Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel Level 3 GCE

Thursday 16 May 2024

Afternoon (Time: 2 hours)

Paper reference **8MA0/01**

Mathematics

Advanced Subsidiary

PAPER 1: Pure Mathematics

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 14 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

1. Find

$$\int \frac{2\sqrt{x} - 3}{x^2}\,dx$$

giving your answer in simplest form.

(4)

$$\int \frac{2x^{\frac{1}{2}}}{x^2} - \frac{3}{x^2} \quad dx$$

$$\int 2x^{-\frac{3}{2}} - 3x^{-2} \quad dx$$

$$\frac{2x^{-\frac{1}{2}}}{-\frac{1}{2}} - \frac{3x^{-1}}{-1} + c$$

$$\underline{-4x^{-\frac{1}{2}} + 3x^{-1} + c}$$

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Question 1 continued

(Total for Question 1 is 4 marks)

Turn over

2. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

$$f(x) = 2x^3 - 3ax^2 + bx + 8a$$

where a and b are constants.

Given that $(x - 4)$ is a factor of $f(x)$, $f(4) = 0$

(a) use the factor theorem to show that

$$10a = 32 + b$$

(2)

Given also that $(x - 2)$ is a factor of $f(x)$, $f(2) = 0$

(b) express $f(x)$ in the form

$$f(x) = (2x + k)(x - 4)(x - 2)$$

where k is a constant to be found.

(4)

(c) Hence,

(i) state the number of real roots of the equation $f(x) = 0$

(ii) write down the largest root of the equation $f\left(\frac{1}{3}x\right) = 0$

(2)

a/ $f(4) = 0$

$2(4)^3 - 3a(4)^2 + b(4) + 8a = 0$

$128 - 48a + 4b + 8a = 0$

$128 - 40a + 4b = 0$

$32 - 10a + b = 0$

$32 + b = 10a$

b/ $f(2) = 0$

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Question 2 continued

$2(2)^3 - 3a(2)^2 + b(2) + 8a = 0$

$16 - 12a + 2b + 8a = 0$

$16 - 4a + 2b = 0$

$8 - 2a + b = 0$

$b = 2a - 8$

$32 + b = 10a$

$b = 10a - 32$

$10a - 32 = 2a - 8$

$8a = 24$

$a = 3$

$b = 2(3) - 8$

$= -2$

$f(x) = 2x^3 - 3ax^2 + bx + 8a$

$f(x) = 2x^3 - 9x^2 - 2x + 24$

$(x-2)$ and $(x-4)$ are factors

$(x-2)(x-4) = x^2 - 6x + 8$

$$
\begin{array}{r|l}
 & 2x + 3 \\
x^2 - 6x + 8 & 2x^3 - 9x^2 - 2x + 24 \\
 & 2x^3 - 12x^2 + 16x \\
 & 3x^2 - 18x + 24 \\
 & 3x^2 - 18x + 24 \\
\hline
 & 0
\end{array}
$$

$f(x) = (x-2)(x-4)(2x+3)$

P 7 4 0 8 7 A 0 5 4 4

Question 2 continued

$f(x) = (2x+3)(x-4)(x-2)$ $\quad$ $k = 3$

c i/ 3 $\quad$ $x = -\frac{3}{2}$ $\quad$ $x = 4$ $\quad$ $x = 2$

ii/ roots or $f(\frac{1}{3}x)$ are $x = -\frac{9}{2}$ $x = 12$ $x = 6$

$x = 12$

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Question 2 continued

(Total for Question 2 is 8 marks)

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3. Relative to a fixed origin O,

- point P has position vector $9\mathbf{i} - 8\mathbf{j}$
- point Q has position vector $3\mathbf{i} - 5\mathbf{j}$

(a) Find $\overrightarrow{PQ}$

(2)

Given that R is the point such that $\overrightarrow{QR} = 9\mathbf{i} + 18\mathbf{j}$

(b) show that angle $PQR = 90°$

(2)

Given also that S is the point such that $\overrightarrow{PS} = 3\overrightarrow{QR}$

(c) find the exact area of $PQRS$

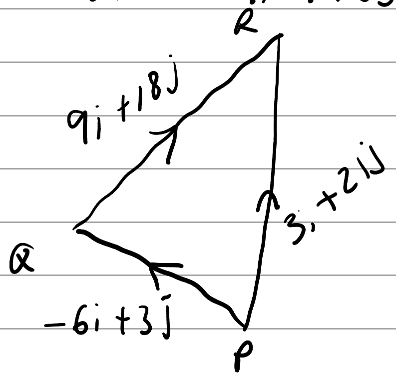
(4)

a/ $\overrightarrow{PQ} = \begin{pmatrix} 3 \\ -5 \end{pmatrix} - \begin{pmatrix} 9 \\ -8 \end{pmatrix}$

$= \begin{pmatrix} -6 \\ 3 \end{pmatrix}$

$\overrightarrow{PQ} = -6i + 3j$

b/ $\overrightarrow{QR} = 9i + 18j$



$\overrightarrow{PR} = \begin{pmatrix} -6 \\ 3 \end{pmatrix} + \begin{pmatrix} 9 \\ 18 \end{pmatrix} = \begin{pmatrix} 3 \\ 21 \end{pmatrix}$

If PQR is $90°$ $a^2 + b^2 = c^2$

$QP = \sqrt{6^2 + 3^2} = \sqrt{45}$

$QR = \sqrt{9^2 + 18^2} = \sqrt{405}$

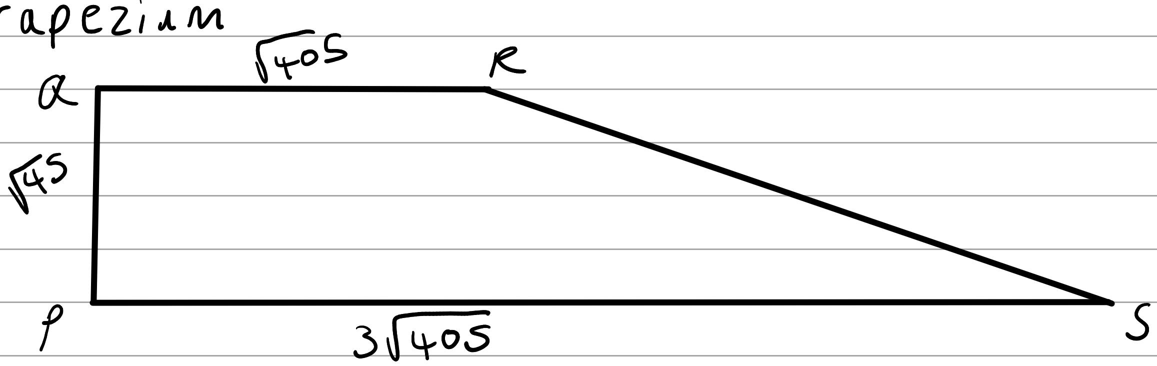
$\overrightarrow{PR} = \sqrt{3^2 + 21^2} = \sqrt{450}$

Question 3 continued

$45 + 405 = 450$ $\therefore PQR = 90^\circ$

c)

Trapezium



$\text{Area} = \frac{1}{2}\left(\sqrt{405} + 3\sqrt{405}\right) \times \sqrt{45}$

$= 2\sqrt{405} \times \sqrt{45}$

$= \underline{\underline{270}}$

(Total for Question 3 is 8 marks)

4.

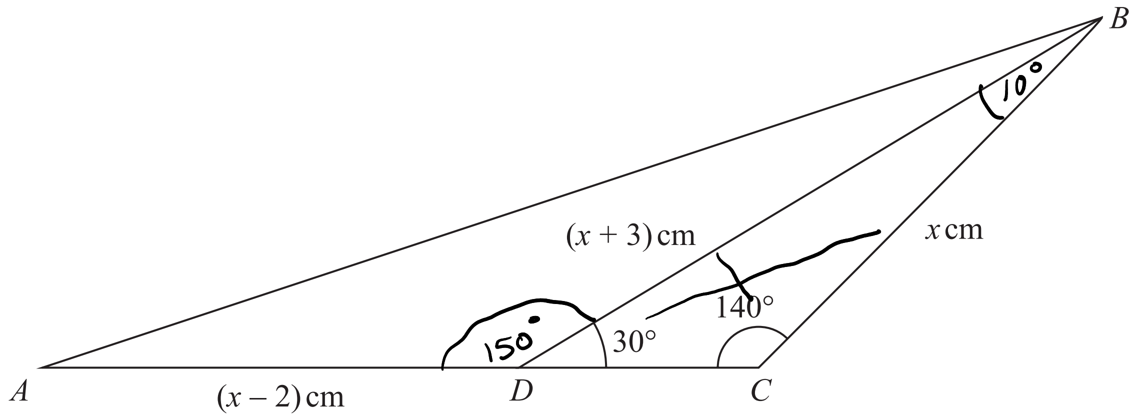


Figure 1

Figure 1 shows a sketch of triangle *ABD* and triangle *BCD*

Given that

- *ADC* is a straight line
- $BD = (x + 3)$ cm
- $BC = x$ cm
- angle $BDC = 30°$
- angle $BCD = 140°$

(a) show that $x = 10.5$ correct to 3 significant figures.

(3)

Given also that $AD = (x - 2)$ cm

(b) find the length of *AB*, giving your answer to 3 significant figures.

(2)

$$\frac{x+3}{\sin 140} = \frac{x}{\sin 30} \qquad \sin 30 = \frac{1}{2}$$

$$\frac{1}{2}(x+3) = x \sin 140$$

$$\frac{1}{2}x + \frac{3}{2} = x \sin 140$$

$$\frac{3}{2} = x \sin 140 - \frac{1}{2}x$$

$$\frac{3}{2} = x\left(\sin 140 - \frac{1}{2}\right)$$

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Question 4 continued

$$x = \frac{1.5}{\sin(140) - \frac{1}{2}}$$

$$= \underline{10.5 \text{ cm}} \quad (3sf)$$

b) $a^2 = b^2 + c^2 - 2bc \cos A$

$a^2 = 13.5^2 + 8.5^2 - 2(13.5)(8.5)\cos(150)$

$= 453.67$

$a = \underline{21.3 \text{ cm}^2} \quad (3sf)$

(Total for Question 4 is 5 marks)

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5. The curve C_1 has equation

$$y = \frac{6}{x} + 3$$

(a) (i) Sketch C_1 stating the coordinates of any points where the curve cuts the coordinate axes.

(ii) State the equations of any asymptotes to the curve C_1

(3)

The curve C_2 has equation

$$y = 3x^2 - 4x - 10$$

(b) Show that C_1 and C_2 intersect when

$$3x^3 - 4x^2 - 13x - 6 = 0$$

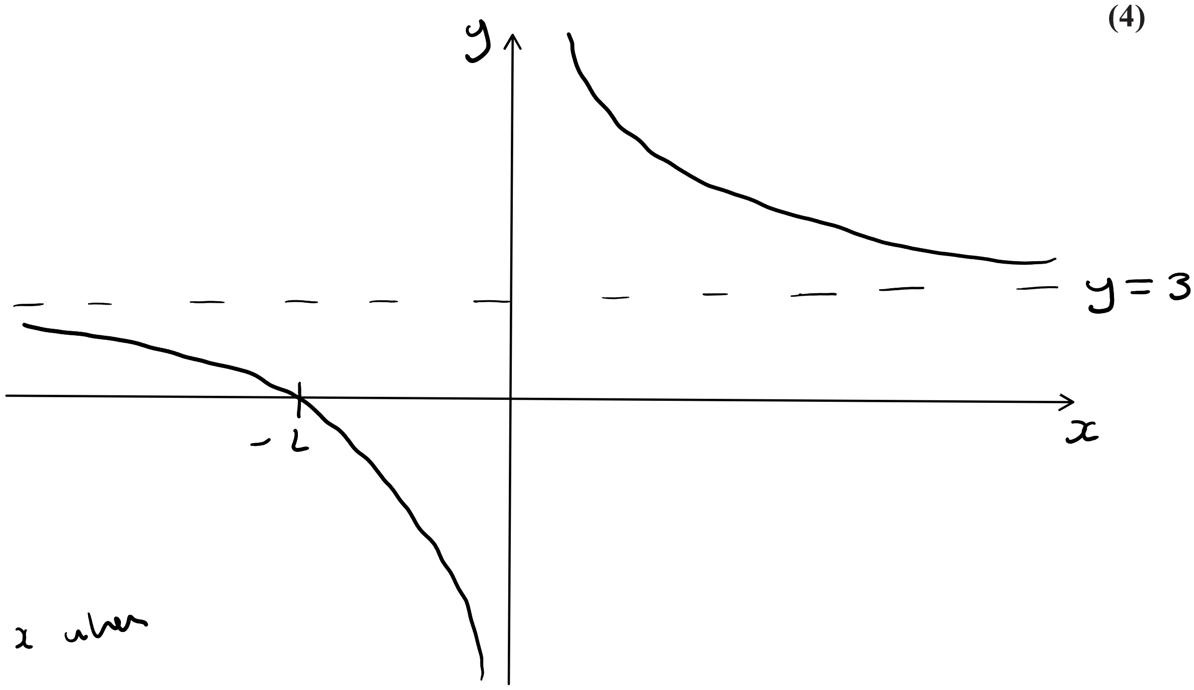
(2)

Given that the x coordinate of one of the points of intersection is $-\frac{2}{3}$

(c) use algebra to find the x coordinates of the other points of intersection between C_1 and C_2

(*Solutions relying on calculator technology are not acceptable.*)

(4)



Crosses x when

$y = 0$

$0 = \frac{6}{x} + 3$

$-3 = \frac{6}{x}$

$x = -2$

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ii/ $\underline{\underline{y=3}}$ and $\underline{\underline{x=0}}$

b/ $y = \frac{6}{x} + 3$ and $y = 3x^2 - 4x - 10$

intersection where:

$$\frac{6}{x} + 3 = 3x^2 - 4x - 10$$

$$6 + 3x = 3x^3 - 4x^2 - 10x$$

$$0 = 3x^3 - 4x^2 - 13x - 6$$

$x = -\frac{2}{3}$ $\therefore$ $(3x+2)$ is a factor

$$\begin{array}{r|l} & x^2 - 2x - 3 \\ \hline 3x+2 & 3x^3 - 4x^2 - 13x - 6 \\ & 3x^3 + 2x^2 \\ \hline & \quad -6x^2 - 13x \\ & \quad -6x^2 - 4x \\ \hline & \qquad -9x - 6 \\ & \qquad -9x - 6 \\ \hline & \qquad\quad 0 \end{array}$$

$$(3x+2)(x^2 - 2x - 3)$$

$$(3x+2)(x-3)(x+1)$$

$x = -\frac{2}{3}$ $\underline{\underline{x = 3}}$ $\underline{\underline{x = -1}}$

Question 5 continued

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Question 5 continued

(Total for Question 5 is 9 marks)

6. The binomial expansion of

$$(1 + ax)^{12}$$

up to and including the term in x^2 is

$$1 - \frac{15}{2}x + kx^2$$

where a and k are constants.

(a) Show that $a = -\frac{5}{8}$

(2)

(b) Hence find the value of k

(2)

Using the expansion and making your method clear,

(c) find an estimate for the value of $\left(\frac{17}{16}\right)^{12}$, giving your answer to 4 decimal places.

(2)

a) $1(1)^{12} + 12(1)^{11}(ax) + 66(1)^{10}(ax)^2$

$1 + 12ax + 66a^2x^2$

$12a = -\frac{15}{2}$

$a = \frac{-15}{24} = \frac{-5}{8}$

b/ $k = 66a^2$

$= 66\left(\frac{-5}{8}\right)^2$

$= \frac{825}{32}$

c/ $\left(1 - \frac{5}{8}x\right)^{12}$

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Question 6 continued

$$1 - \frac{5}{8}x = \frac{17}{16}$$

$$-\frac{1}{16} = \frac{5}{8}x$$

$$x = -\frac{1}{10}$$

substitute $-\frac{1}{10}$ into the expansion

$$1 - \frac{15}{2}\left(-\frac{1}{10}\right) + \frac{825}{32}\left(-\frac{1}{10}\right)^2 = \underline{\underline{2.0078}}$$

(Total for Question 6 is 6 marks)

7.

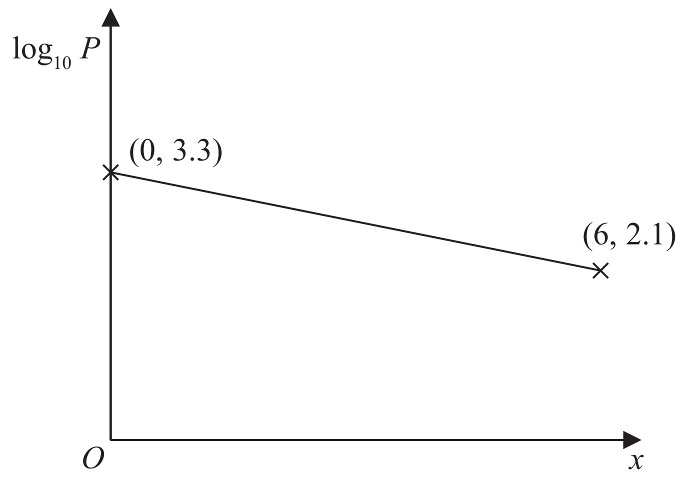

Figure 2

A chimney emits smoke particles.

On a particular day, the concentration of smoke particles in the air emitted by this chimney, P parts per million, is measured at various distances, x km, from the chimney.

Figure 2 shows a sketch of the linear relationship between $\log_{10} P$ and x that is used to model this situation.

The line passes through the point (0, 3.3) and the point (6, 2.1)

x_1 y_1 x_2 y_2

(a) Find a complete equation for the model in the form

$$P = ab^x$$

where a and b are constants. Give the value of a and the value of b each to 4 significant figures.

(4)

(b) With reference to the model, interpret the value of ab

(1)

$y = mx + c$ $\quad m = \frac{2.1 - 3.3}{6}$

$\log_{10} P = mx + 3.3$ $\quad = -\frac{1}{5}$

$\log_{10} P = -\frac{1}{5}x + 3.3$

$P = 10^{-\frac{1}{5}x + 3.3}$

$= 10^{3.3} \cdot \left(10^{-\frac{1}{5}}\right)^x$

Question 7 continued

$a = 10^{3.3}$ $\qquad$ $b = 10^{-\frac{1}{5}}$

$= 1995$ $\qquad$ $= 0.6310$

$p = 1995(0.6310)^x$

b) ab is the concentration of smoke particles 1 km from the chimney.

(Total for Question 7 is 5 marks)

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8.

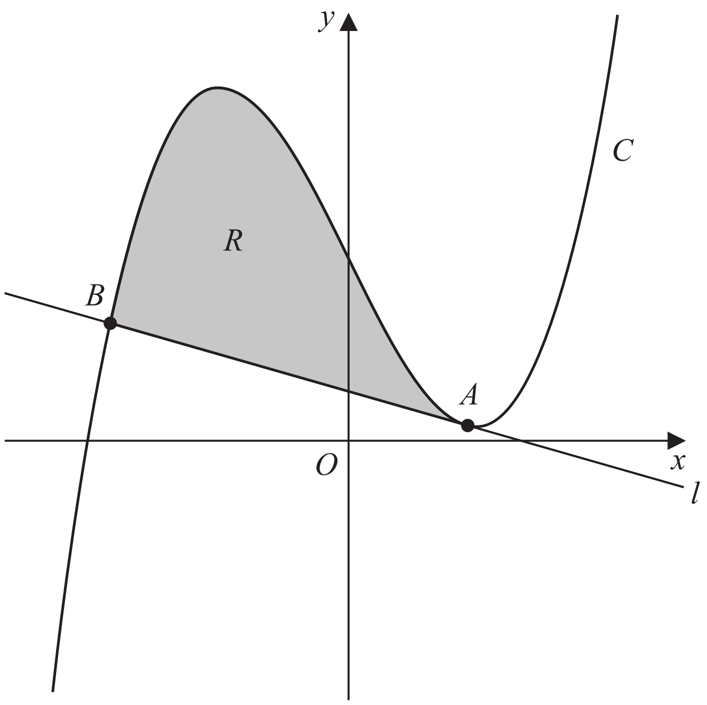

Figure 3

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 3 shows a sketch of the curve C with equation

$$y = x^3 - 14x + 23$$

The line l is the tangent to C at the point A, also shown in Figure 3.

Given that l has equation $y = -2x + 7$ $\quad \frac{dy}{dx} = -2$

(a) show, using calculus, that the x coordinate of A is 2

(3)

The line l cuts C again at the point B.

(b) Verify that the x coordinate of B is -4

(2)

The finite region, R, shown shaded in Figure 3, is bounded by C and l.

Using algebraic integration,

(c) show that the area of R is 108

(5)

a/ $\quad \frac{dy}{dx} = 3x^2 - 14 \qquad \frac{dy}{dx} = -2$

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Question 8 continued

$$3x^2 - 14 = -2$$
$$3x^2 = 12$$
$$x^2 = 4$$
$$x = \pm 2$$

$\therefore$ the x coordinate of A is 2.

b) Intersection where

$$x^3 - 14x + 23 = -2x + 7$$

$$x^3 - 12x + 16 = 0$$

$$(-4)^3 - 12(-4) + 16 = 0 \quad ✓$$

c) $$\int_{-4}^{2} x^3 - 14x + 23 - (-2x + 7) \, dx$$

$$\int_{-4}^{2} x^3 - 12x + 16 \, dx$$

$$\left[\frac{1}{4}x^4 - 6x^2 + 16x\right]_{-4}^{2}$$

$$\left(\frac{1}{4}(2)^4 - 6(2)^2 + 16(2)\right) - \left(\frac{1}{4}(-4)^4 - 6(-4)^2 + 16(-4)\right)$$

<u>108 units^2</u>

Question 8 continued

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Question 8 continued

(Total for Question 8 is 10 marks)

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9.

$$p = \log_a 16$$

$$q = \log_a 25$$

where a is a constant.

Find in terms of p and/or q,

(a) $\log_a 256$ **(1)**

(b) $\log_a 100$ **(2)**

(c) $\log_a 80 \times \log_a 3.2$ **(2)**

a/ $16^2 = 256$

$\log_a 16 + \log_a 16 = \log_a 256$

$p + p = \log_a 256$

$\underline{\underline{2p}} = \log_a 256$

b/

$\log_a 16 = p$

$a^p = 16$

$(a^p)^{\frac{1}{2}} = 4$

$a^{\frac{1}{2}p} = 4$ $\quad \log_a 4 = \frac{1}{2}p$

$\log_a 4 + \log_a 25 = \log_a 100$

$\underline{\underline{\frac{1}{2}p + q}} = \log_a 100$

c/ $\frac{1}{2}p = \log_a 4$ $\quad \frac{1}{2}q = \log_a 5$

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Question 9 continued

$\log_a 80 = \log_a 5 + \log_a 16$

$= \frac{1}{2}q + p$

$\log_a 3.2 = \log_a 16 - \log_a 5$

$= p - \frac{1}{2}q$

$\left(\frac{1}{2}q + p\right)\left(p - \frac{1}{2}q\right)$

$\left(p + \frac{1}{2}q\right)\left(p - \frac{1}{2}q\right)$

$p^2 - \frac{1}{4}q^2$

(Total for Question 9 is 5 marks)

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10.

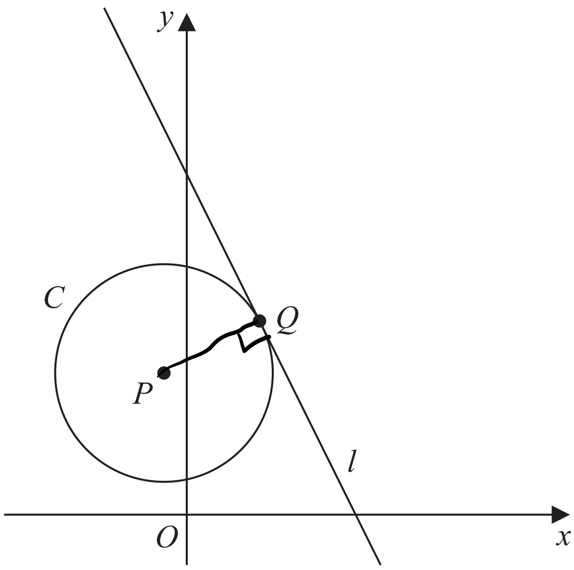

Figure 4

Figure 4 shows a sketch of the circle C

- the point $P\,(-1, k+8)$ is the centre of C
  $x_1 \quad y_1$
- the point $Q\,(3, k^2 - 2k)$ lies on C
  $x_2 \quad y_2$
- k is a positive constant
- the line l is the tangent to C at Q

Given that the gradient of l is -2 gradient of radius $= \frac{1}{2}$

(a) show that

$$k^2 - 3k - 10 = 0$$

(4)

(b) Hence find an equation for C

(4)

$\frac{1}{2} = \frac{k^2 - 2k - (k+8)}{3 - -1}$

$\frac{1}{2} = \frac{k^2 - 3k - 8}{4}$

$2 = k^2 - 3k - 8$

$0 = k^2 - 3k - 10$

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Question 10 continued

b/ $0 = (k-5)(k+2)$

k is positive $\therefore$ $\underline{k = 5}$

centre $(-1, k+8)$

$\underline{(-1, 13)}$

$Q:$ $(3, k^2 - 2k)$

$(3, 15)$

$(x+1)^2 + (y-13)^2 = r^2$

$(3+1)^2 + (15-13)^2 = 20$

$\underline{(x+1)^2 + (y-13)^2 = 20}$

Question 10 continued

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Question 10 continued

(Total for Question 10 is 8 marks)

11. The prices of two precious metals are being monitored.

The price per gram of metal A, £ V_A, is modelled by the equation

$$V_A = 100 + 20e^{0.04t}$$

where t is the number of months after monitoring began.

The price per gram of metal B, £ V_B, is modelled by the equation

$$V_B = pe^{-0.02t}$$

where p is a positive constant and t is the number of months after monitoring began.

Given that $V_B = 2V_A$ when $t = 0$

(a) find the value of p

(2)

When $t = T$, the rate of **increase** in the price per gram of metal A was equal to the rate of **decrease** in the price per gram of metal B

(b) Find the value of T, giving your answer to one decimal place.

(*Solutions based entirely on calculator technology are not acceptable.*)

(4)

a/ when $t = 0$ $\quad V_A = 120$

$\therefore V_B = 240$

$\underline{240 = p}$

b/ $\dfrac{dV_A}{dt} = \dfrac{4}{5}e^{0.04t}$

$\dfrac{dV_B}{dt} = -\dfrac{24}{5}e^{-0.02t}$

when $t = T$ $\quad \dfrac{dV_A}{dt} = -\dfrac{dV_B}{dt}$

$\dfrac{4}{5}e^{0.04T} = \dfrac{24}{5}e^{-0.02T}$

Question 11 continued

$$\frac{e^{0.04T}}{e^{-0.02T}} = 6$$

$$e^{0.06T} = 6$$

$$0.06T = \ln 6$$

$$T = \frac{\ln 6}{0.06}$$

$$= \underline{\underline{29.9}} \text{ months}$$

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Question 11 continued

Question 11 continued

(Total for Question 11 is 6 marks)

12.

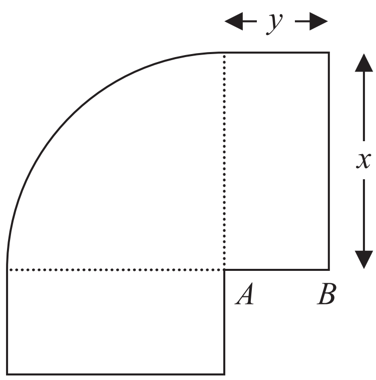

Figure 5

Figure 5 shows the plan view of the design for a swimming pool.

The pool is modelled as a quarter of a circle joined to two equal sized rectangles as shown.

Given that

- the quarter circle has radius x metres
- the rectangles each have length x metres and width y metres
- the total surface area of the swimming pool is $100\,\text{m}^2$

(a) show that, according to the model, the perimeter P metres of the swimming pool is given by

$$P = 2x + \frac{200}{x}$$

(5)

(b) Use calculus to find the value of x for which P has a stationary value.

(4)

(c) Prove, by further calculus, that this value of x gives a minimum value for P

(2)

Access to the pool is by side AB shown in Figure 5.

Given that AB must be at least one metre,

(d) determine, according to the model, whether the swimming pool with the minimum perimeter would be suitable.

(2)

$100 = 2xy + \frac{1}{4}\pi x^2$

$P = 2x + 4y + \frac{1}{4}(2\pi x)$

Question 12 continued

$$100 - \frac{1}{4}\pi x^2 = 2xy$$

$$y = \frac{100}{2x} - \frac{1}{8}\pi x$$

$$P = 2x + 4\left(\frac{100}{2x} - \frac{1}{8}\pi x\right) + \frac{1}{4}2\pi x$$

$$= 2x + \frac{200}{x} - \frac{1}{2}\pi x + \frac{1}{2}\pi x$$

$$P = 2x + \frac{200}{x}$$

b) $P = 2x + 200x^{-1}$

$$\frac{dP}{dx} = 2 - 200x^{-2}$$

stationary value when $\frac{dP}{dx} = 0$

$$2 - \frac{200}{x^2} = 0$$

$$2x^2 - 200 = 0$$

$$2x^2 = 200$$

$$x^2 = 100$$

$$x = 10$$

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Question 12 continued

c/ $\frac{d^2P}{dx^2} = 400x^{-3}$

when $x = 10$ $\quad \frac{d^2P}{dx^2} = \frac{400}{10^3} = 0.4$

$\frac{d^2P}{dx^2} > 0 \quad \therefore$ Minimum

d/ $y = \frac{100}{2x} - \frac{1}{8}\pi x$

$y = \frac{100}{2(10)} - \frac{1}{8}\pi(10)$

$= 1.07m$ $\therefore$ It would be suitable

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Question 12 continued

(Total for Question 12 is 13 marks)

13. **In this question you must show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that the equation

$$\sin\theta(7\sin\theta - 4\cos\theta) = 4$$

can be written as

$$3\tan^2\theta - 4\tan\theta - 4 = 0$$

(4)

(b) Hence solve, for $0 < x < 360°$

$$\sin x(7\sin x - 4\cos x) = 4$$

giving your answers to one decimal place.

(4)

(c) Hence find the smallest solution of the equation

$$\sin 4\alpha(7\sin 4\alpha - 4\cos 4\alpha) = 4$$

in the range $720° < \alpha < 1080°$, giving your answer to one decimal place.

(1)

a/ $7\sin^2\theta - 4\cos\theta\sin\theta = 4$

$\sin^2\theta + \cos^2\theta = 1$

$4\sin^2\theta + 4\cos^2\theta = 4$

$7\sin^2\theta - 4\cos\theta\sin\theta = 4\sin^2\theta + 4\cos^2\theta$

$3\sin^2\theta - 4\cos\theta\sin\theta - 4\cos^2\theta = 0$

$3\dfrac{\sin^2\theta}{\cos^2\theta} - \dfrac{4\cos\theta\sin\theta}{\cos^2\theta} - \dfrac{4\cos^2\theta}{\cos^2\theta} = 0$

$3\tan^2\theta - 4\tan\theta - 4 = 0$

b/ $3\tan^2 x - 4\tan x - 4 = 0$

$(3\tan x + 2)(\tan x - 2) = 0$

P 7 4 0 8 7 A 0 3 8 4 4

Question 13 continued

$\tan x = -\frac{2}{3}$ $\tan x = 2$

$x = -33.7, 146.3, 326.3$ $x = 63.4, 243.4$

$63.4°, 146.3°, 243.4°, 326.3°$

c) $4\alpha = 63.4°, 146.3°, 243.4°, 326.3°$

$\alpha = 15.9°, 36.6°, \ldots, 735.9°$

$735.9°$

Question 13 continued

Question 13 continued

(Total for Question 13 is 9 marks)

14. Prove, using algebra, that

$$n^2 + 5n$$

is even for all $n \in \mathbb{N}$

(4)

if n is even $n = 2m$

$(2m)^2 + 5(2m)$

$4m^2 + 10m = 2(2m^2 + 5m)$ $\therefore$ even

if n is odd $n = 2m + 1$

$(2m+1)^2 + 5(2m+1)$

$4m^2 + 2m + 2m + 1 + 10m + 5$

$4m^2 + 14m + 6$

$2(2m^2 + 7m + 3)$ $\therefore$ even

$\therefore$ $n^2 + 5m$ is even for all $n \in N$

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Question 14 continued

Question 14 continued

(Total for Question 14 is 4 marks)

TOTAL FOR PAPER IS 100 MARKS