Write your name here

Surname	Other names

Pearson Edexcel GCE

Centre Number | Candidate Number

Core Mathematics C2

Advanced Subsidiary

Wednesday 25 May 2016 – Morning
Time: 1 hour 30 minutes

Paper Reference
6664/01

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B). Coloured pencils and highlighter pens must not be used.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided *– there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information

- The total mark for this paper is 75.
- The marks for **each** question are shown in brackets *– use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

PEARSON

Leave blank

1. A geometric series has first term a and common ratio $r = \frac{3}{4}$

The sum of the first 4 terms of this series is 175

(a) Show that $a = 64$ **(2)**

(b) Find the sum to infinity of the series. **(2)**

(c) Find the difference between the 9th and 10th terms of the series.
Give your answer to 3 decimal places. **(3)**

1a) $S_n = \frac{a(1-r^n)}{1-r}$

$S_4 = 175$

$175 = \frac{a\left(1-\left(\frac{3}{4}\right)^4\right)}{1-\frac{3}{4}}$

$175 = \frac{\frac{175}{256}a}{1/4}$

$\frac{175}{4} = \frac{175}{256}a$

$a = 64$

b) $S_\infty = \frac{a}{1-r}$

$= \frac{64}{1-\frac{3}{4}}$

$= 256$

Question 1 continued

c/ $U_n = ar^{n-1}$

$U_9 = 64\left(\frac{3}{4}\right)^8$

$= \frac{6561}{1024}$

$U_{10} = 64\left(\frac{3}{4}\right)^9$

$= 4.805419922$

$U_9 - U_{10} = 1.601806641$

$= 1.602$ (3dp)

Q1

(Total 7 marks)

Leave blank

Leave blank

2. The curve C has equation

$$y = 8 - 2^{x-1}, \qquad 0 \leqslant x \leqslant 4$$

(a) Complete the table below with the value of y corresponding to $x = 1$

x	0	1	2	3	4
y	7.5	7	6	4	0

(1)

(b) Use the trapezium rule, with all the values of y in the completed table, to find an approximate value for $\int_0^4 \left(8 - 2^{x-1}\right)\,dx$

(3)

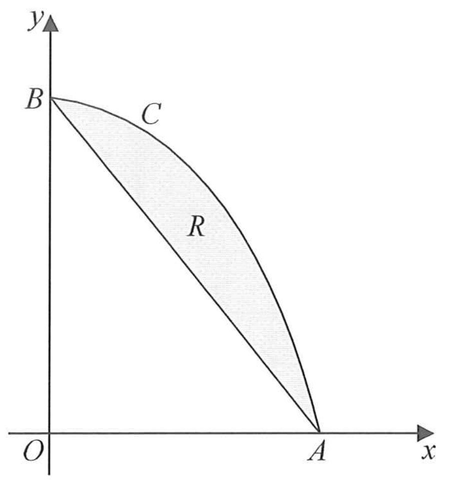


Figure 1

Figure 1 shows a sketch of the curve C with equation $y = 8 - 2^{x-1}$, $0 \leqslant x \leqslant 4$

The curve C meets the x-axis at the point A and meets the y-axis at the point B.

The region R, shown shaded in Figure 1, is bounded by the curve C and the straight line through A and B.

(c) Use your answer to part (b) to find an approximate value for the area of R.

(2)

b/ $1\left(\frac{7.5}{2} + 7 + 6 + 4 + \frac{0}{2}\right)$

$= \underline{\underline{20.75}}$

c/ Area of triangle $= \frac{1}{2}$ base $\times$ height

$= \frac{1}{2}(4)(7.5)$

$= 15$

P 4 6 7 1 6 A 0 4 3 2

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Leave blank

Question 2 continued

$20.75 - 15 = \underline{\underline{5.75 \text{ units}^2}}$

3.

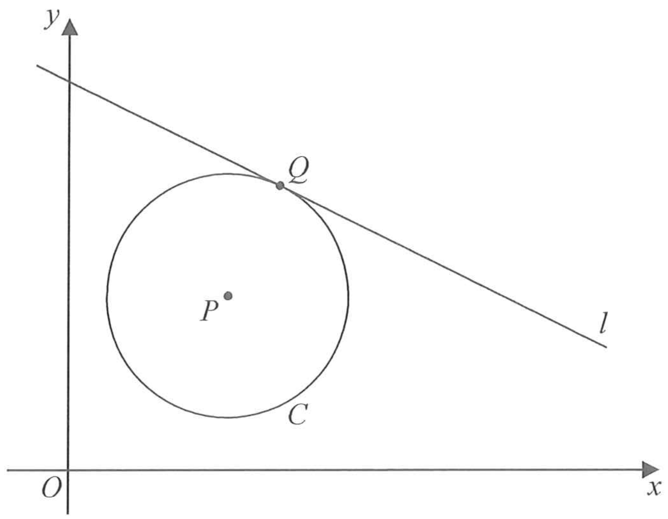


Figure 2

The circle C has centre $P(7, 8)$ and passes through the point $Q(10, 13)$, as shown in Figure 2.

(a) Find the length PQ, giving your answer as an exact value.

(2)

(b) Hence write down an equation for C.

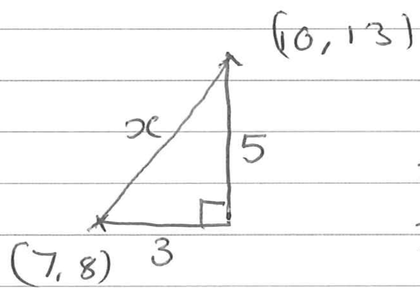
(2)

The line l is a tangent to C at the point Q, as shown in Figure 2.

(c) Find an equation for l, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

(4)

a)



$x^2 = 5^2 + 3^2$

$x = \sqrt{34}$

b) $(x - 7)^2 + (y - 8)^2 = 34$

c) gradient of PQ. $\quad \overset{x_1 \ y_1}{(7, 8)} \quad \overset{x_2 \ y_2}{(10, 13)}$

$m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{13 - 8}{10 - 7} = \dfrac{5}{3}$

Leave blank

Question 3 continued

perp. gradient $= -\frac{3}{5}$

$y = -\frac{3}{5}x + c$ $\quad \overset{x\ \ y}{(10, 13)}$

$13 = -\frac{3}{5}(10) + c$

$13 = -6 + c$

$c = 19$

$y = -\frac{3}{5}x + 19$

$5y = -3x + 95$

$3x + 5y - 95 = 0$

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

P 4 6 7 1 6 A 0 9 3 2

Leave blank

4.

$$f(x) = 6x^3 + 13x^2 - 4$$

(a) Use the remainder theorem to find the remainder when f(x) is divided by $(2x + 3)$.
(2)

(b) Use the factor theorem to show that $(x + 2)$ is a factor of f(x).
(2)

(c) Factorise f(x) completely.
(4)

a) $f(-3/2) = 6\left(\frac{-3}{2}\right)^3 + 13\left(\frac{-3}{2}\right)^2 - 4$
$= \underline{\underline{5}}$

b/ $f(-2) = 6(-2)^3 + 13(-2)^2 - 4$
$\underline{\underline{= 0}}$ $\therefore (x+2)$ is a factor

c/

	$6x^2$	x	-2
x	$6x^3$	x^2	$-2x$
$+2$	$12x^2$	$2x$	-4

$(x+2)(6x^2 + x - 2)$

$(x+2)(3x+2)(2x-1)$

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

P 4 6 7 1 6 A 0 1 2 3 2

Leave blank

5. (a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$(2 - 9x)^4$$

giving each term in its simplest form.

(4)

$$f(x) = (1 + kx)(2 - 9x)^4, \text{ where } k \text{ is a constant}$$

The expansion, in ascending powers of x, of $f(x)$ up to and including the term in x^2 is

$$A - 232x + Bx^2$$

where A and B are constants.

(b) Write down the value of A.

(1)

(c) Find the value of k.

(2)

(d) Hence find the value of B.

(2)

4^{th} line $\quad nC0 \quad nC1 \quad nC2$

$1 \quad 4 \quad 6$

$1(2)^4 + 4(2)^3(-9x) + 6(2)^2(-9x)^2$

$16 - 288x + 1944x^2$

b/ $(1+kx)(16 - 288x + 1944x^2)$

$16 - 288x + 1944x^2 + 16kx - 288kx^2 + 1944kx^3$

$16 - 288x + 16kx + 1944x^2 - 288kx^2$

$A = 16$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Leave blank

Question 5 continued

Equating x terms

c/ $-288 + 16k = -232$

$16k = 56$

$k = \frac{7}{2}$

d/ $1944 - 288\left(\frac{7}{2}\right) = B$

$936 = B$

6. (i) Solve, for $-\pi < \theta \leqslant \pi$,

RADIANS

$$1 - 2\cos\left(\theta - \frac{\pi}{5}\right) = 0$$

giving your answers in terms of π.

(3)

(ii) Solve, for $0 \leqslant x < 360°$,

DEGREES

$$4\cos^2 x + 7\sin x - 2 = 0$$

giving your answers to one decimal place.

(Solutions based entirely on graphical or numerical methods are not acceptable.)

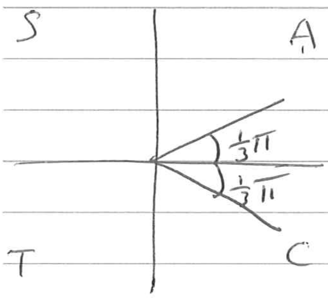
(6)

i/ $1 - 2\cos\left(\theta - \frac{\pi}{5}\right) = 0$

$1 = 2\cos\left(\theta - \frac{\pi}{5}\right)$

$\frac{1}{2} = \cos\left(\theta - \frac{\pi}{5}\right)$

$\cos^{-1}\left(\frac{1}{2}\right) = \theta - \frac{\pi}{5}$



$\frac{1}{3}\pi, \ 2\pi - \frac{1}{3}\pi$

$\theta - \frac{\pi}{5} = \frac{1}{3}\pi, \ \frac{5}{3}\pi, \ -\frac{1}{3}\pi$

$\theta = \frac{8}{15}\pi, \ -\frac{2}{15}\pi$

ii/ $4\cos^2 x + 7\sin x - 2 = 0$

$4(1 - \sin^2 x) + 7\sin x - 2 = 0$

$4 - 4\sin^2 x + 7\sin x - 2 = 0$

$-4\sin^2 x + 7\sin x + 2 = 0$

$4\sin^2 x - 7\sin x - 2 = 0$

$(4\sin x + 1)(\sin x - 2) = 0$

$\sin x = -\frac{1}{4}$ $\qquad$ $\sin x = 2$

$\times$

$x = -14.4775121 9$

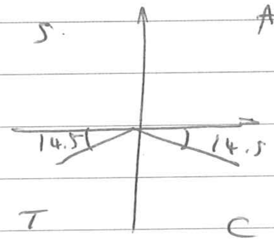
P 4 6 7 1 6 A 0 1 8 3 2

Leave blank

Question 6 continued

$x = -14.5,\ 194.5,\ 345.5$

$= \underline{\underline{194.5^\circ}},\ \underline{\underline{345.5^\circ}}$



Leave blank

7.

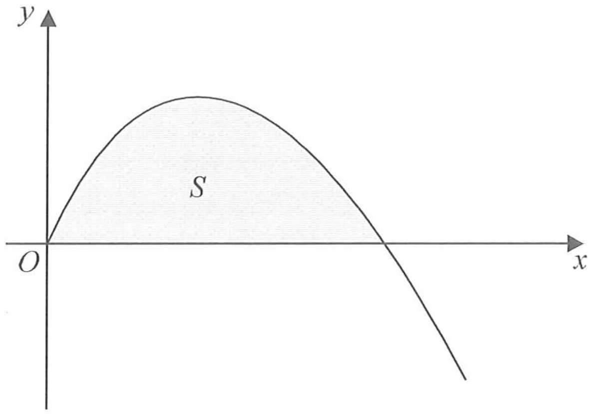


Figure 3

Figure 3 shows a sketch of part of the curve with equation

$$y = 3x - x^{\frac{3}{2}}, \qquad x \geqslant 0$$

The finite region S, bounded by the x-axis and the curve, is shown shaded in Figure 3.

(a) Find

$$\int \left(3x - x^{\frac{3}{2}}\right) \mathrm{d}x$$

(3)

(b) Hence find the area of S.

(3)

a) $\int 3x - x^{\frac{3}{2}} \, dx$

$\frac{3x^2}{2} - \frac{x^{\frac{5}{2}}}{5/2} + c$

$\frac{3}{2}x^2 - \frac{2}{5}x^{\frac{5}{2}} + c$

b/ crosses x when $y=0$

$0 = 3x - x^{3/2}$

$0 = x(3 - x^{\frac{1}{2}})$

$x = 0 \qquad x^{\frac{1}{2}} = 3$

$x = 9$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 7 continued

$$\left[\frac{3}{2}x^2 - \frac{2}{5}x^{\frac{5}{2}} + c\right]_0^9$$

$$\left(\frac{3}{2}(9)^2 - \frac{2}{5}(9)^{\frac{5}{2}}\right) - (0)$$

$$= 24.3 \text{ units}^2$$

P46716A02332

Turn over

8. (i) Given that

$$\log_3(3b + 1) - \log_3(a - 2) = -1, \qquad a > 2$$

express b in terms of a.

(3)

(ii) Solve the equation

$$2^{2x+5} - 7(2^x) = 0$$

giving your answer to 2 decimal places.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

i/ $\log_3\left(\frac{3b+1}{a-2}\right) = -1$

$\frac{3b+1}{a-2} = 3^{-1}$

$\frac{3b+1}{a-2} = \frac{1}{3}$

$3(3b+1) = a-2$

$9b + 3 = a - 2$

$9b = a - 5$

$b = \frac{a-5}{9}$

ii/ $2^5(2^{2x}) - 7(2^x) = 0$

$32(2^{2x}) - 7(2^x) = 0$

$32y^2 - 7y = 0$ let $2^x = y$

$y(32y - 7) = 0$

$y = 0 \quad y = 7/32$

$2^x = 0 \quad 2^x = \frac{7}{32}$

$\times \quad x = \log_2 7/32$

$= -2.19$

9.

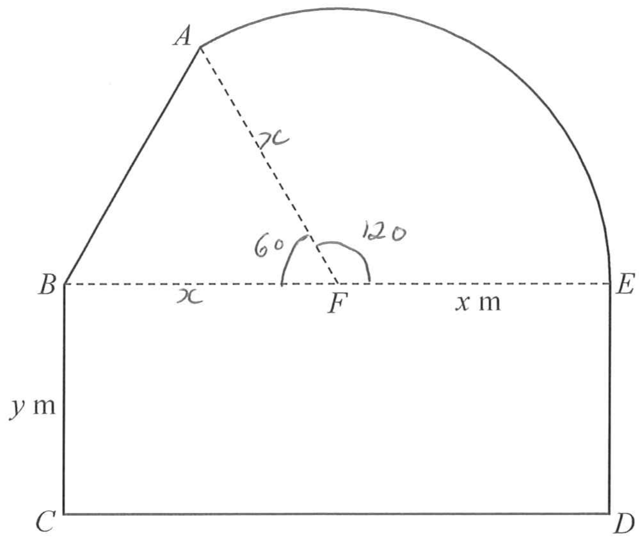


Diagram not drawn to scale

Figure 4

Figure 4 shows a plan view of a sheep enclosure.

The enclosure $ABCDEA$, as shown in Figure 4, consists of a rectangle $BCDE$ joined to an equilateral triangle BFA and a sector FEA of a circle with radius x metres and centre F.

The points B, F and E lie on a straight line with $FE = x$ metres and $10 \leqslant x \leqslant 25$

(a) Find, in m^2, the exact area of the sector FEA, giving your answer in terms of x, in its simplest form.
(2)

Given that $BC = y$ metres, where $y > 0$, and the area of the enclosure is $1000\ \text{m}^2$,

(b) show that

$$y = \frac{500}{x} - \frac{x}{24}\left(4\pi + 3\sqrt{3}\right)$$

(3)

(c) Hence show that the perimeter P metres of the enclosure is given by

$$P = \frac{1000}{x} + \frac{x}{12}\left(4\pi + 36 - 3\sqrt{3}\right)$$

(3)

(d) Use calculus to find the minimum value of P, giving your answer to the nearest metre.
(5)

(e) Justify, by further differentiation, that the value of P you have found is a minimum.
(2)

Leave blank

Question 9 continued

a) $\frac{120}{360} \times \pi x^2$

$\frac{1}{3}\pi x^2$

b) Area of Rectangle $= 2xy$

Area of Triangle $= \frac{1}{2}ab\sin C$

$= \frac{1}{2}(x)(x)\sin(60)$

$= \frac{\sqrt{3}}{4}x^2$

Total Area $= \frac{\sqrt{3}}{4}x^2 + 2xy + \frac{1}{3}\pi x^2$

$1000 = \frac{\sqrt{3}}{4}x^2 + 2xy + \frac{1}{3}\pi x^2$

$1000 - \frac{\sqrt{3}}{4} - \frac{1}{3}\pi x^2 = 2xy$

$2xy = 1000 - \frac{\sqrt{3}}{4}x^2 - \frac{1}{3}\pi x^2$

$y = \frac{500}{x} - \frac{\sqrt{3}x^2}{8x} - \frac{\frac{1}{3}\pi x^2}{2x}$

$= \frac{500}{x} - \frac{\sqrt{3}x^2}{8x} - \frac{\pi x}{6}$

$= \frac{500}{x} - \frac{\sqrt{3}x}{8} - \frac{\pi x}{6}$

$= \frac{500}{x} - \frac{3\sqrt{3}x}{24} - \frac{4\pi x}{24}$

$= \frac{500}{x} - \frac{x}{24}(3\sqrt{3} + 4\pi)$

$= \frac{500}{x} - \frac{x}{24}(4\pi + 3\sqrt{3})$

Question 9 continued

c/ $\text{perimeter} = x + y + 2x + y + \frac{120}{360} \cdot 2\pi(x)$

$$= 3x + 2y + \frac{2}{3}\pi x$$

$$= 3x + 2\left(\frac{500}{x} - \frac{x}{24}\left(4\pi + 3\sqrt{3}\right)\right) + \frac{2}{3}\pi x$$

$$= 3x + \frac{1000}{x} - \frac{x}{12}\left(4\pi + 3\sqrt{3}\right) + \frac{2}{3}\pi x$$

$$= 3x + \frac{1000}{x} - \frac{4\pi x}{12} - \frac{3\sqrt{3}x}{12} + \frac{2}{3}\pi x$$

$$= 3x + \frac{1000}{x} - \frac{\pi x}{3} - \frac{\sqrt{3}x}{4} + \frac{2}{3}\pi x$$

$$= \frac{1000}{x} + 3x + \frac{1}{3}\pi x - \frac{\sqrt{3}}{4}x$$

$$= \frac{1000}{x} + \frac{x}{12}\left(36 + 4\pi\cancel{x} - 3\sqrt{3}\cancel{x}\right)$$

d/ $\frac{dP}{dx} = -1000x^{-2} + \frac{36 + 4\pi - 3\sqrt{3}}{12}$

$$0 = \frac{-1000}{x^2} + \frac{36 + 4\pi - 3\sqrt{3}}{12}$$

$$\frac{1000}{x^2} = \frac{36 + 4\pi - 3\sqrt{3}}{12}$$

$$\frac{12000}{x^2} = 36 + 4\pi - 3\sqrt{3}$$

$$\frac{12000}{36 + 4\pi - 3\sqrt{3}} = x^2$$

$$x^2 = 276.6875635$$

$$x = 16.63392808$$

$$P = \frac{1000}{\text{Ans}} + \frac{\text{Ans}}{12}\left(4\pi + 36 - 3\sqrt{3}\right) = 120.2361817$$

Question 9 continued

$= 120$ m (nearest metre)

e) $\frac{d^2P}{dx^2} = 2000x^{-3}$

when $x = 16.6$ $= 2000(16.6)^{-3}$ $= 0.4372...$

$=$ positive answer $\therefore$ minimum

Leave blank

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA