right) - \left(\pi \mp 2\sqrt{3} + \frac{\sqrt{3}}{2} \right) \right]$$

$$\pi \left[4\pi + 4\sqrt{3} - \sqrt{3} \right]$$

$$\pi \left(4\pi + 3\sqrt{3} \right)$$

$$\underline{\underline{4\pi^2 + 3\sqrt{3}\,\pi}}$$

$$\underline{\underline{4\pi^2 + 3\pi\sqrt{3}}}$$

7. With respect to a fixed origin O, the lines l_1 and l_2 are given by the equations

$$l_1 : \mathbf{r} = (9\mathbf{i} + 13\mathbf{j} - 3\mathbf{k}) + \lambda(\mathbf{i} + 4\mathbf{j} - 2\mathbf{k})$$

$$l_2 : \mathbf{r} = (2\mathbf{i} - \mathbf{j} + \mathbf{k}) + \mu(2\mathbf{i} + \mathbf{j} + \mathbf{k})$$

where λ and μ are scalar parameters.

(a) Given that l_1 and l_2 meet, find the position vector of their point of intersection.
(5)

(b) Find the acute angle between l_1 and l_2, giving your answer in degrees to 1 decimal place.
(3)

Given that the point A has position vector $4\mathbf{i} + 16\mathbf{j} - 3\mathbf{k}$ and that the point P lies on l_1 such that AP is perpendicular to l_1,

(c) find the exact coordinates of P.
(6)

a. i// $9 + \lambda = 2 + 2\mu$ ①

j// $13 + 4\lambda = -1 + \mu$ ②

$26 + 8\lambda = -2 + 2\mu$ ~~①~~ ②×2

$9 + \lambda = 2 + 2\mu$ ①

$17 + 7\lambda = -4$

$7\lambda = -21$

$\underline{\underline{\lambda = -3}}$

$9 + (-3) = 2 + 2\mu$

$6 = 2 + 2\mu$

$\underline{\underline{\mu = 2}}$

l_1 $\begin{matrix} 9 \\ 13 \\ -3 \end{matrix} + -3\begin{pmatrix} 1 \\ 4 \\ -2 \end{pmatrix} = \begin{pmatrix} 6 \\ 1 \\ 3 \end{pmatrix}$

l_2 $\begin{matrix} 2 \\ -1 \\ 1 \end{matrix} + 2\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 6 \\ 1 \\ 3 \end{pmatrix}$

$[6i + j + 3k]$

Question 7 continued

b) $\cos\theta = \dfrac{a.b}{|a||b|}$

$a.b = 1(2) + 4(1) + -2(1)$

$= 2 + 4 - 2$

$= 4$

$|a| = \sqrt{1^2 + 4^2 + 2^2}$ $\qquad$ $|b| = \sqrt{2^2 + 1^2 + 1^2}$

$= \sqrt{21}$ $\qquad$ $= \sqrt{6}$

$\cos\theta = \dfrac{4}{\sqrt{21}\sqrt{6}}$

$= \underline{\underline{69.1}}^{\circ}$ (1dp)

c) p (lies on l_1) : $\begin{pmatrix}9\\13\\-3\end{pmatrix} + \lambda\begin{pmatrix}1\\4\\-2\end{pmatrix}$

$\overrightarrow{AP} = \begin{pmatrix}9\\13\\-3\end{pmatrix} + \lambda\begin{pmatrix}1\\4\\-2\end{pmatrix} - \begin{pmatrix}4\\16\\-3\end{pmatrix}$

$= \begin{pmatrix}5\\-3\\0\end{pmatrix} + \lambda\begin{pmatrix}1\\4\\-2\end{pmatrix}$

perpendicular so $a.b = 0$

$1(5+\lambda) + 4(-3+4\lambda) - 2(-2\lambda) = 0$

$5 + \lambda - 12 + 16\lambda + 4\lambda = 0$

$21\lambda - 7 = 0$

$\lambda = 1/3$

$\therefore p \begin{pmatrix}9\\13\\-3\end{pmatrix} + \frac{1}{3}\begin{pmatrix}1\\4\\-2\end{pmatrix} = \begin{pmatrix}9\,1/3\\14\,1/3\\-3\,2/3\end{pmatrix}$

$\frac{28}{3}i + ~~14~~ \frac{43}{3}j - \frac{11}{3}k$

Leave blank

8. A bottle of water is put into a refrigerator. The temperature inside the refrigerator remains constant at 3 °C and t minutes after the bottle is placed in the refrigerator the temperature of the water in the bottle is θ °C.

The rate of change of the temperature of the water in the bottle is modelled by the differential equation,

$$\frac{d\theta}{dt} = \frac{(3-\theta)}{125}$$

(a) By solving the differential equation, show that,

$$\theta = Ae^{-0.008t} + 3$$

where A is a constant.

(4)

Given that the temperature of the water in the bottle when it was put in the refrigerator was 16 °C,

(b) find the time taken for the temperature of the water in the bottle to fall to 10 °C, giving your answer to the nearest minute.

(5)

a/ $\int \frac{1}{3-\theta} \, d\theta = \int \frac{1}{125} \, dt$

$-\ln(3-\theta) = \frac{t}{125} + c$

$\ln(3-\theta) = \frac{-t}{125} + c$

$3-\theta = e^{\frac{-t}{125}+c}$

$3-\theta = e^{\frac{-t}{125}} e^{c}$

Let $e^{c} = -A$.

$3-\theta = -Ae^{\frac{-t}{125}}$

$\theta = 3 + Ae^{\frac{-t}{125}}$

$\theta = Ae^{-0.008t} + 3$

$t=0 \quad \theta = 16 \quad \therefore$

b/ $A = 13$

Question 8 continued

$\theta = 13e^{-0.008t} + 3$

$10 = 13e^{-0.008t} + 3$

$7 = 13e^{-0.008t}$

$\frac{7}{13} = e^{-0.008t}$

$\ln \frac{7}{13} = -0.008t$

$t = -125 \ln\left(\frac{7}{13}\right)$

$= 77$ minutes