

Please check the examination details below before entering your candidate information

Candidate surname

Other names

Centre Number

Candidate Number

--	--	--	--	--

--	--	--	--	--

## Pearson Edexcel Level 3 GCE

**Thursday 18 May 2023**

Afternoon (Time: 2 hours)

Paper  
reference

**8MA0/01**

### Mathematics

#### Advanced Subsidiary

#### PAPER 1: Pure Mathematics

**You must have:**

Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

**Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

### Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear.
- Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

### Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 17 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

### Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

P72839A

©2023 Pearson Education Ltd.  
N:1/1/1/1/1/



  
Pearson

1. A curve has equation

$$y = \frac{2}{3}x^3 - \frac{7}{2}x^2 - 4x + 5$$

(a) Find  $\frac{dy}{dx}$  writing your answer in simplest form.

(2)

(b) Hence find the range of values of  $x$  for which  $y$  is decreasing.

(4)

a/  $\frac{dy}{dx} = 2x^2 - 7x - 4$

b/ decreasing when  $\frac{dy}{dx} < 0$

$$2x^2 - 7x - 4 < 0$$

$$\underline{-\frac{1}{2} < x < 4}$$



Question 1 continued

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 1 is 6 marks)



2.

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

Using the substitution  $u = \sqrt{x}$  or otherwise, solve

$$6x + 7\sqrt{x} - 20 = 0$$

(4)

$$6u^2 + 7u - 20 = 0$$

$$u = \frac{4}{3}, \quad -\frac{5}{2}$$

$$\sqrt{x} = \frac{4}{3} \quad \sqrt{x} = -\frac{5}{2}$$

$$x = \frac{16}{9}$$

$\sqrt{x}$  cannot be negative

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



**Question 2 continued**

Lined area for writing the answer to Question 2.

DO NOT WRITE IN THIS AREA

**(Total for Question 2 is 4 marks)**



P 7 2 8 3 9 A 0 5 4 4

3.

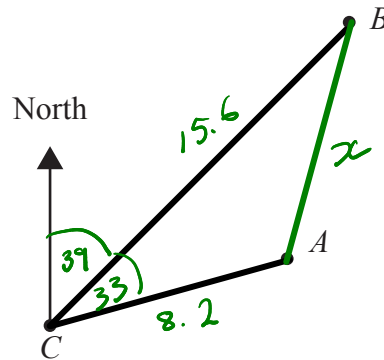


Figure 1

Figure 1 is a sketch showing the position of three phone masts,  $A$ ,  $B$  and  $C$ .

The masts are identical and their bases are assumed to lie in the same horizontal plane.

From mast  $C$

- mast  $A$  is 8.2 km away on a bearing of  $072^\circ$
- mast  $B$  is 15.6 km away on a bearing of  $039^\circ$

$$72 - 39 = 33$$

- (a) Find the distance between masts  $A$  and  $B$ , giving your answer in km to one decimal place.

(3)

An engineer needs to travel from mast  $A$  to mast  $B$ .

- (b) Give a reason why the answer to part (a) is unlikely to be an accurate value for the distance the engineer travels.

(1)

$$a/ \quad a^2 = b^2 + c^2 - 2bc \cos A$$

$$x^2 = 8.2^2 + 15.6^2 - 2(8.2)(15.6) \cos(33)$$

$$x^2 = 96.03$$

$$x = \underline{9.8 \text{ km}}$$

b/ There will probably not be a road directly from  $A$  to  $B$ . The engineer will probably travel further on indirect roads.



DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

Question 3 continued

Lined writing area for the answer to Question 3.

(Total for Question 3 is 4 marks)



4. (a) Sketch the curve with equation

$$y = \frac{k}{x} \quad x \neq 0$$

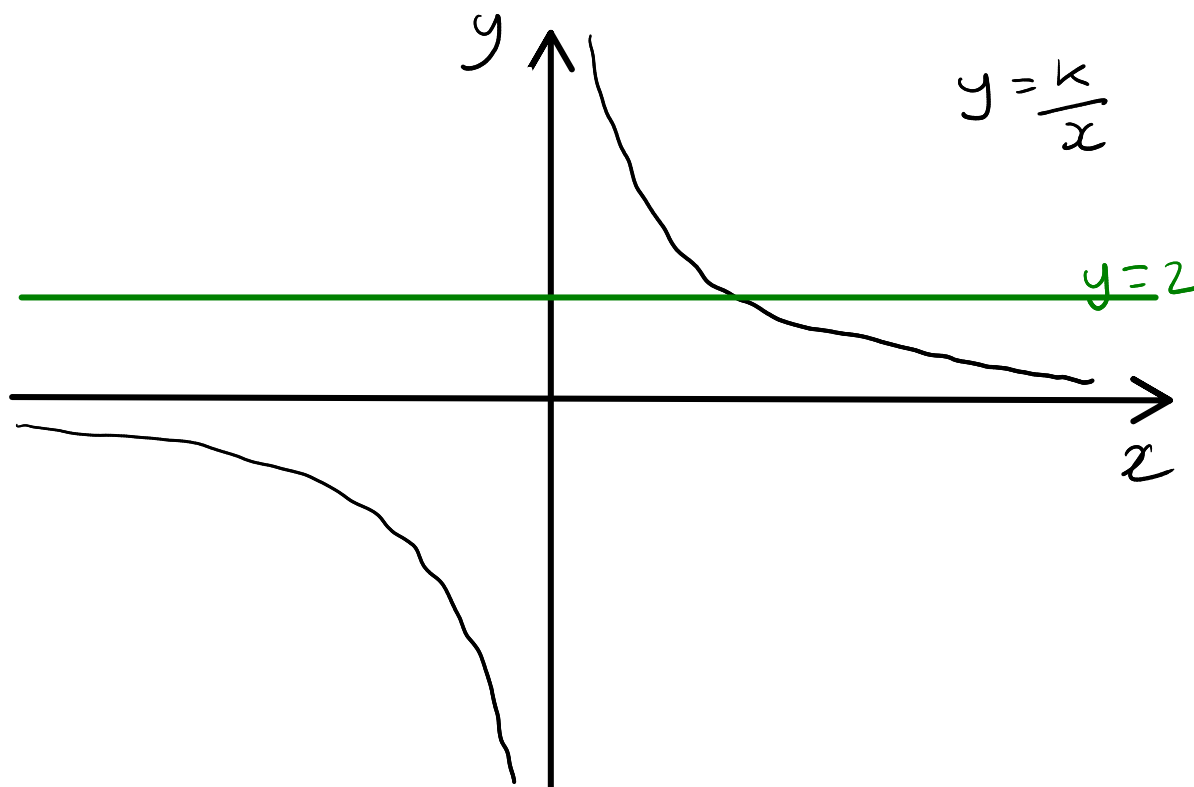
where  $k$  is a positive constant.

(2)

(b) Hence or otherwise, solve

$$\frac{16}{x} \leq 2$$

(3)





Question 4 continued

$$b) \frac{16}{x} \leq 2$$

$$\underline{\underline{x \geq 8}} \quad \text{or} \quad \underline{\underline{x < 0}} \quad (\text{see graph})$$

(Total for Question 4 is 5 marks)

DO NOT WRITE IN THIS AREA



P 7 2 8 3 9 A 0 9 4 4

5.

In this question you must show all stages of your working.

Solutions relying on calculator technology are not acceptable.

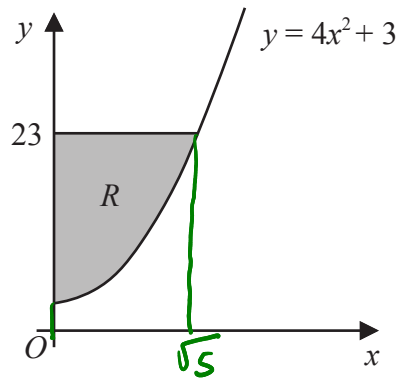


Figure 2

The finite region  $R$ , shown shaded in Figure 2, is bounded by the curve with equation  $y = 4x^2 + 3$ , the  $y$ -axis and the line with equation  $y = 23$

Show that the exact area of  $R$  is  $k\sqrt{5}$  where  $k$  is a rational constant to be found.

(5)

$$23 = 4x^2 + 3$$

$$20 = 4x^2$$

$$5 = x^2$$

$$x = \sqrt{5}$$

$$\text{Area of rectangle} = 23 \times \sqrt{5}$$

$$= 23\sqrt{5}$$

$$\int_0^{\sqrt{5}} (4x^2 + 3) dx$$

$$\left[ \frac{4x^3}{3} + 3x \right]_0^{\sqrt{5}}$$

$$\left( \frac{4(\sqrt{5})^3}{3} + 3\sqrt{5} \right) - (0)$$

$$\text{Area of } R = 23\sqrt{5} - \frac{4(\sqrt{5})^3}{3} - 3\sqrt{5}$$

$$= \frac{40\sqrt{5}}{3}$$





6. The circle  $C$  has equation

$$x^2 + y^2 - 6x + 10y + k = 0$$

where  $k$  is a constant.

(a) Find the coordinates of the centre of  $C$ .

(2)

Given that  $C$  does not cut or touch the  $x$ -axis,

(b) find the range of possible values for  $k$ .

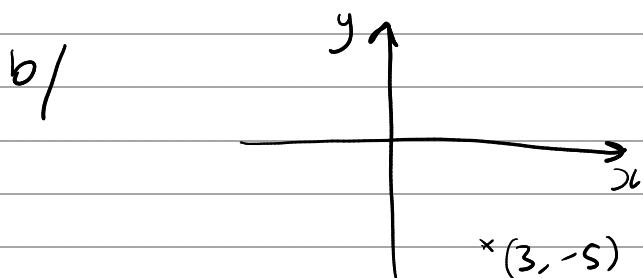
(3)

$$a) \quad x^2 - 6x + y^2 + 10y + k = 0$$

$$(x-3)^2 - 9 + (y+5)^2 - 25 + k = 0$$

$$(x-3)^2 + (y+5)^2 = 34 - k$$

$$\underline{\underline{(3, -5)}}$$



$r$  must be less than 5

$$34 - k < 25$$

$$9 < k$$

and  $k$  must be less than 34 or  $r$  would be zero/negative.

$$\underline{\underline{9 < k < 34}}$$





7. The distance a particular car can travel in a journey starting with a full tank of fuel was investigated.

- From a full tank of fuel, 40 litres remained in the car's fuel tank after the car had travelled 80 km  $d_1$   $V_1$
- From a full tank of fuel, 25 litres remained in the car's fuel tank after the car had travelled 200 km  $d_2$   $V_2$

Using a **linear model**, with  $V$  litres being the volume of fuel remaining in the car's fuel tank and  $d$  km being the distance the car had travelled,

- (a) find an equation linking  $V$  with  $d$ .  $V = m d + c$  (4)

Given that, on a particular journey

- the fuel tank of the car was initially full
- the car continued until it ran out of fuel

find, according to the model,

- (b) (i) the initial volume of fuel that was in the fuel tank of the car,  
 (ii) the distance that the car travelled on this journey. (3)

In fact the car travelled 320 km on this journey.

- (c) Evaluate the model in light of this information. (1)

$$a/ \quad m = \frac{40 - 25}{80 - 200}$$

$$= -\frac{1}{8}$$

$$V = -\frac{1}{8}d + c$$

$$40 = -\frac{1}{8}(80) + c$$

$$c = 50$$

$$V = -\frac{1}{8}d + 50$$

b i / 50 litres.



Question 7 continued

$$\text{ii/ } 0 = -\frac{1}{8}d + 50$$

$$\frac{1}{8}d = 50$$

$$\underline{\underline{d = 400 \text{ km}}}$$

c/ The model does not seem to work as 320 is significantly lower than 400.

(Total for Question 7 is 8 marks)



P 7 2 8 3 9 A 0 1 5 4 4

8.

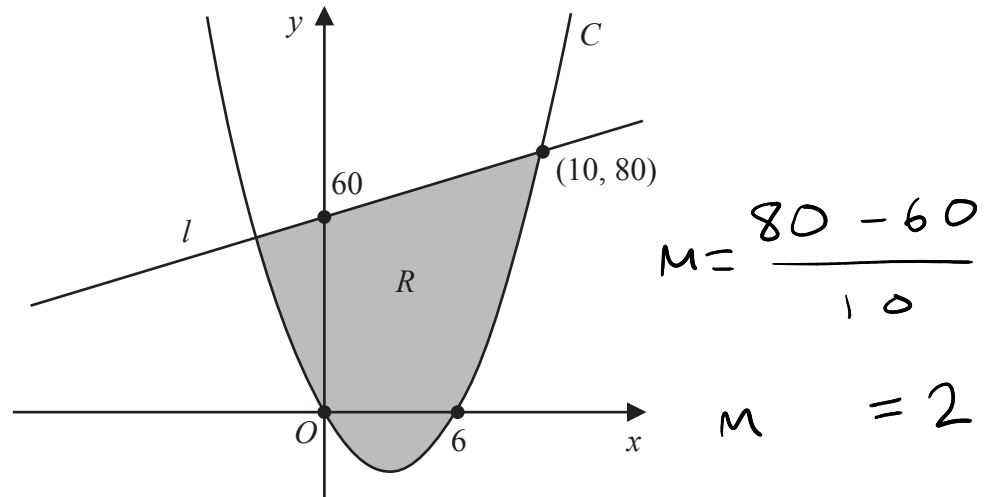


Figure 3

Figure 3 shows a sketch of a curve  $C$  and a straight line  $l$ .

Given that

- $C$  has equation  $y = f(x)$  where  $f(x)$  is a quadratic expression in  $x$
- $C$  cuts the  $x$ -axis at 0 and 6
- $l$  cuts the  $y$ -axis at 60 and intersects  $C$  at the point  $(10, 80)$

use inequalities to define the region  $R$  shown shaded in Figure 3.

(5)

$$y = kx(x - 6)$$

$$80 = k(10)(10 - 6)$$

$$k = 2$$

$$\underline{y = 2x(x - 6)}$$

$$y = mx + 60$$

$$\underline{y = 2x + 60}$$

$$\underline{y \geq 2x(x - 6)} \quad \text{and} \quad \underline{y \leq 2x + 60}$$







9. Using the laws of logarithms, solve the equation

$$2\log_5(3x-2) - \log_5 x = 2$$

(5)

$$\log_5(3x-2)^2 - \log_5 x = 2$$

$$\log_5 \left( \frac{(3x-2)^2}{x} \right) = 2$$

$$\frac{(3x-2)^2}{x} = 5^2$$

$$(3x-2)^2 = 25x$$

$$9x^2 - 12x + 4 = 25x$$

$$9x^2 - 37x + 4 = 0$$

$$x = 4, \quad x = \frac{1}{9}$$

↓  
does not work for  
the question.

$$3\left(\frac{1}{9}\right) - 2 < 0$$

$$\underline{\underline{x = 4}}$$





10.

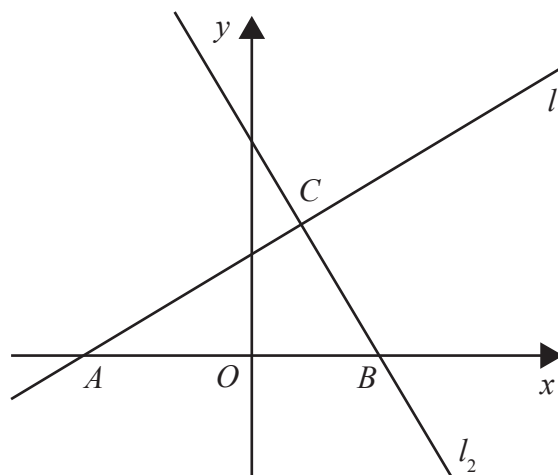


Figure 4

The line  $l_1$  has equation  $y = \frac{3}{5}x + 6$

The line  $l_2$  is perpendicular to  $l_1$  and passes through the point  $B(8,0)$ , as shown in the sketch in Figure 4.

(a) Show that an equation for line  $l_2$  is

$$5x + 3y = 40 \quad (3)$$

Given that

- lines  $l_1$  and  $l_2$  intersect at the point  $C$
- line  $l_1$  crosses the  $x$ -axis at the point  $A$

(b) find the exact area of triangle  $ABC$ , giving your answer as a fully simplified

fraction in the form  $\frac{p}{q}$  (5)

$$\text{perpendicular gradient} = -\frac{5}{3}$$

$$y - y_1 = m(x - x_1)$$

$$y - 0 = -\frac{5}{3}(x - 8)$$

$$y = -\frac{5}{3}x + \frac{40}{3}$$

$$3y = -5x + 40$$

$$5x + 3y = 40$$



Question 10 continued

$$b/ \quad y = \frac{3}{5}x + 6$$

crosses  $x$  when  $0 = \frac{3}{5}x + 6$

$$-6 = \frac{3}{5}x$$

$$x = -\frac{30}{3}$$

$$= -10 \quad \underline{A}$$

intersection where

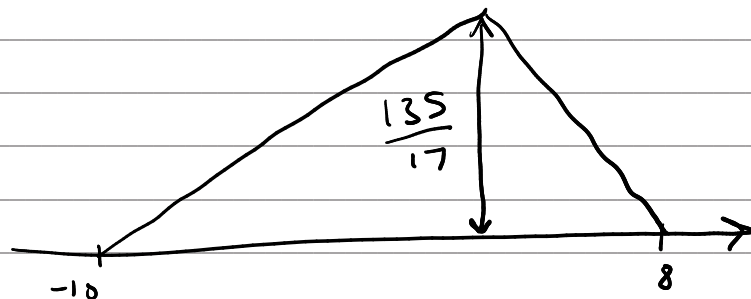
$$\frac{3}{5}x + 6 = -\frac{5}{3}x + \frac{40}{3}$$

$$\frac{34}{15}x = \frac{22}{3}$$

$$x = \frac{55}{17}$$

$$y = \frac{3}{5}\left(\frac{55}{17}\right) + 6$$

$$= \frac{135}{17} \quad \underline{C}$$



$$\text{Area} = \frac{1}{2} (18) \left( \frac{135}{17} \right) = \frac{1215}{17}$$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA







11. The height,  $h$  metres, of a plant,  $t$  years after it was first measured, is modelled by the equation

$$h = 2.3 - 1.7e^{-0.2t} \quad t \in \mathbb{R} \quad t \geq 0$$

Using the model,

- (a) find the height of the plant when it was first measured, (2)

- (b) show that, exactly 4 years after it was first measured, the plant was growing at approximately 15.3 cm per year. (3)

According to the model, there is a limit to the height to which this plant can grow.

- (c) Deduce the value of this limit. (1)

a/ when  $t = 0$

$$h = 2.3 - 1.7$$

$$= \underline{\underline{0.6 \text{ m}}}$$

b/  $\frac{dh}{dt} = -1.7(-0.2)e^{-0.2t}$

$$\frac{dh}{dt} = \frac{17}{50} e^{-0.2t}$$

when  $t = 4$

$$\frac{dh}{dt} = \frac{17}{50} e^{-0.2(4)}$$

$$= 0.15277 \text{ m per year}$$

$$= 15.3 \text{ cm per year.}$$

c/ As  $t$  gets bigger  $e^{-0.2t}$  gets closer to zero

$$\therefore \text{max height} = \underline{\underline{2.3 \text{ m}}}$$







12.

In this question you must show detailed reasoning.

Solutions relying entirely on calculator technology are not acceptable.

(a) Show that the equation

$$4 \tan x = 5 \cos x$$

can be written as

$$5 \sin^2 x + 4 \sin x - 5 = 0 \quad (3)$$

(b) Hence solve, for  $0 < x \leq 360^\circ$ 

$$4 \tan x = 5 \cos x$$

giving your answers to one decimal place.

(4)

(c) Hence find the **number of solutions** of the equation

$$4 \tan 3x = 5 \cos 3x$$

in the interval  $0 < x \leq 1800^\circ$ , explaining briefly the reason for your answer.

(2)

$$a/ \quad \frac{4 \sin x}{\cos x} = 5 \cos x$$

$$4 \sin x = 5 \cos^2 x$$

$$4 \sin x = 5(1 - \sin^2 x)$$

$$4 \sin x = 5 - 5 \sin^2 x$$

$$\underline{5 \sin^2 x + 4 \sin x - 5 = 0}$$

$$\sin x = \frac{-2 + \sqrt{29}}{5}$$

$$\sin x = \frac{-2 - \sqrt{29}}{5}$$

X

$$x = \underline{\underline{42.6^\circ}}, \underline{\underline{137.4^\circ}}$$



Question 12 continued

c/ 2 solutions for original 0 to 360

For  $\sin 3x$  there will be 6 solutions 0 to 360

$$6 \times 5 = 30 \text{ solutions } 0 < x < 1800$$

$$\begin{array}{r} \uparrow \\ 1800 \\ \hline 360 \\ \hline = 5 \end{array}$$

DO NOT WRITE IN THIS AREA







13. Relative to a fixed origin  $O$ 

- point  $A$  has position vector  $10\mathbf{i} - 3\mathbf{j}$
- point  $B$  has position vector  $-8\mathbf{i} + 9\mathbf{j}$
- point  $C$  has position vector  $-2\mathbf{i} + p\mathbf{j}$  where  $p$  is a constant

(a) Find  $\vec{AB}$  (2)

(b) Find  $|\vec{AB}|$  giving your answer as a fully simplified surd. (2)

Given that points  $A$ ,  $B$  and  $C$  lie on a straight line,

- (c) (i) find the value of  $p$ ,  
 (ii) state the ratio of the area of triangle  $AOC$  to the area of triangle  $AOB$ . (3)

$$a/ \begin{pmatrix} -8 \\ 9 \end{pmatrix} - \begin{pmatrix} 10 \\ -3 \end{pmatrix} = \begin{pmatrix} -18 \\ 12 \end{pmatrix}$$

$$\vec{AB} = -18\mathbf{i} + 12\mathbf{j}$$

$$b/ \sqrt{18^2 + 12^2} = \underline{\underline{6\sqrt{13}}}$$

$$c/ \vec{BC} = \begin{pmatrix} -2 \\ p \end{pmatrix} - \begin{pmatrix} -8 \\ 9 \end{pmatrix} = \begin{pmatrix} 6 \\ p-9 \end{pmatrix}$$

$$\vec{AB} = k\vec{BC}$$

$$\begin{pmatrix} -18 \\ 12 \end{pmatrix} = k \begin{pmatrix} 6 \\ p-9 \end{pmatrix}$$

$$k = -3$$

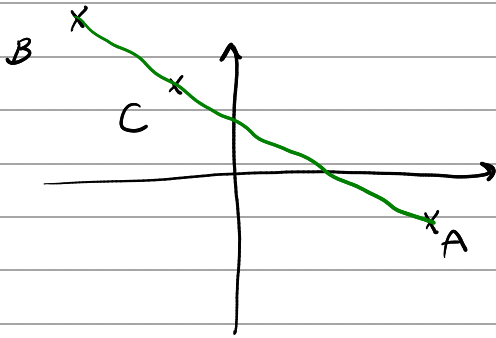
$$12 = -3(p-9)$$

$$-4 = p-9$$

$$\underline{\underline{p = 5}}$$



Question 13 continued



$$|\vec{AC}| = \sqrt{12^2 + 8^2} \\ = 4\sqrt{13}$$

$$\therefore AC : AB \\ 4 : 6 \\ 2 : 3$$

$$\therefore \triangle AOC : \triangle AOB \\ \underline{\underline{2 : 3}}$$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



P 7 2 8 3 9 A 0 3 1 4 4







14. Find, in simplest form, the coefficient of  $x^5$  in the expansion of

$$(5 + 8x^2)\left(3 - \frac{1}{2}x\right)^6 \quad (5)$$

$$(5 + 8x^2)\left(3^6 + \dots + {}^6C_3(3)^3\left(-\frac{1}{2}x\right)^3 + \dots + {}^6C_5(3)\left(-\frac{1}{2}x\right)^5 + \dots\right)$$

$$(5 + 8x^2)\left({}^6C_3(3)^3\left(-\frac{1}{2}x\right)^3 + {}^6C_5(3)\left(-\frac{1}{2}x\right)^5\right)$$

$$(5 + 8x^2)\left(-\frac{135}{2}x^3 - \frac{9}{16}x^5\right)$$

$$-540x^3 - \frac{45}{16}x^5$$

$$- \frac{8685}{16}$$

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA



**Question 14 continued**

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

DO NOT WRITE IN THIS AREA

(Total for Question 14 is 5 marks)

DO NOT WRITE IN THIS AREA



P 7 2 8 3 9 A 0 3 5 4 4

15.

In this question you must show detailed reasoning.

Solutions relying on calculator technology are not acceptable.

The curve  $C_1$  has equation  $y = 8 - 10x + 6x^2 - x^3$ The curve  $C_2$  has equation  $y = x^2 - 12x + 14$ (a) Verify that when  $x = 1$  the curves  $C_1$  and  $C_2$  intersect.

(2)

The curves also intersect when  $x = k$ .Given that  $k < 0$ (b) use algebra to find the exact value of  $k$ .

(5)

$$\text{when } x = 1 \quad C_1: y = 8 - 10(1) + 6(1)^2 - (1)^3$$

$$= 3$$

$$C_2: y = (1)^2 - 12(1) + 14$$

$$= 3$$

Both at  $(1, 3) \therefore$  intersection.

$$8 - 10x + 6x^2 - x^3 = x^2 - 12x + 14$$

$$-6 + 2x + 5x^2 - x^3 = 0$$

$$x^3 - 5x^2 - 2x + 6 = 0$$

 $(x - 1)$  is a factor

$$\begin{array}{r}
 x^2 - 4x - 6 \\
 x - 1 \overline{) x^3 - 5x^2 - 2x + 6} \\
 \underline{x^3 - x^2} \phantom{- 2x + 6} \\
 -4x^2 - 2x \phantom{+ 6} \\
 \underline{-4x^2 + 4x} \phantom{+ 6} \\
 -6x + 6 \\
 \underline{-6x + 6} \\
 0
 \end{array}$$



Question 15 continued

$$(x-1)(x^2-4x-6)$$

$$x=1 \quad x=2+\sqrt{10} \quad x=2-\sqrt{10}$$

$$k < 0 \quad \therefore k = 2 - \sqrt{10}$$

DO NOT WRITE IN THIS AREA



P 7 2 8 3 9 A 0 3 7 4 4





16. A curve has equation  $y = f(x)$ ,  $x \geq 0$

Given that

- $f'(x) = 4x + a\sqrt{x} + b$ , where  $a$  and  $b$  are constants
- the curve has a stationary point at  $(4, 3)$
- the curve meets the  $y$ -axis at  $-5$  when  $x = 0$

find  $f(x)$ , giving your answer in simplest form.

(6)

$$f'(4) = 0$$

$$0 = 4(4) + a\sqrt{4} + b$$

$$0 = 16 + 2a + b$$

$$2a + b = -16 \quad (1)$$

$$f'(x) = 4x + ax^{\frac{1}{2}} + b$$

$$f(x) = 2x^2 + \frac{2}{3}ax^{\frac{3}{2}} + bx + c$$

$$(4, 3) \quad 3 = 2(4)^2 + \frac{2}{3}a(4)^{\frac{3}{2}} + b(4) + c$$

$$3 = 32 + \frac{16}{3}a + 4b + c$$

$$c = -29 - \frac{16}{3}a - 4b$$

$c$  is  $-5$  (where the curve crosses  $y$ )

$$-5 = -29 - \frac{16}{3}a - 4b$$

$$\frac{16}{3}a + 4b = -24 \quad (2)$$

$$a = -15$$

$$b = 14$$





Question 16 continued

$$f(x) = 2x^2 + \frac{2a}{3}x^{\frac{3}{2}} + bx + c$$

$$f(x) = 2x^2 - 10x^{\frac{3}{2}} + 14x - 5$$

(Total for Question 16 is 6 marks)



P 7 2 8 3 9 A 0 4 1 4 4





