

1. Write

$$\sqrt{75} - \sqrt{27}$$

in the form $k\sqrt{x}$, where k and x are integers.

(2)

$$\sqrt{75} - \sqrt{27}$$

$$\sqrt{25}\sqrt{3} - \sqrt{9}\sqrt{3}$$

$$5\sqrt{3} - 3\sqrt{3}$$

$$\underline{\underline{2\sqrt{3}}}$$

Q1

(Total 2 marks)

2. Find

$$\int (8x^3 + 6x^{\frac{1}{2}} - 5) dx$$

giving each term in its simplest form.

(4)

$$\frac{8x^4}{4} + \frac{6x^{\frac{3}{2}}}{\frac{3}{2}} - 5x + C$$

$$\underline{\underline{2x^4 + 4x^{\frac{3}{2}} - 5x + C}}$$

Q2

(Total 4 marks)

3. Find the set of values of x for which

(a) $3(x-2) < 8-2x$

(2)

(b) $(2x-7)(1+x) < 0$

(3)

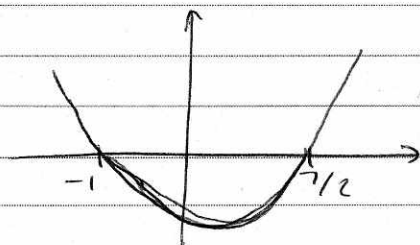
(c) both $3(x-2) < 8-2x$ and $(2x-7)(1+x) < 0$

(1)

$$\begin{aligned} \text{a)} \quad & 3(x-2) < 8-2x \\ & 3x-6 < 8-2x \\ & 5x-6 < 8 \\ & 5x < 14 \\ & \underline{\underline{x < 14/5}} \end{aligned}$$

$$\text{b)} \quad (2x-7)(1+x) < 0$$

$$x = \frac{7}{2} \quad x = -1$$



$$\underline{\underline{-1 < x < 7/2}}$$

$$\text{c)} \quad \underline{\underline{-1 < x < 14/5}}$$

4. (a) Show that $x^2 + 6x + 11$ can be written as

$$(x + p)^2 + q$$

where p and q are integers to be found.

(2)

- (b) In the space at the top of page 7, sketch the curve with equation $y = x^2 + 6x + 11$, showing clearly any intersections with the coordinate axes.

(2)

- (c) Find the value of the discriminant of $x^2 + 6x + 11$

(2)

a) $x^2 + 6x + 11$

$$(x + 3)^2 - (3)^2 + 11$$

$$(x + 3)^2 - 9 + 11$$

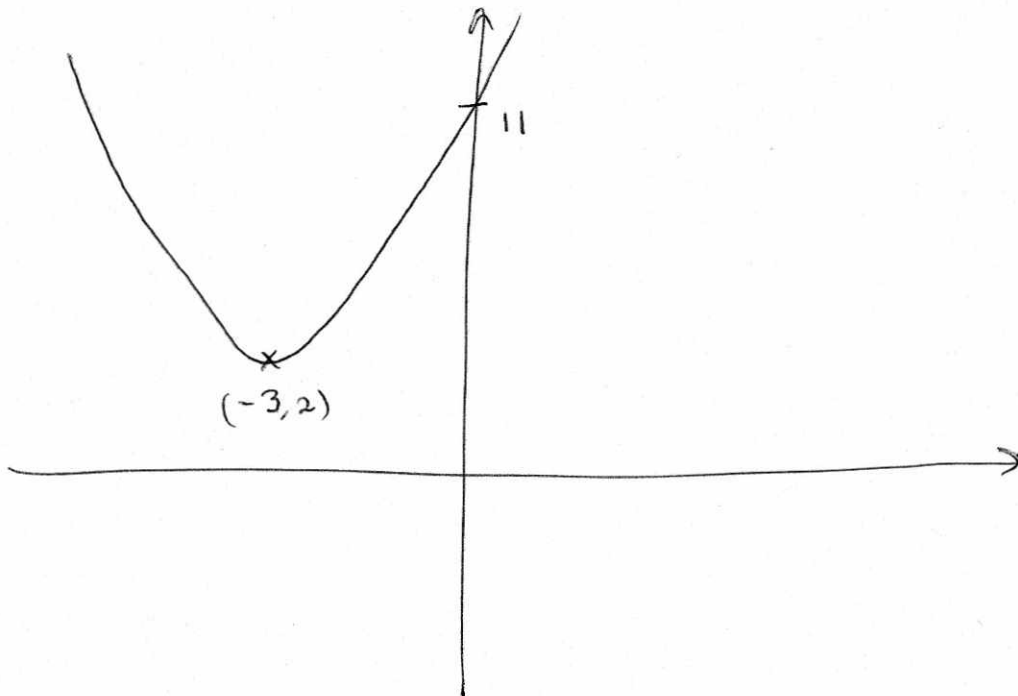
$$\underline{(x + 3)^2 + 2}$$

b) Min point at $(-3, 2)$

crosses y when $x = 0$

$$y = (0)^2 + 6(0) + 11$$
$$= 11$$

Question 4 continued



$$c) \quad b^2 - 4ac$$

$$a=1 \quad b=6 \quad c=11$$

$$(6)^2 - 4(1)(11)$$

$$36 - 44$$

$$b^2 - 4ac = \underline{\underline{-8}}$$

Q4

(Total 6 marks)

5. A sequence of positive numbers is defined by

$$a_{n+1} = \sqrt{a_n^2 + 3}, \quad n \geq 1,$$
$$a_1 = 2$$

(a) Find a_2 and a_3 , leaving your answers in surd form.

(2)

(b) Show that $a_5 = 4$

(2)

$$\begin{aligned} \text{a)} \quad a_2 &= \sqrt{(a_1)^2 + 3} \\ &= \sqrt{(2)^2 + 3} \\ &= \sqrt{7} \end{aligned}$$

$$\begin{aligned} a_3 &= \sqrt{(a_2)^2 + 3} \\ &= \sqrt{(\sqrt{7})^2 + 3} \\ &= \sqrt{10} \end{aligned}$$

$$\begin{aligned} \text{b)} \quad a_4 &= \sqrt{(a_3)^2 + 3} \\ &= \sqrt{(\sqrt{10})^2 + 3} \\ &= \sqrt{13} \end{aligned}$$

$$\begin{aligned} a_5 &= \sqrt{(a_4)^2 + 3} \\ &= \sqrt{(\sqrt{13})^2 + 3} \\ &= \sqrt{16} \\ &= \underline{\underline{4}} \end{aligned}$$

6.

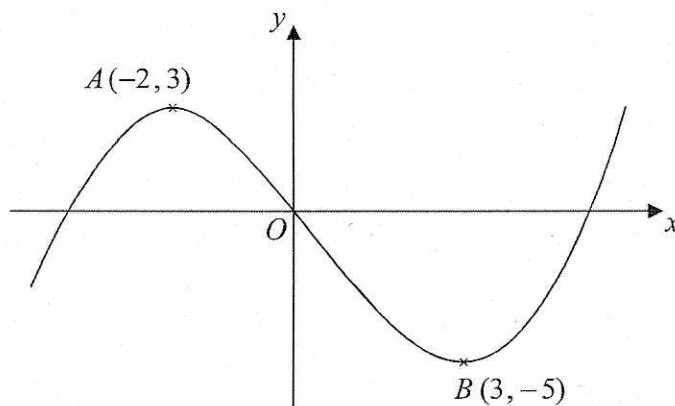


Figure 1

Figure 1 shows a sketch of the curve with equation $y = f(x)$. The curve has a maximum point A at $(-2, 3)$ and a minimum point B at $(3, -5)$.

On separate diagrams sketch the curve with equation

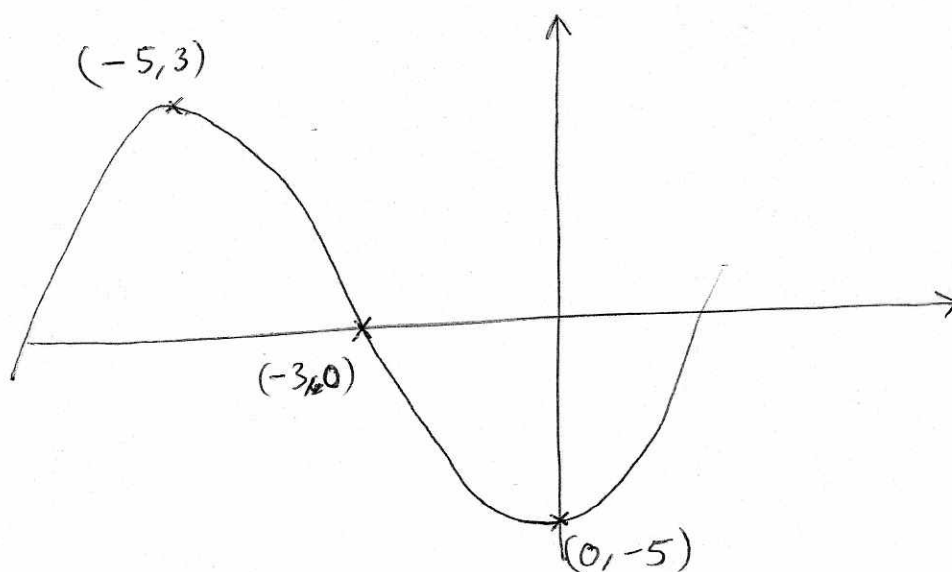
(a) $y = f(x+3)$ (3)

(b) $y = 2f(x)$ (3)

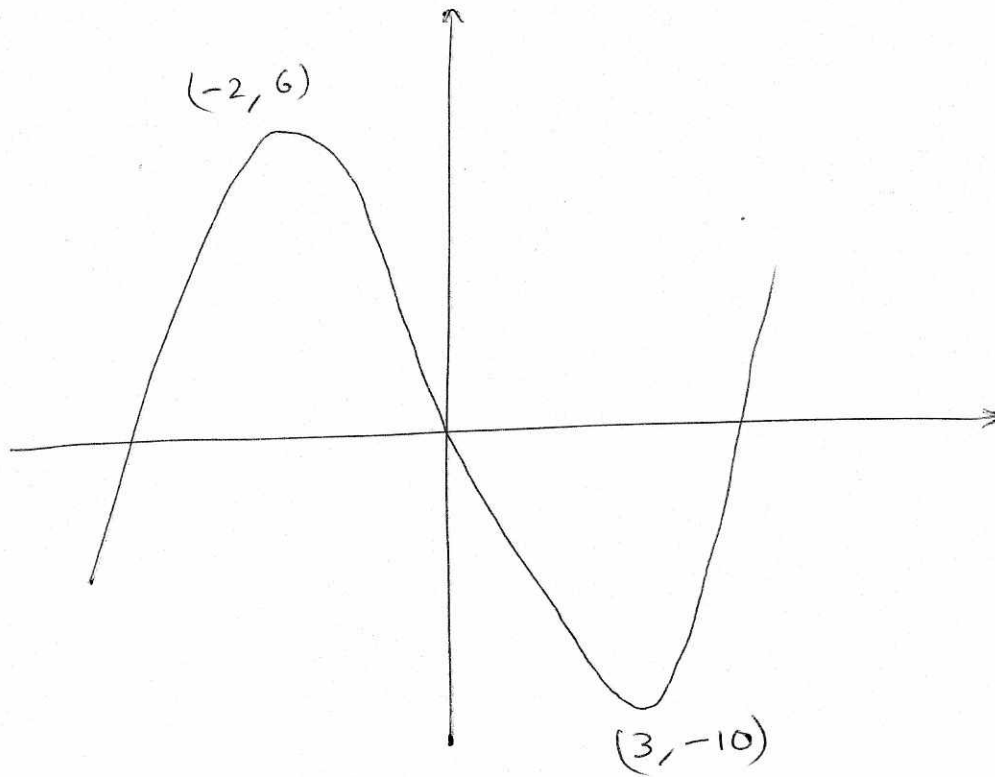
On each diagram show clearly the coordinates of the maximum and minimum points.

The graph of $y = f(x) + a$ has a minimum at $(3, 0)$, where a is a constant.

(c) Write down the value of a . (1)



Question 6 continued



c/ a = 5

(Total 7 marks)

Q6

7. Given that

$$y = 8x^3 - 4\sqrt{x} + \frac{3x^2 + 2}{x}, \quad x > 0$$

find $\frac{dy}{dx}$.

(6)

$$y = 8x^3 - 4x^{1/2} + 3x + 2x^{-1}$$

$$\frac{dy}{dx} = 24x^2 + 2x^{-1/2} + 3 - 2x^{-2}$$

8. (a) Find an equation of the line joining $A(7, 4)$ and $B(2, 0)$, giving your answer in the form $ax+by+c=0$, where a, b and c are integers. (3)

(b) Find the length of AB , leaving your answer in surd form. (2)

The point C has coordinates $(2, t)$, where $t > 0$, and $AC = AB$.

(c) Find the value of t . (1)

(d) Find the area of triangle ABC . (2)

$$a) \quad m = \frac{y_2 - y_1}{x_2 - x_1} \quad \begin{matrix} x_1, y_1 \\ (2, 0) \end{matrix} \quad \begin{matrix} x_2, y_2 \\ (7, 4) \end{matrix}$$

$$= \frac{4 - 0}{7 - 2} = \frac{4}{5}$$

$$y = \frac{4}{5}x + c \quad (2, 0)$$

$$0 = \frac{4}{5}(2) + c$$

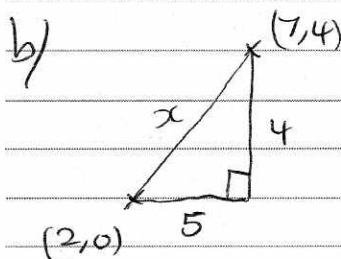
$$0 = \frac{8}{5} + c$$

$$c = -\frac{8}{5}$$

$$y = \frac{4}{5}x - \frac{8}{5}$$

$$5y = 4x - 8$$

$$\underline{0 = 4x - 5y - 8}$$



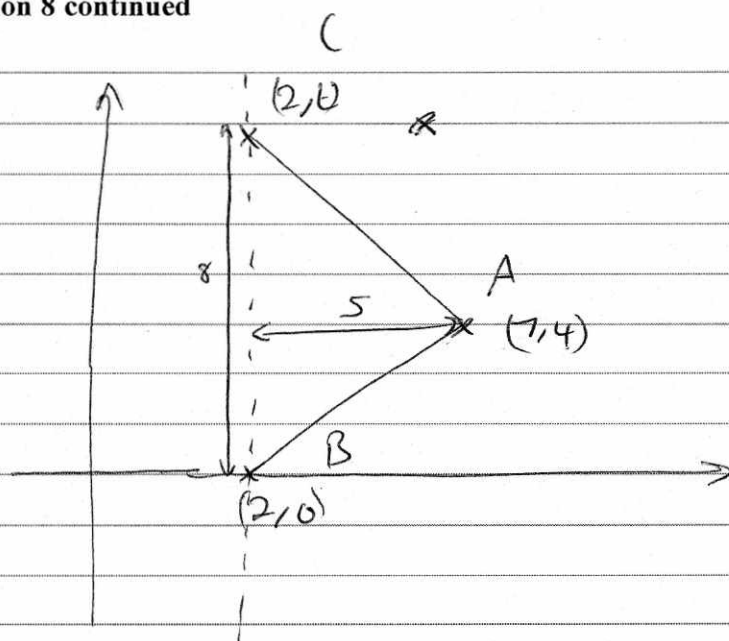
$$x^2 = 5^2 + 4^2$$

$$x^2 = 25 + 16$$

$$x^2 = 41$$

$$\underline{x = \sqrt{41}}$$

Question 8 continued



$$\underline{\underline{h=8}}$$

$$d/ \text{ Area} = \frac{1}{2} \text{ base} \times \text{height}$$

$$= \frac{1}{2} \times 8 \times 5$$

$$= \underline{\underline{20 \text{ units}^2}}$$

9. A farmer has a pay scheme to keep fruit pickers working throughout the 30 day season. He pays £ a for their first day, £ $(a+d)$ for their second day, £ $(a+2d)$ for their third day, and so on, thus increasing the daily payment by £ d for each extra day they work.

A picker who works for all 30 days will earn £40.75 on the final day.

- (a) Use this information to form an equation in a and d .

(2)

A picker who works for all 30 days will earn a total of £1005

- (b) Show that $15(a+40.75) = 1005$

(2)

- (c) Hence find the value of a and the value of d .

(4)

$$\begin{aligned} \text{a/} \quad u_{30} &= 40.75 \\ u_n &= a + (n-1)d \\ 40.75 &= a + 29d \end{aligned}$$

$$\begin{aligned} \text{b/} \quad S_n &= \frac{n}{2}(2a + (n-1)d) \\ S_{30} &= \frac{30}{2}(2a + 29d) \end{aligned}$$

$$\begin{aligned} 1005 &= 15(2a + 29d) \\ 1005 &= 15(a + (a + 29d)) \\ &\quad \quad \quad \uparrow \\ &\quad \quad \quad 40.75 = a + 29d \\ 1005 &= 15(a + 40.75) \end{aligned}$$

$$\begin{aligned} \text{c/} \quad \frac{1005}{15} &= a + 40.75 \\ 67 &= a + 40.75 \\ \underline{\underline{\pounds 26.25}} &= a \end{aligned}$$

$$\begin{aligned} 40.75 &= 26.25 + 29d \\ 14.50 &= 29d \\ \underline{\underline{d}} &= \underline{\underline{50p}} \end{aligned}$$

10. (a) On the axes below sketch the graphs of

(i) $y = x(4-x)$

(ii) $y = x^2(7-x)$

showing clearly the coordinates of the points where the curves cross the coordinate axes.

(5)

(b) Show that the x -coordinates of the points of intersection of

$$y = x(4-x) \quad \text{and} \quad y = x^2(7-x)$$

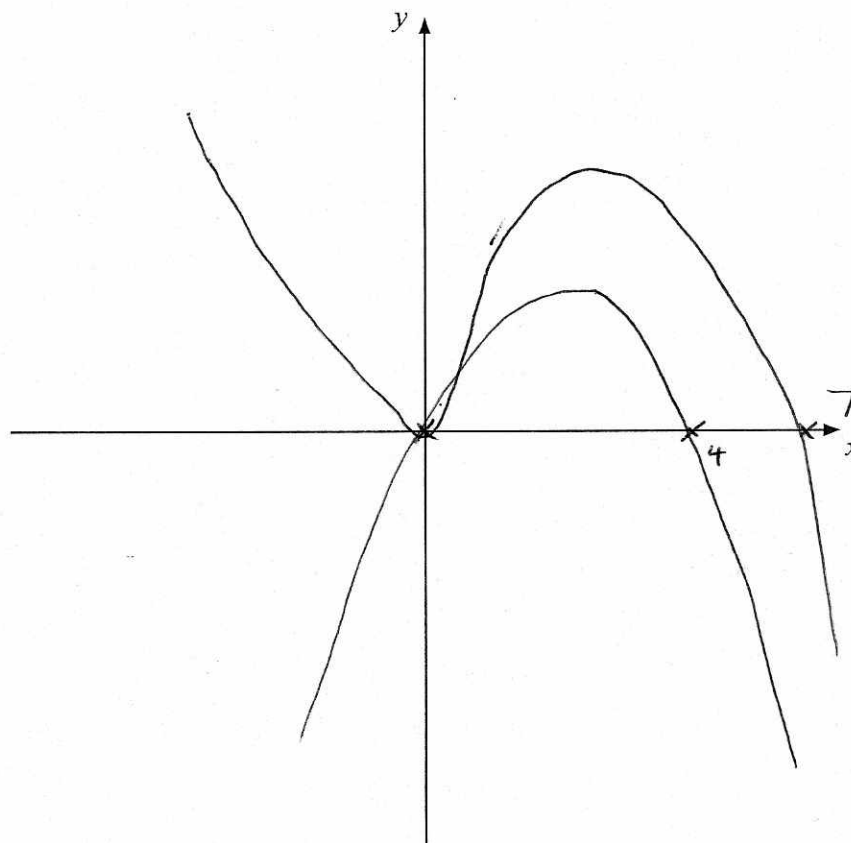
are given by the solutions to the equation $x(x^2 - 8x + 4) = 0$

(3)

The point A lies on both of the curves and the x and y coordinates of A are both positive.

(c) Find the exact coordinates of A , leaving your answer in the form $(p + q\sqrt{3}, r + s\sqrt{3})$, where p, q, r and s are integers.

(7)



Question 10 continued

$$y = x(4-x) \quad y = x^2(7-x)$$

$$x(4-x) = x^2(7-x)$$

$$4x - x^2 = 7x^2 - x^3$$

$$x^3 - x^2 + 4x = 7x^2$$

$$x^3 - 8x^2 + 4x = 0$$

$$\underline{x(x^2 - 8x + 4) = 0}$$

$$c) \quad x^2 - 8x + 4 = 0$$

$$(x-4)^2 - (4)^2 + 4 = 0$$

$$(x-4)^2 - 16 + 4 = 0$$

$$(x-4)^2 - 12 = 0$$

$$(x-4)^2 = 12$$

$$x-4 = \pm\sqrt{12}$$

$$x = 4 \pm\sqrt{12}$$

$$= 4 \pm\sqrt{4\sqrt{3}}$$

$$= 4 \pm 2\sqrt{3}$$

$$y = x(4-x)$$

$$= (4-2\sqrt{3})(4-(4-2\sqrt{3}))$$

$$= (4-2\sqrt{3})(4-4+2\sqrt{3})$$

$$= (4-2\sqrt{3})(2\sqrt{3})$$

$$= 8\sqrt{3} - 12$$

$$\underline{(4-2\sqrt{3}, -12+8\sqrt{3})}$$

11. The curve C has equation $y=f(x)$, $x > 0$, where

$$\frac{dy}{dx} = 3x - \frac{5}{\sqrt{x}} - 2$$

Given that the point $P(4, 5)$ lies on C , find

(a) $f(x)$,

(5)

(b) an equation of the tangent to C at the point P , giving your answer in the form $ax+by+c=0$, where a , b and c are integers.

(4)

$$\begin{aligned} \text{11a)} \quad \frac{dy}{dx} &= 3x - 5x^{-1/2} - 2 \\ y &= \frac{3x^2}{2} - \frac{5x^{1/2}}{1/2} - 2x + C \\ \text{2 y} \\ (4, 5) \quad y &= \frac{3x^2}{2} - 10x^{1/2} - 2x + C \\ 5 &= \frac{3(4)^2}{2} - 10(4)^{1/2} - 2(4) + C \\ 5 &= \frac{48}{2} - 20 - 8 + C \\ 5 &= 24 - 20 - 8 + C \\ 5 &= -4 + C \\ 9 &= C \\ f(x) &= \frac{3x^2}{2} - 10x^{1/2} - 2x + 9 \end{aligned}$$

$$\begin{aligned} \text{b/ when } x=4 \quad \frac{dy}{dx} &= 3(4) - \frac{5}{\sqrt{4}} - 2 \\ &= 12 - \frac{5}{2} - 2 \\ &= 10 - \frac{5}{2} \\ &= \frac{15}{2} \end{aligned}$$

$$m = \frac{15}{2}$$

Question 11 continued

$$y = \frac{15}{2}x + c \quad (4, 5)$$

$$5 = \frac{15}{2}(4) + c$$

$$5 = 30 + c$$

$$-25 = c$$

$$y = \frac{15}{2}x - 25$$

$$2y = 15x - 50$$

$$\underline{\underline{15x - 2y - 50 = 0}}$$