



1. Using calculus, find the coordinates of the stationary point on the curve with equation

$$y = 2x + 3 + \frac{8}{x^2}, \quad x > 0$$

(6)

$$y = 2x + 3 + 8x^{-2}$$

$$\frac{dy}{dx} = 2 - 16x^{-3}$$

Stationary point is where  $\frac{dy}{dx} = 0$

$$2 - 16x^{-3} = 0$$

$$2 - \frac{16}{x^3} = 0$$

$$2 = \frac{16}{x^3}$$

$$2x^3 = 16$$

$$x^3 = 8$$

$$x = 2$$

when  $x = 2$ :

$$y = 2(2) + 3 + \frac{8}{(2)^2}$$

$$= 9$$

(2, 9)



2.

$$y = \frac{x}{\sqrt{1+x}}$$

(a) Complete the table below with the value of  $y$  corresponding to  $x = 1.3$ , giving your answer to 4 decimal places.

(1)

$x$	1	1.1	1.2	1.3	1.4	1.5
$y$	0.7071	0.7591	0.8090	0.8572	0.9037	0.9487

(b) Use the trapezium rule, with all the values of  $y$  in the completed table, to obtain an approximate value for

$$\int_1^{1.5} \frac{x}{\sqrt{1+x}} dx$$

giving your answer to 3 decimal places.

You must show clearly each stage of your working.

(4)

b/  $0.1 \left( \frac{0.7071 + 0.7591 + 0.8090 + 0.8572 + 0.9037 + 0.9487}{2} \right)$

$= 0.41569$

$= 0.416 \text{ units}^2 \text{ (3 dp)}$



3. Find the first 4 terms, in ascending powers of  $x$ , of the binomial expansion of

$$\left(2 - \frac{1}{2}x\right)^8$$

giving each term in its simplest form.

(4)

$$1 \quad 8 \quad 28 \quad 56$$

$$1(2)^8 + 8(2)^7\left(-\frac{1}{2}x\right) + 28(2)^6\left(-\frac{1}{2}x\right)^2 + 56(2)^5\left(-\frac{1}{2}x\right)^3$$

$$256 - 512x + 448x^2 - 224x^3$$



4.  $f(x) = ax^3 - 11x^2 + bx + 4$ , where  $a$  and  $b$  are constants.

When  $f(x)$  is divided by  $(x - 3)$  the remainder is 55

When  $f(x)$  is divided by  $(x + 1)$  the remainder is  $-9$

(a) Find the value of  $a$  and the value of  $b$ .

(5)

Given that  $(3x + 2)$  is a factor of  $f(x)$ ,

(b) factorise  $f(x)$  completely.

(4)

$$\begin{aligned} \text{a)} \quad f(3) &= 55 \\ f(-1) &= -9 \end{aligned}$$

$$a(3)^3 - 11(3)^2 + b(3) + 4 = 55$$

$$27a - 99 + 3b + 4 = 55$$

$$27a + 3b = 150$$

$$9a + b = 50 \quad (1)$$

$$a(-1)^3 - 11(-1)^2 + b(-1) + 4 = -9$$

$$-a - 11 - b + 4 = -9$$

$$-a - b = -2$$

$$a + b = 2 \quad (2)$$

$$(1) - (2) \quad 8a = 48$$

$$a = 6$$

$$b = -4$$

b)

$$\begin{array}{r} 2x^2 - 5x + 2 \\ 3x+2 \overline{) 6x^3 - 11x^2 - 4x + 4} \\ \underline{6x^3 + 4x^2} \phantom{+ 4} \\ -15x^2 - 4x \phantom{+ 4} \\ \underline{-15x^2 - 10x} \phantom{+ 4} \\ 6x + 4 \\ \underline{6x + 4} \\ 0 \end{array}$$



## Question 4 continued

$$(3x+2)(2x^2 - 5x + 2)$$

$$(3x+2)(2x-1)(x-2)$$



D 4 2 8 2 6 A 0 0 2 2

5. The first three terms of a geometric series are  $4p$ ,  $(3p + 15)$  and  $(5p + 20)$  respectively, where  $p$  is a **positive** constant.

(a) Show that  $11p^2 - 10p - 225 = 0$  (4)

(b) Hence show that  $p = 5$  (2)

(c) Find the common ratio of this series. (2)

(d) Find the sum of the first ten terms of the series, giving your answer to the nearest integer. (3)

$$a) \quad \frac{3p+15}{4p} = \frac{5p+20}{3p+15}$$

$$(3p+15)(3p+15) = 4p(5p+20)$$

$$9p^2 + 90p + 225 = 20p^2 + 80p$$

$$0 = 11p^2 - 10p - 225$$

$$b) \quad (11p + 45)(p - 5) = 0$$

$$p = \frac{-45}{11} \quad p = 5$$

$p$  cannot be negative  $\therefore p = 5$

$$c) \quad r = \frac{3p+15}{4p}$$

$$= \frac{3(5)+15}{4(5)} = \frac{3}{2}$$

$$d) \quad S_n = \frac{a(1-r^n)}{1-r} \quad a = 4p$$

$$S_{10} = \frac{20(1 - (\frac{3}{2})^{10})}{1 - (\frac{3}{2})} \quad = 4(5)$$

$$= 20$$

$$= 2267 \quad (\text{nearest integer})$$



6. Given that  $\log_3 x = a$ , find in terms of  $a$ ,

(a)  $\log_3(9x)$  (2)

(b)  $\log_3\left(\frac{x^5}{81}\right)$  (3)

giving each answer in its simplest form.

(c) Solve, for  $x$ ,

$$\log_3(9x) + \log_3\left(\frac{x^5}{81}\right) = 3$$

giving your answer to 4 significant figures. (4)

$$\begin{aligned} \text{a) } \log_3 9x &= \log_3 9 + \log_3 x \\ &= 2 + a \end{aligned}$$

$$\begin{aligned} \text{b) } \log_3\left(\frac{x^5}{81}\right) &= \log_3 x^5 - \log_3 81 \\ &= 5 \log_3 x - \log_3 81 \\ &= 5a - 4 \end{aligned}$$

$$\begin{aligned} \text{c) } 2 + a + 5a - 4 &= 3 \\ 6a - 2 &= 3 \\ 6a &= 5 \\ a &= \frac{5}{6} \end{aligned}$$

$$\log_3 x = \frac{5}{6}$$

$$\begin{aligned} x &= 3^{\frac{5}{6}} \\ &= 2.498 \quad (4 \text{ sf}) \end{aligned}$$





7.

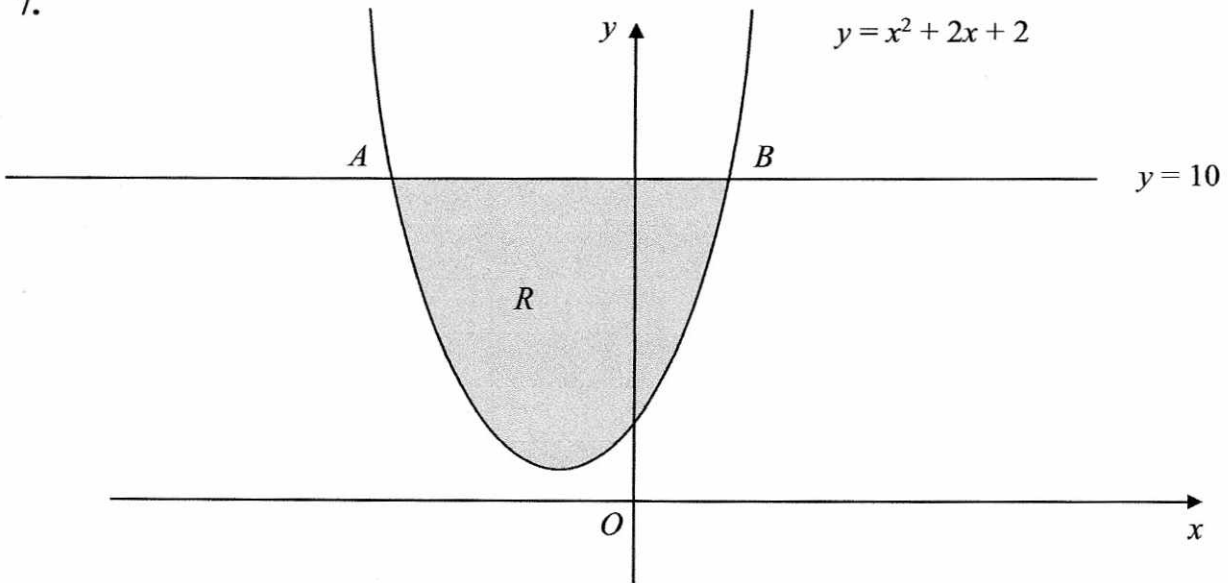


Figure 1

The line with equation  $y = 10$  cuts the curve with equation  $y = x^2 + 2x + 2$  at the points  $A$  and  $B$  as shown in Figure 1. The figure is not drawn to scale.

- (a) Find by calculation the  $x$ -coordinate of  $A$  and the  $x$ -coordinate of  $B$ . (2)

The shaded region  $R$  is bounded by the line with equation  $y = 10$  and the curve as shown in Figure 1.

- (b) Use calculus to find the exact area of  $R$ . (7)

$$\begin{aligned}
 \text{a)} \quad & 10 = x^2 + 2x + 2 \\
 & 0 = x^2 + 2x - 8 \\
 & 0 = (x + 4)(x - 2) \\
 & \quad \underline{\underline{x = -4}} \quad \quad \underline{\underline{x = 2}} \\
 & \quad \quad \quad \underline{\underline{A}} \quad \quad \quad \underline{\underline{B}}
 \end{aligned}$$

$$\begin{aligned}
 \text{b)} \quad & \int_{-4}^2 (x^2 + 2x - 8) \, dx \\
 & \left[ \frac{x^3}{3} + \frac{2x^2}{2} - 8x + C \right]_{-4}^2 \\
 & \left[ \frac{1}{3}x^3 + x^2 - 8x \right]_{-4}^2
 \end{aligned}$$

$$\text{or} \parallel \int_{-4}^2 (10 - x^2 - 2x - 2) \, dx$$



## Question 7 continued

$$\left[ \frac{1}{3}(2)^3 + (2)^2 - 8(2) \right] - \left[ \frac{1}{3}(-4)^3 + (-4)^2 - 8(-4) \right]$$

$$\left[ \frac{-28}{3} \right] - \left[ \frac{80}{3} \right] = -36$$

36 units<sup>2</sup>



8.

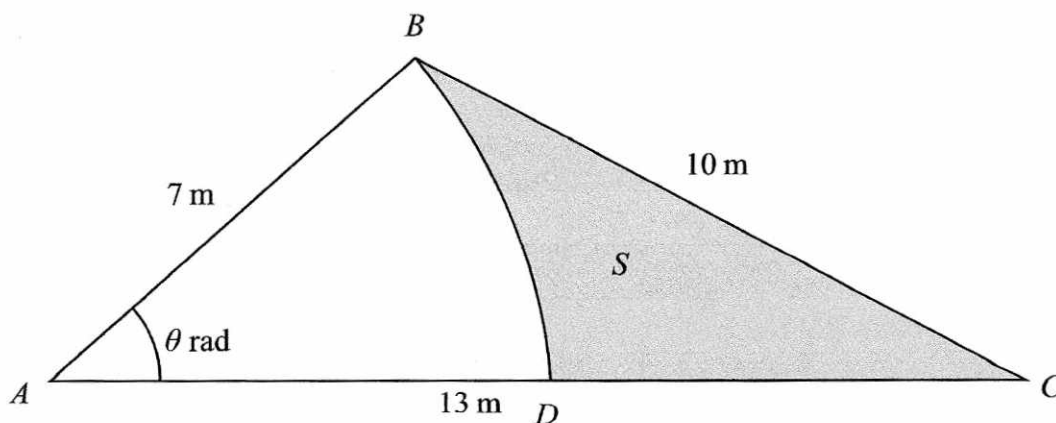


Figure 2

Figure 2 shows the design for a triangular garden  $ABC$  where  $AB = 7$  m,  $AC = 13$  m and  $BC = 10$  m.

Given that angle  $BAC = \theta$  radians,

- (a) show that, to 3 decimal places,  $\theta = 0.865$  (3)

The point  $D$  lies on  $AC$  such that  $BD$  is an arc of the circle centre  $A$ , radius 7 m.

The shaded region  $S$  is bounded by the arc  $BD$  and the lines  $BC$  and  $DC$ . The shaded region  $S$  will be sown with grass seed, to make a lawned area.

Given that 50 g of grass seed are needed for each square metre of lawn,

- (b) find the amount of grass seed needed, giving your answer to the nearest 10 g. (7)

$$\begin{aligned} \text{a) } \cos \theta &= \frac{b^2 + c^2 - a^2}{2bc} \\ &= \frac{(7)^2 + (13)^2 - (10)^2}{2(7)(13)} \\ \cos \theta &= 59/91 \\ \theta &= 0.8653789549^\circ \\ &= 0.865 \text{ (3dp)} \end{aligned}$$

$$\text{b) Area of } S = \text{Area of triangle} - \text{Area of sector}$$

$$\begin{aligned} \text{Area of triangle} &= \frac{1}{2} ab \sin C \\ &= \frac{1}{2} (7)(13) \sin(0.865) \\ &= 34.6298345 \text{ m}^2 \end{aligned}$$



## Question 8 continued

$$\begin{aligned}\text{Area of sector} &= \frac{\theta}{2} r^2 \\ &= \frac{0.865}{2} (7)^2 \\ &= 21.1925 \text{ m}^2\end{aligned}$$

$$\text{Area of } S = 13.437... \text{ m}^2$$

$$\begin{aligned}\text{Grass seed needed} &= 13.437 \times 50 \\ &= 671.8667... \text{ g} \\ &= 670 \text{ g (nearest 10g)}\end{aligned}$$



9. (i) Solve, for  $0 \leq \theta < 180^\circ$

$$\sin(2\theta - 30^\circ) + 1 = 0.4$$

giving your answers to 1 decimal place.

(5)

(ii) Find all the values of  $x$ , in the interval  $0 \leq x < 360^\circ$ , for which

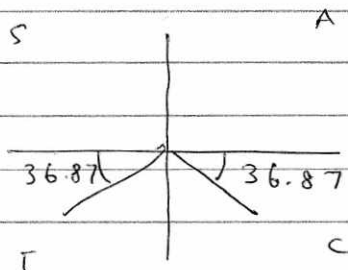
$$9\cos^2 x - 11\cos x + 3\sin^2 x = 0$$

giving your answers to 1 decimal place.

(7)

You must show clearly how you obtained your answers.

i/  $\sin(2\theta - 30) = -0.6$   
 $2\theta - 30 = -36.87$



$$2\theta - 30 = 216.87, 323.13$$

$$\theta = \underline{123.4}, \underline{176.6}$$

ii/  $\cos^2 \theta + \sin^2 \theta = 1$   $[\sin^2 \theta = 1 - \cos^2 \theta]$

$$9\cos^2 x - 11\cos x + 3(1 - \cos^2 x) = 0$$

$$9\cos^2 x - 11\cos x + 3 - 3\cos^2 x = 0$$

$$6\cos^2 x - 11\cos x + 3 = 0$$

$$(3\cos x - 1)(2\cos x - 3) = 0$$

$$\cos x = \frac{1}{3} \quad \cos x = \frac{3}{2}$$

no solutions

$$x = \underline{70.5}, \underline{289.5}$$

