| Centre No. | | | | | | Paper Reference | | | | | | | Surname | Initial(s) |
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| Candidate No. | | | | | | 6 | 6 | 6 | 5 | / | 0 | 1 | Signature | |

Paper Reference(s)

# 6665/01

# Edexcel GCE

## Core Mathematics C3

## Advanced

Thursday 16 June 2011 – Afternoon

Time: 1 hour 30 minutes

| Examiner's use only | | |
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| Team Leader's use only | | |
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| Question Number | Leave Blank |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| 6 | |
| 7 | |
| 8 | |
| Total | |

| Materials required for examination | Items included with question papers |
|---|---|
| Mathematical Formulae (Pink) | Nil |

**Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation or symbolic differentiation/integration, or have retrievable mathematical formulae stored in them.**

### Instructions to Candidates

In the boxes above, write your centre number, candidate number, your surname, initials and signature.
Check that you have the correct question paper.
Answer ALL the questions.
You must write your answer for each question in the space following the question.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

### Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
The marks for individual questions and the parts of questions are shown in round brackets: e.g. **(2)**.
There are 8 questions in this question paper. The total mark for this paper is 75.
There are 24 pages in this question paper. Any blank pages are indicated.

### Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You should show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.





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**1.** Differentiate with respect to $x$

(a) $\ln\left(x^2+3x+5\right)$

**(2)**

(b) $\dfrac{\cos x}{x^2}$

**(3)**

1a) $\dfrac{1}{x^2+3x+5} \times 2x+3$

$\dfrac{2x+3}{x^2+3x+5}$

b) $u=\cos x$ $\qquad v=x^2$

$\dfrac{du}{dx}=-\sin x$ $\qquad \dfrac{dv}{dx}=2x$

$\dfrac{-x^2\sin x - 2x\cos x}{(x^2)^2}$

$\dfrac{-x^2\sin x - 2x\cos x}{x^4}$

$\dfrac{-x\sin x - 2\cos x}{x^3}$

Leave blank

**2.** $$f(x) = 2\sin(x^2) + x - 2, \qquad 0 \leqslant x < 2\pi$$

(a) Show that $f(x) = 0$ has a root $\alpha$ between $x = 0.75$ and $x = 0.85$

**(2)**

The equation $f(x) = 0$ can be written as $x = \left[\arcsin(1 - 0.5x)\right]^{\frac{1}{2}}$.

(b) Use the iterative formula

$$x_{n+1} = \left[\arcsin(1 - 0.5x_n)\right]^{\frac{1}{2}}, \quad x_0 = 0.8$$

to find the values of $x_1$, $x_2$ and $x_3$, giving your answers to 5 decimal places.

**(3)**

(c) Show that $\alpha = 0.80157$ is correct to 5 decimal places.

**(3)**

a) $f(0.75) = -0.1833946529$
$f(0.85) = 0.172524 2475$

change of sign $\therefore f(x) = 0$ has a root between 0.75 and 0.85

b/ $x_0 = 0.8$
$x_1 = [\sin^{-1}(1 - 0.5(0.8)]^{1/2} = 0.80219$
$x_2 = 0.80133$
$x_3 = 0.80167$

c/ $f(0.801565) = -2.704865539 \times 10^{-5}$
$f(0.801575) = 8.62055286 \times 10^{-6}$

change of sign $\therefore \alpha = 0.80157$ to 5dp

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**3.**

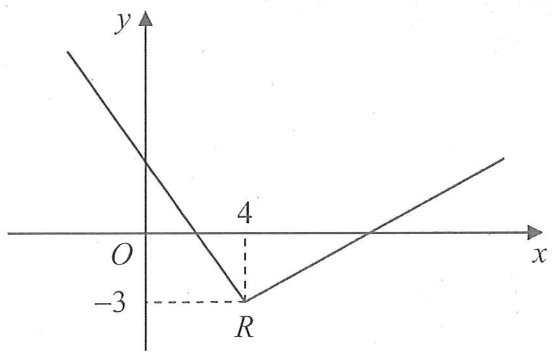



**Figure 1**

Figure 1 shows part of the graph of $y = \mathrm{f}(x)$, $x \in \mathbb{R}$.

The graph consists of two line segments that meet at the point $R(4, -3)$, as shown in Figure 1.

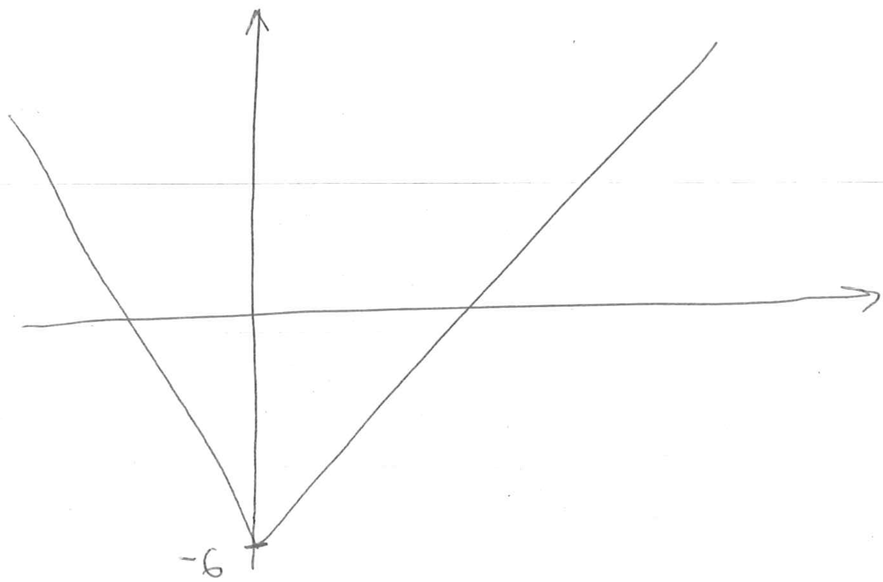
Sketch, on separate diagrams, the graphs of

(a) $y = 2\mathrm{f}(x+4)$, **(3)**

(b) $y = |\mathrm{f}(-x)|$. **(3)**

On each diagram, show the coordinates of the point corresponding to $R$.

a)



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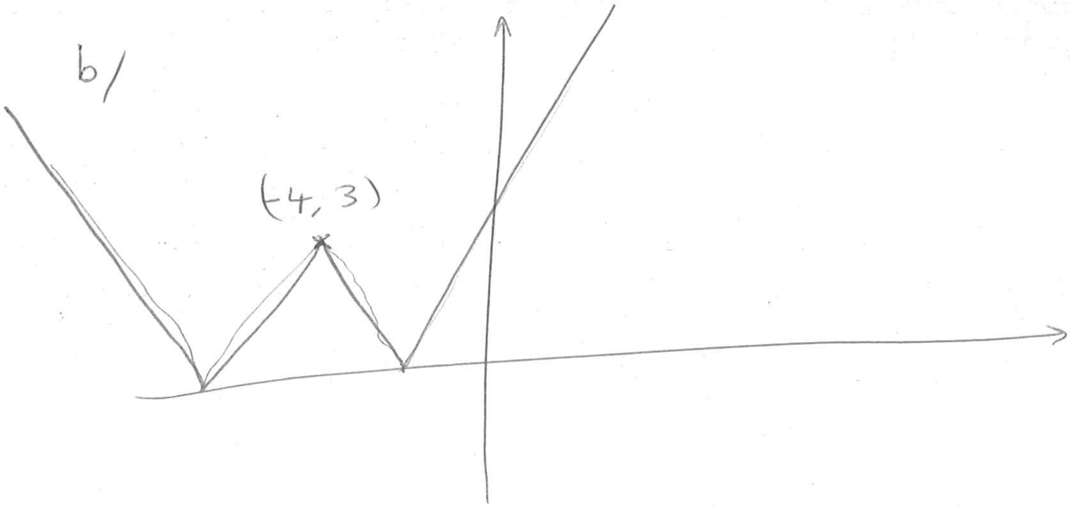
**Question 3 continued**

b/




Q3

**(Total 6 marks)**

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4. The function f is defined by

$$f: x \mapsto 4 - \ln(x+2), \quad x \in \mathbb{R}, \ x \geqslant -1$$

(a) Find $f^{-1}(x)$. **(3)**

(b) Find the domain of $f^{-1}$. **(1)**

The function g is defined by

$$g: x \mapsto e^{x^2} - 2, \quad x \in \mathbb{R}$$

(c) Find $fg(x)$, giving your answer in its simplest form. **(3)**

(d) Find the range of fg. **(1)**

a) $y = 4 - \ln(x+2)$

$x = 4 - \ln(y+2)$

$\ln(y+2) = 4 - x$

$y + 2 = e^{4-x}$

$y = e^{4-x} - 2$

$f^{-1}(x) = e^{4-x} - 2$

b) $x \leq 4$

c) $fg(x) = 4 - \ln(e^{x^2} - 2 + 2)$

$= 4 - \ln e^{x^2}$

$= 4 - x^2$

d) $fg(x) \leq 4$

5. The mass, $m$ grams, of a leaf $t$ days after it has been picked from a tree is given by

$$m = pe^{-kt}$$

where $k$ and $p$ are positive constants.

When the leaf is picked from the tree, its mass is 7.5 grams and 4 days later its mass is 2.5 grams.

(a) Write down the value of $p$.
**(1)**

(b) Show that $k = \frac{1}{4}\ln 3$.
**(4)**

(c) Find the value of $t$ when $\frac{dm}{dt} = -0.6\ln 3$.
**(6)**

a) $p = 7.5$

b) $m = 7.5e^{-kt}$

$2.5 = 7.5e^{-k(4)}$

$\frac{1}{3} = e^{-4k}$

$\ln\frac{1}{3} = -4k$

$-\frac{1}{4}\ln\frac{1}{3} = k$

$\frac{1}{4}\ln\left(\frac{1}{3}\right)^{-1} = k$

$\frac{1}{4}\ln 3 = k$

c) $\frac{dm}{dt} = -7.5ke^{-kt}$

$-0.6\ln 3 = -7.5ke^{-\frac{1}{4}\ln 3\cdot t}$

$-0.6\ln 3 = -7.5(\frac{1}{4}\ln 3)e^{-1/4\ln 3\cdot t}$

$0.32 = e^{-1/4\ln 3\cdot t}$

$\ln 0.32 = -1/4\ln 3\cdot t$

$t = 4.15$ (3sf)

Leave blank

6. (a) Prove that

$$\frac{1}{\sin 2\theta} - \frac{\cos 2\theta}{\sin 2\theta} = \tan\theta, \quad \theta \neq 90n°, \ n \in \mathbb{Z}$$

**(4)**

(b) Hence, or otherwise,

(i) show that $\tan 15° = 2 - \sqrt{3}$,

**(3)**

(ii) solve, for $0 < x < 360°$,

$$\operatorname{cosec} 4x - \cot 4x = 1$$

**(5)**

6a) $\dfrac{1 - \cos 2\theta}{\sin 2\theta}$

$\dfrac{1 - (\cos^2\theta - \sin^2\theta)}{2\sin\theta\cos\theta}$

$\dfrac{1 - (1 - \sin^2\theta - \sin^2\theta)}{2\sin\theta\cos\theta}$

$\dfrac{\cancel{2}\sin^2\theta}{\cancel{2}\cancel{\sin\theta}\cos\theta}$

$\dfrac{\sin\theta}{\cos\theta} = \underline{\tan\theta}$

b) $\tan 15 = \dfrac{1}{\sin 30} - \dfrac{\cos 30}{\sin 30}$

$= \dfrac{1}{0.5} - \dfrac{\sqrt{3}/2}{0.5}$

$= \underline{2 - \sqrt{3}}$

c) $\operatorname{cosec} 4x - \cot 4x = 1$

$\tan 2\cancel{\theta}^{x} = 1$

Leave blank

**Question 6 continued**

$\tan 2x = 1$

$2x = 45, 225, 405, 585$

$x = 22.5, 112.5, 202.5, 292.5$

Leave blank

**7.** $$\mathrm{f}(x)=\frac{4x-5}{(2x+1)(x-3)}-\frac{2x}{x^2-9}, \qquad x \neq \pm 3,\ x \neq -\frac{1}{2}$$

(a) Show that

$$\mathrm{f}(x)=\frac{5}{(2x+1)(x+3)}$$

**(5)**

The curve $C$ has equation $y=\mathrm{f}(x)$. The point $P\left(-1, -\frac{5}{2}\right)$ lies on $C$.

(b) Find an equation of the normal to $C$ at $P$.

**(8)**

a) $$\frac{4x-5}{(2x+1)(x-3)} - \frac{2x}{(x+3)(x-3)}$$

$$\frac{(4x-5)(x+3)}{(2x+1)(x-3)(x+3)} - \frac{2x\,(\cancel{4x-5})(2x+1)}{(2x+1)(x-3)(x+3)}$$

$$\frac{(4x-5)(x+3) - 2x(2x+1)}{(2x+1)(x-3)(x+3)}$$

$$\frac{4x^2+12x-5x-15-4x^2-2x}{(2x+1)(x-3)(x+3)}$$

$$\frac{5x-15}{(2x+1)(x-3)(x+3)}$$

$$\frac{5(x-3)}{(2x+1)(x-3)(x+3)}$$

$$\frac{5}{(2x+1)(x+3)}$$

b) $\mathrm{f}(x) = 5(2x^2+7x+3)^{-1}$

$\mathrm{f}'(x) = -5(2x^2+7x+3)^{-2}\,(4x+7)$

$\mathrm{f}'(-1) = \dfrac{-5(3)}{4} = \dfrac{-15}{4}$

Leave blank

**Question 7 continued**

$x \quad y$

$y = \frac{4}{15}x + c \qquad (-1, -\frac{5}{2})$

$-\frac{5}{2} = \frac{4}{15}(-1) + c$

$-5/2 = -4/15 + c$

$c = \frac{-67}{30}$

$y = \frac{4}{15}x - \frac{67}{30}$



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**8.** (a) Express $2\cos 3x - 3\sin 3x$ in the form $R\cos(3x+\alpha)$, where $R$ and $\alpha$ are constants, $R>0$ and $0<\alpha<\frac{\pi}{2}$. Give your answers to 3 significant figures.

**(4)**

$$f(x) = e^{2x}\cos 3x$$

(b) Show that $f'(x)$ can be written in the form

$$f'(x) = R e^{2x}\cos(3x+\alpha)$$

where $R$ and $\alpha$ are the constants found in part (a).

**(5)**

(c) Hence, or otherwise, find the smallest positive value of $x$ for which the curve with equation $y = f(x)$ has a turning point.

**(3)**

8a) $\cos(A+B) = \cos A\cos B - \sin A\sin B$

$R\cos(3x+\alpha) = R\cos 3x\cos\alpha - R\sin 3x\sin\alpha$

$2 = R\cos\alpha \qquad 3 = R\sin\alpha$

$\tan\alpha = 3/2 \qquad R^2 = 2^2+3^2$

$\alpha = 0.983 \qquad R = \sqrt{13}$

$= 3.61$

$3.61\cos(3x+0.983)$

b) $u = e^{2x} \qquad v = \cos 3x$

$\frac{du}{dx} = 2e^{2x} \qquad \frac{dv}{dx} = -3\sin 3x$

$f'(x) = 2e^{2x}\cos 3x - 3e^{2x}\sin 3x$

$= e^{2x}(2\cos 3x - 3\sin 3x)$

$= e^{2x}(\sqrt{13}\cos(3x+0.983))$

$= \sqrt{13}\, e^{2x}\cos(3x+0.983)$

Question 8 continued

turning point where $\frac{dy}{dx} = 0$

c) $0 = (\sqrt{13}e^{2x})\cos(3x + 0.983)$

$0 = \cos(3x + 0.983)$

$\cos^{-1}(0) = 3x + 0.983$

$\frac{1}{2}\pi = \cancel{\cos}\ 3x + 0.983$

$x = 0.196 \quad (3sf)$