ALGEBRA

C1

a Factorise fully the expression

 $20x - 2x^2 - 6x^3$.

b Hence, find all solutions to the equation

$$20x - 2x^2 - 6x^3 = 0$$

- 2 *A* is the point (-2, 1) and *B* is the point (6, *k*). **a** Show that $AB^2 = k^2 - 2k + 65$. Given also that AB = 10,
 - **b** find the possible values of *k*.
- **3** Solve the equations

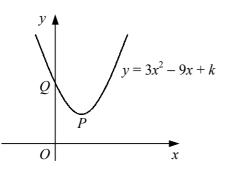
a
$$x - \frac{5}{x} = 4$$

b $\frac{9}{5-x} - 1 = 2x$

- **a** Find the coordinates of the turning point of the curve with equation $y = 3 5x 2x^2$.
 - **b** Sketch the curve $y = 3 5x 2x^2$, showing the coordinates of any points of intersection with the coordinate axes.
- 5 Find in the form $k\sqrt{2}$ the solutions of the equation

$$2x^2 + 5\sqrt{2}x - 6 = 0.$$

6



The diagram shows the curve with equation $y = 3x^2 - 9x + k$ where k is a constant.

a Find the *x*-coordinate of the turning point of the curve, *P*.

Given that the *y*-coordinate of *P* is $\frac{17}{4}$,

- **b** find the coordinates of the point Q where the curve crosses the y-axis.
- 7 By letting $y = 2^x$, or otherwise, solve the equation $2^{2x} - 10(2^x) + 16 = 0.$
- 8 Given that the equation

$$kx^2 - 2x + 3 - 2k = 0$$

has equal roots, find the possible values of the constant k.

9

$$f(x) \equiv 3 + 4x - x^2$$

- **a** Express f(x) in the form $a(x+b)^2 + c$.
- **b** State the coordinates of the turning point of the curve y = f(x).
- **c** Solve the equation f(x) = 2, giving your answers in the form $d + e\sqrt{5}$.
- 10 Giving your answers in terms of surds, solve the equations
 - **a** $3x^2 5x + 1 = 0$
 - $\mathbf{b} \quad \frac{x}{x+2} = \frac{3}{x-1}$
- 11 **a** By completing the square, find, in terms of k, the solutions of the equation $x^2 - 4kx + 6 = 0.$
 - **b** Using your answers to part **a**, solve the equation

$$x^2 - 12x + 6 = 0.$$

12 a Find in the form $a + b\sqrt{3}$, where a and b are integers, the values of x such that

$$2x^2 - 12x = 6.$$

b Solve the equation

$$2y^3 + y^2 - 15y = 0.$$

Labelling the coordinates of any points of intersection with the coordinate axes, sketch the curves
a y = (x + 1)(x − p) where p > 0,
b y = (x + q)² where q < 0.

14

$$f(x) \equiv 2x^2 - 6x + 5$$

a Find the values of A, B and C such that

$$\mathbf{f}(x) \equiv A(x+B)^2 + C$$

- **b** Hence deduce the minimum value of f(x).
- 15 **a** Given that $t = x^{\frac{1}{3}}$ express $x^{\frac{2}{3}}$ in terms of t.
 - **b** Hence, or otherwise, solve the equation

$$2\,x^{\frac{2}{3}}\,+\,x^{\frac{1}{3}}\,-\,6\,=\,0.$$

- 16 a Express $k^2 8k + 20$ in the form $a(k+b)^2 + c$, where a, b and c are constants.
 - **b** Hence prove that the equation

$$x^2 - kx + 2k = 5$$

has real and distinct roots for all real values of k.

17 a Show that

 $(x^{2} + 2x - 3)(x^{2} - 3x - 4) \equiv x^{4} - x^{3} - 13x^{2} + x + 12.$

- **b** Hence solve the equation
 - $x^4 x^3 13x^2 + x + 12 = 0.$