Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Wednesday 15 May 2019

Morning (Time: 2 hours) | Paper Reference **8MA0/01**

Mathematics

Advanced Subsidiary
Paper 1: Pure Mathematics

You must have:
Mathematical Formulae and Statistical Tables, calculator

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 16 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

Pearson

1. The line l_1 has equation $2x + 4y - 3 = 0$

The line l_2 has equation $y = mx + 7$, where m is a constant.

Given that l_1 and l_2 are perpendicular,

(a) find the value of m.

(2)

The lines l_1 and l_2 meet at the point P.

(b) Find the x coordinate of P.

(2)

a/ $2x + 4y - 3 = 0$

$4y = -2x + 3$

$y = -\frac{1}{2}x + \frac{3}{4}$

l_1: $m = -\frac{1}{2}$ $\quad$ l_2: $\underline{\underline{m = 2}}$

b/ l_1: $y = -\frac{1}{2}x + \frac{3}{4}$ $\quad$ l_2: $y = 2x + 7$

$-\frac{1}{2}x + \frac{3}{4} = 2x + 7$

$\frac{3}{4} = \frac{5}{2}x + 7$

$-\frac{25}{4} = \frac{5}{2}x$

$\underline{\underline{x = -\frac{5}{2}}}$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

2. Find, using algebra, all real solutions to the equation

(i) $16a^2 = 2\sqrt{a}$

(4)

(ii) $b^4 + 7b^2 - 18 = 0$

(4)

i/ $16a^2 = 2a^{\frac{1}{2}}$

$16a^{\frac{3}{2}} = 2$

$a^{\frac{3}{2}} = \frac{1}{8}$

$a^{\frac{1}{2}} = \frac{1}{2}$

$a = \frac{1}{4}$ OR $a = 0$

ii/ $(b^2+9)(b^2-2) = 0$

$b^2 = -9$ $b^2 = 2$

NO SOL. $b = \pm\sqrt{2}$

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

3. (a) Given that k is a constant, find

$$\int\left(\frac{4}{x^3}+kx\right)dx$$

simplifying your answer.

(3)

(b) Hence find the value of k such that

$$\int_{0.5}^{2}\left(\frac{4}{x^3}+kx\right)dx=8$$

(3)

a/ $\int 4x^{-3} + kx \; dx$

$-2x^{-2} + \frac{1}{2}kx^2 + c$

b/ $\left[-2x^{-2} + \frac{1}{2}kx^2\right]_{0.5}^{2} = 8$

$\left(-2(2)^{-2} + \frac{1}{2}k(2)^2\right) - \left(-2(0.5)^{-2} + \frac{1}{2}k(0.5)^2\right) = 8$

$-\frac{1}{2} + 2k - \left(-8 + \frac{1}{8}k\right) = 8$

$-\frac{1}{2} + 2k + 8 - \frac{1}{8}k = 8$

$\frac{15}{8}k = \frac{1}{2}$

$k = \frac{8}{30}$

$= \frac{4}{15}$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

4. A tree was planted in the ground.
Its height, H metres, was measured t years after planting.

Exactly 3 years after planting, the height of the tree was 2.35 metres.
Exactly 6 years after planting, the height of the tree was 3.28 metres.

Using a linear model,

(a) find an equation linking H with t.

(3)

The height of the tree was approximately 140 cm when it was planted.

(b) Explain whether or not this fact supports the use of the linear model in part (a).

(2)

a) $H = mt + c$

$2.35 = 3m + c$

$3.28 = 6m + c$
$2.35 = 3m + c$

$0.93 = 3m$
$m = 0.31$

$2.35 = 3(0.31) + c$
$2.35 = 0.93 + c$
$c = 1.42$

$H = 0.31t + 1.42$

b) Yes, with the model the height of the tree is 142 cm when $t = 0$.
(140 cm is close to 142 cm)

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

5. A curve has equation

$$y = 3x^2 + \frac{24}{x} + 2 \qquad x > 0$$

(a) Find, in simplest form, $\frac{dy}{dx}$

(3)

(b) Hence find the exact range of values of x for which the curve is increasing.

(2)

a/ $y = 3x^2 + 24x^{-1} + 2$

$\frac{dy}{dx} = 6x - 24x^{-2}$

b/ Increasing when $\frac{dy}{dx} > 0$

$6x - 24x^{-2} > 0$

$6x^3 - 24 > 0$

$x^3 - 4 > 0$

$x^3 > 4$

$x > \sqrt[3]{4}$

6.

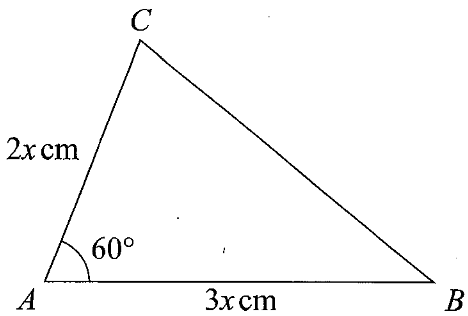


Figure 1

Figure 1 shows a sketch of a triangle ABC with $AB = 3x$ cm, $AC = 2x$ cm and angle $CAB = 60°$

Given that the area of triangle ABC is $18\sqrt{3}$ cm^2

(a) show that $x = 2\sqrt{3}$

(3)

(b) Hence find the exact length of BC, giving your answer as a simplified surd.

(3)

a/ $A = \frac{1}{2}ab \sin C$

$18\sqrt{3} = \frac{1}{2}(2x)(3x)\sin(60)$

$18\cancel{\sqrt{3}} = 3x^2 \cdot \frac{\cancel{\sqrt{3}}}{2}$

$6 = \frac{1}{2}x^2$

$12 = x^2$

$x = \sqrt{12} = \underline{\underline{2\sqrt{3}}}$

$b = 4\sqrt{3}$

$c = 6\sqrt{3}$

b/ $a^2 = b^2 + c^2 - 2bc \cos A$

$= (4\sqrt{3})^2 + (6\sqrt{3})^2 - 2(4\sqrt{3})(6\sqrt{3})\cos(60)$

$a^2 = 84$

$a = \sqrt{84} = \underline{\underline{2\sqrt{21}}}$ cm

7. The curve C has equation

$$y = \frac{k^2}{x} + 1 \qquad x \in \mathbb{R},\ x \neq 0$$

where k is a constant.

(a) Sketch C stating the equation of the horizontal asymptote.

(3)

The line l has equation $y = -2x + 5$

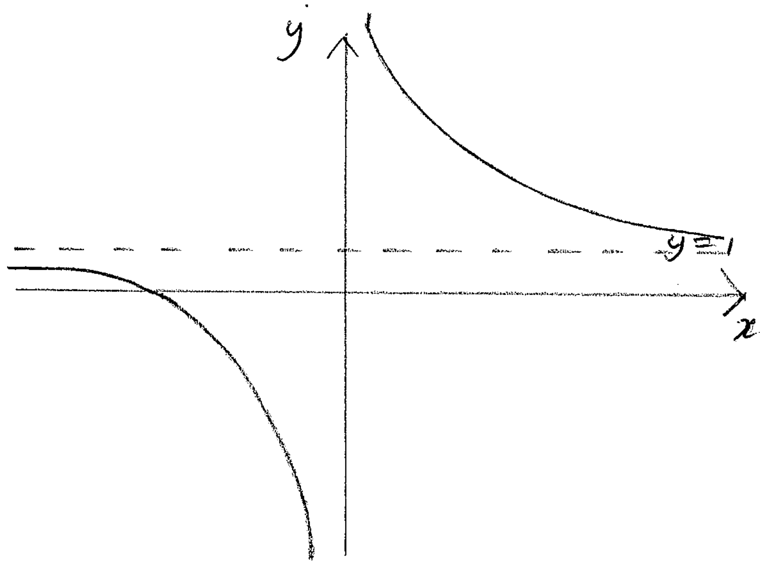
(b) Show that the x coordinate of any point of intersection of l with C is given by a solution of the equation

$$2x^2 - 4x + k^2 = 0$$

(2)

(c) Hence find the exact values of k for which l is a tangent to C.

(3)



Question 7 continued

b/ $\frac{k^2}{x} + 1 = -2x + 5$

$k^2 + x = -2x^2 + 5x$

$2x^2 + x + k^2 = 5x$

$2x^2 - 4x + k^2 = 0$

c/ tangent where there is 1 solution

$b^2 - 4ac = 0$

$(-4)^2 - 4(2)(k^2) = 0$

$16 - 8k^2 = 0$

$16 = 8k^2$

$2 = k^2$

$k = \pm\sqrt{2}$

(Total for Question 7 is 8 marks)

8. (a) Find the first 3 terms, in ascending powers of x, of the binomial expansion of

$$\left(2+\frac{3x}{4}\right)^6$$

giving each term in its simplest form.

(4)

(b) Explain how you could use your expansion to estimate the value of 1.925^6
You do not need to perform the calculation.

(1)

1 6 15

a/ $(2)^6 + 6(2)^5\left(\frac{3x}{4}\right) + 15(2)^4\left(\frac{3x}{4}\right)^2$

$64 + 144x + 135x^2$

b/ $2 + \frac{3x}{4} = 1.925$

$\frac{3}{4}x = -\frac{3}{40}$

$x = -0.1$

We could substitute $x=-0.1$ into the expansion

9. A company started mining tin in Riverdale on 1st January 2019.

A model to find the total mass of tin that will be mined by the company in Riverdale is given by the equation

$$T = 1200 - 3(n - 20)^2$$

where T tonnes is the total mass of tin mined in the n years after the start of mining.

Using this model,

(a) calculate the mass of tin that will be mined up to 1st January 2020, **(1)**

(b) deduce the maximum total mass of tin that could be mined, **(1)**

(c) calculate the mass of tin that will be mined in 2023. **(2)**

(d) State, giving reasons, the limitation on the values of n. **(2)**

a/ $T = 1200 - 3(1-20)^2$

$= \underline{117 \text{ tonnes}}$

b/ $\underline{1200 \text{ tonnes}}$

c/ Up to 1 Jan 2023 $= 1200 - 3(4-20)^2$
$= 432$ tonnes

Up to 1 Jan 2024 $= 1200 - 3(5-20)^2$
$= 525$ tonnes

$525 - 432 = \underline{93 \text{ tonnes}}$

total

d/ After 20 years the total amount will start decreasing, this is not possible.

$\therefore \; n \leqslant 20$

10. A circle C has equation

$$x^2 + y^2 - 4x + 8y - 8 = 0$$

(a) Find

(i) the coordinates of the centre of C,

(ii) the exact radius of C.

(3)

The straight line with equation $x = k$, where k is a constant, is a tangent to C.

(b) Find the possible values for k.

(2)

a) $x^2 - 4x + y^2 + 8y - 8 = 0$

$(x-2)^2 - 4 + (y+4)^2 - 16 - 8 = 0$

$(x-2)^2 + (y+4)^2 = 28$

i) $\cancel{x}$ $(2, -4)$

ii) $\sqrt{28}$ or $2\sqrt{7}$

b) $x = 2 + \sqrt{28}$ or $x = 2 - \sqrt{28}$

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

11. $$f(x) = 2x^3 - 13x^2 + 8x + 48$$

(a) Prove that $(x - 4)$ is a factor of $f(x)$.

(2)

(b) Hence, using algebra, show that the equation $f(x) = 0$ has only two distinct roots.

(4)

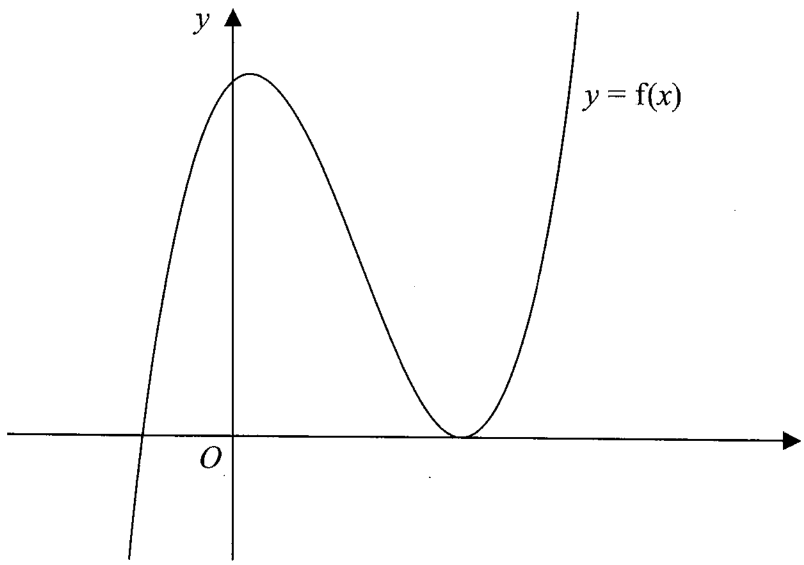


Figure 2

Figure 2 shows a sketch of part of the curve with equation $y = f(x)$.

(c) Deduce, giving reasons for your answer, the number of real roots of the equation

$$2x^3 - 13x^2 + 8x + 46 = 0$$

(2)

Given that k is a constant and the curve with equation $y = f(x + k)$ passes through the origin,

(d) find the two possible values of k.

(2)

a) $f(4) = 2(4)^3 - 13(4)^2 + 8(4) + 48$
$= 0$

$f(4) = 0$ therefore $(x - 4)$ is a factor of $f(x)$

Question 11 continued

b/

$$\begin{array}{r|l} & 2x^2 - 5x - 12 \\ x-4 & 2x^3 - 13x^2 + 8x + 48 \\ & 2x^3 - 8x^2 \\ & -5x^2 + 8x \\ & -5x^2 + 20x \\ & -12x + 48 \\ & -12x + 48 \\ & 0 \end{array}$$

$(x-4)(2x^2 - 5x - 12) = 0$

$(x-4)(x-4)(2x+3) = 0$

$(2x+3)(x-4)^2 = 0$

$x = -\frac{3}{2}$ $x = 4$

c/ $f(x) - 2$

Graph moves down by 2 units $\therefore$ 3 real roots (3 points or intersection with x axis)

d/ $f(x - \frac{3}{2})$ or $f(x+4)$

$k = -\frac{3}{2}$ or $k = 4$

12. (a) Show that

$$\frac{10\sin^2\theta - 7\cos\theta + 2}{3 + 2\cos\theta} \equiv 4 - 5\cos\theta$$

(4)

(b) Hence, or otherwise, solve, for $0 \leqslant x < 360°$, the equation

$$\frac{10\sin^2 x - 7\cos x + 2}{3 + 2\cos x} = 4 + 3\sin x$$

(3)

a/ $\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$$\frac{10(1-\cos^2\theta) - 7\cos\theta + 2}{3 + 2\cos\theta}$$

$$\frac{10 - 10\cos^2\theta - 7\cos\theta + 2}{3 + 2\cos\theta}$$

$$\frac{-10\cos^2\theta - 7\cos\theta + 12}{3 + 2\cos\theta}$$

$$\frac{(-2\cos\theta - 3)(5\cos\theta - 4)}{3 + 2\cos\theta}$$

$$-\frac{(\cancel{3 + 2\cos\theta})(5\cos\theta - 4)}{\cancel{3 + 2\cos\theta}}$$

$$-(5\cos\theta - 4)$$

$$-5\cos\theta + 4$$

$$\underline{\underline{4 - 5\cos\theta}}$$

b/ $4 - 5\cos x = 4 + 3\sin x$

$-5\cos x = 3\sin x$

$\frac{-5}{3} = \tan x$

Question 12 continued

$\tan x = -\frac{5}{3}$

$x = \tan^{-1}\left(-\frac{5}{3}\right)$

$= -59.0,\ \underline{121.0^\circ},\ \underline{301.0^\circ}$

$\underline{121.0^\circ},\ \underline{301.0^\circ}$

(Total for Question 12 is 7 marks)

13.

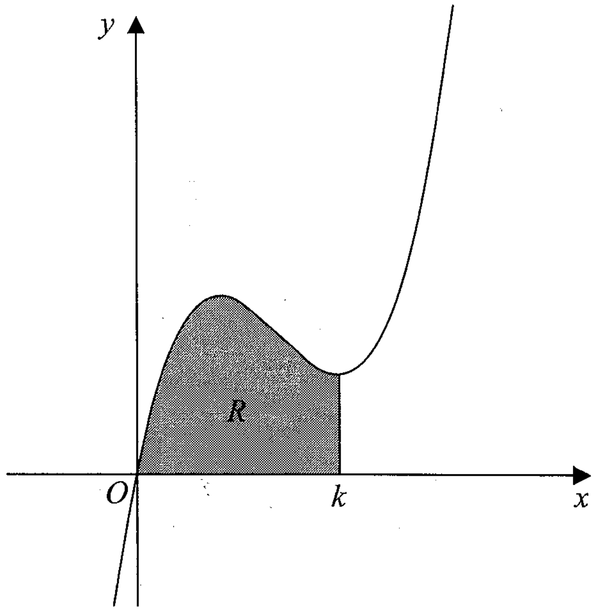


Figure 3

Figure 3 shows a sketch of part of the curve with equation

$$y = 2x^3 - 17x^2 + 40x$$

The curve has a minimum turning point at $x = k$.

The region R, shown shaded in Figure 3, is bounded by the curve, the x-axis and the line with equation $x = k$.

Show that the area of R is $\dfrac{256}{3}$

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(7)

Min point where $\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = 6x^2 - 34x + 40$$

$$6x^2 - 34x + 40 = 0$$
$$3x^2 - 17x + 20 = 0$$
$$\cancel{(3x \quad 20)(x \quad 1) = 0}$$
$$(3x - 5)(x - 4) = 0$$
$$x = \frac{5}{3} \quad x = 4$$

$$\underline{k = 4}$$

DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA DO NOT WRITE IN THIS AREA

Question 13 continued

$$\int_0^4 2x^3 - 17x^2 + 40x \; dx =$$

$$\left[\frac{1}{2}x^4 - \frac{17}{3}x^3 + 20x^2\right]_0^4$$

$$\frac{1}{2}(4)^4 - \frac{17}{3}(4)^3 + 20(4)^2 - 0$$

$$= \frac{256}{3}$$

(Total for Question 13 is 7 marks)

14. The value of a car, £V, can be modelled by the equation

$$V = 15700e^{-0.25t} + 2300 \qquad t \in \mathbb{R},\ t \geqslant 0$$

where the age of the car is t years.

Using the model,

(a) find the initial value of the car.

(1)

Given the model predicts that the value of the car is decreasing at a rate of £500 per year at the instant when $t = T$,

(b) (i) show that

$$3925e^{-0.25T} = 500$$

(ii) Hence find the age of the car at this instant, giving your answer in years and months to the nearest month.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(6)

The model predicts that the value of the car approaches, but does not fall below, £A.

(c) State the value of A.

(1)

(d) State a limitation of this model.

(1)

a/ $15700 + 2300 = \text{£}18000$

b/ when $t = T$ $\quad \frac{dV}{dt} = -500$

$$\frac{dV}{dt} = -3925e^{-0.25t}$$

$$-3925e^{-0.25T} = -500$$

$$3925e^{-0.25T} = 500$$

ii/ $$e^{-0.25T} = \frac{20}{157}$$

$$-0.25T = \ln\frac{20}{157}$$

Question 14 continued

$T = -4 \ln \frac{20}{157}$

$= 8.242$ years

$0.242 \times 12 = 2.9$

$= 8$ years 3 months

c/ £2300

d/ The model only takes the age of the car into account - other factors such as mileage are ignored.

(Total for Question 14 is 9 marks)

15. Given $n \in \mathbb{N}$, prove that $n^3 + 2$ is not divisible by 8

(4)

if n is even

$(2m)^3 + 2$

$8m^3 + 2$

n^3+2 is 2 more than a multiple of 8

if n is odd

$(2m+1)^3 + 2$

~~(2m+1)(2m+1)(2m+1~~

$8m^3 + 3(2m)^2 + 3(2m) + 1 + 2$

$8m^3 + 12m^2 + 6m + 1 + 2$

$8m^3 + 12m^2 + 6m + 3$

~~f2~~ $2(4m^3 + 6m^2 + 3m) + 3$

even + 3 = odd

$n^3 + 2$ is odd, so not divisible by 8.

$\therefore$ n^3+2 is not divisible by 8 (given $n \in \mathbb{N}$)

DO NOT WRITE IN THIS AREA

16. (i) Two non-zero vectors, **a** and **b**, are such that

$$|\mathbf{a} + \mathbf{b}| = |\mathbf{a}| + |\mathbf{b}|$$

Explain, geometrically, the significance of this statement.

(1)

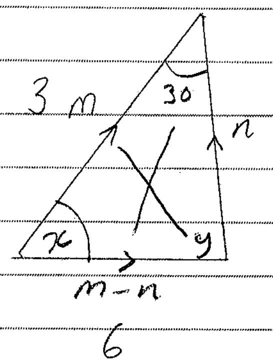
(ii) Two different vectors, **m** and **n**, are such that $|\mathbf{m}| = 3$ and $|\mathbf{m} - \mathbf{n}| = 6$
The angle between vector **m** and vector **n** is 30°

Find the angle between vector **m** and vector **m** – **n**, giving your answer, in degrees, to one decimal place.

(4)

i/ a and b must be in the same direction

ii/



$\frac{\sin y}{3} = \frac{\sin 30}{6}$

$\sin y = \frac{\sin 30}{6} \times 3$

$\sin y = \frac{1}{4}$

$y = 14.5$

$x = 180 - 30 - 14.5 = \underline{\underline{135.5°}}$