1 The third term of an arithmetic series is -10 and the sum of the first eight terms of the series is 16.

a Find the first term and common difference of the series.

b Find the smallest value of n for which the nth term of the series is greater than 300.

2 The third and seventh terms of an arithmetic series are $\frac{5}{6}$ and $2\frac{1}{3}$ respectively.

a Find the first term and common difference of the series.

b Show that the sum of the first n terms of the series is given by

$$kn(9n - 5),$$

where k is an exact fraction to be found.

3 An arithmetic series has first term a and common difference d.

Given that the sum of the first nine terms of the series is 126,

a show that $a + 4d = 14$.

Given also that the sum of the first 15 terms of the series is 277.5,

b find the values of a and d,

c find the sum of the first 32 terms of the series.

4 Three consecutive terms of an arithmetic series are $(7k - 1)$, $(5k + 3)$ and $(4k + 1)$ respectively.

a Find the value of the constant k.

b Find the smallest positive term in the series.

Given also that the series has r positive terms,

c show that the sum of the positive terms of the series is given by $r(4r - 3)$.

5 **a** Evaluate

$$\sum_{r=1}^{30} 4r .$$

b Using your answer to part **a**, or otherwise, evaluate

i $\sum_{r=1}^{30} (4r+1)$,

ii $\sum_{r=1}^{30} (8r-5)$.

6 Ahmed begins making annual payments into a savings scheme. He pays in £500 in the first year and the amount he pays in increases by £40 in each subsequent year.

a Find the amount that Ahmed pays into the scheme in the eighth year.

b Show that during the first n years, Ahmed pays in a total amount, in pounds, of $20n(n + 24)$.

Carol starts making payments into a similar scheme at the same time as Ahmed. She pays in £400 in the first year, with the amount increasing by £60 each year.

c By forming and solving a suitable equation, find the number of years of paying into the schemes after which Carol and Ahmed will have paid in the same amount in total.

7 **a** Prove from first principles that the sum of the first n positive even numbers is $n(n + 1)$.

b Find the sum of the integers between 200 and 800 that are not divisible by 4.

8 **a** State the formula for the sum, S_n, of the first n terms of an arithmetic series with first term a and common difference d.

b Prove that for any arithmetic series,

$$S_8 = 2(S_6 - S_2).$$

c An arithmetic series has first term 40 and common difference −3.

Find the sum of the positive terms of the series.

9 The first and fourth terms of an arithmetic series are x and $(2x + 3)$ respectively.

a Find expressions in terms of x for

i the seventh term of the series,

ii the common difference of the series,

iii the sum of the first ten terms of the series.

Given also that the 20th term of the series is 52,

b find the value of x.

10 An arithmetic series has first term a and common difference d.

Given that the sum of the first twenty terms of the series is equal to the sum of the next ten terms of the series, show that the ratio $a : d = 11 : 2$.

11 The sum, S_n, of the first n terms of a series is given by

$$S_n = 2n(16 - n).$$

a Show that the sixth term of the series is 10.

b Find an expression for the nth term of the series in the form $a + bn$.

c Hence, prove that the series is arithmetic.

12 A publisher decides to start producing calendars.

The publisher sells 2400 calendars during the first year of production and forecasts that the number it will sell in subsequent years will increase by 250 each year.

a According to this forecast,

i find how many calendars the publisher will sell during the sixth year of production,

ii show that the publisher will sell a total of 35 250 calendars during the first ten years of production.

b If the number of calendars sold in subsequent years increases by C each year, instead of by 250, find to the nearest unit the value of C such that the publisher would sell a total of 40 000 calendars during the first ten years of production.

13 **a** Prove that the sum, S_r, of the first r terms of an arithmetic series with first term a and rth term l is given by

$$S_r = \tfrac{1}{2}r(a + l).$$

b The 18th term of an arithmetic series is 68 and the sum of the first 18 terms of the series is 153.

Find the first term of the series.