1 $f(x) = 2x^3 - 7x^2 - 17x + 10$

Use the factor theorem and division to factorise f(x) completely.

$f(2) = -36$

$f(-2) = 0 \quad \therefore \quad (x+2)$ is a factor

$$\begin{array}{r|l} & 2x^2 - 11x + 5 \\ \hline x+2 & 2x^3 - 7x^2 - 17x + 10 \\ & \underline{2x^3 + 4x^2} \\ & \quad -11x^2 - 17x \\ & \quad \underline{-11x^2 - 22x} \\ & \qquad 5x + 10 \\ & \qquad \underline{5x + 10} \\ & \qquad\quad 0 \end{array}$$

$f(x) = (x+2)(2x^2 - 11x + 5)$

$= (x+2)(2x-1)(x-5)$

2 $g(x) = 4x^3 - 8x^2 - 35x + 75$

(a) Use the factor theorem to show that $(x + 3)$ is a factor of g(x) **(2)**

(b) Hence show that g(x) can be written in the form $g(x) = (x + 3)(ax + b)^2$, where a and b are constants to be found. **(4)**

a/ $f(-3) = 4(-3)^3 - 8(-3)^2 - 35(-3) + 75$

$= 0 \quad \therefore \quad (x+3)$ is a factor

b/
$$\begin{array}{r|l} & 4x^2 - 20x + 25 \\ \hline x+3 & 4x^3 - 8x^2 - 35x + 75 \\ & \underline{4x^3 + 12x^2} \\ & \quad -20x^2 - 35x \\ & \quad \underline{-20x^2 - 60x} \\ & \qquad 25x + 75 \\ & \qquad \underline{25x + 75} \\ & \qquad\quad 0 \end{array}$$

$g(x) = (x+3)(4x^2 - 20x + 25)$

$g(x) = (x+3)(2x-5)^2$

3 $\quad f(x) = x^3 + 6x^2 + px + q$

Given that $f(4) = 0$ and $f(-5) = 36$

(a) Find the values of p and q **(3)**

(b) Factorise $f(x)$ completely. **(4)**

a/ $(4)^3 + 6(4)^2 + 4p + q = 0$

$4p + q = -160$ (1)

$(-5)^3 + 6(-5)^2 - 5p + q = 36$

$-5p + q = 11$ (2)

Solve simultaneous equations $p = -19$ $q = -84$

b/ $(x-4)$ is a factor of $x^3 + 6x^2 - 19x - 84$

$$\begin{array}{r|l} & x^2 + 10x + 21 \\ \hline x - 4 & x^3 + 6x^2 - 19x - 84 \\ & \underline{x^3 - 4x^2} \\ & \qquad 10x^2 - 19x \\ & \qquad \underline{10x^2 - 40x} \\ & \qquad\qquad 21x - 84 \\ & \qquad\qquad \underline{21x - 84} \\ & \qquad\qquad\qquad 0 \end{array}$$

$f(x) = (x-4)(x^2 + 10x + 21)$

$= (x-4)(x+7)(x+3)$

4 $\mathrm{f}(x) = 2x^3 - x^2 - 13x + 14$

(a) Use the factor theorem to show that $(x - 2)$ is a factor of $\mathrm{f}(x)$ **(2)**

(b) Hence, or otherwise, solve the equation $2x^3 - x^2 - 13x + 14 = 0$ giving your answers to 2 decimal places where appropriate **(5)**

a/ $f(2) = 2(2)^3 - (2)^2 - 13(2) + 14$

$= 0$

$f(2) = 0 \quad \therefore \quad (x-2)$ is a factor

b/

$$\begin{array}{r|l} & 2x^2 + 3x - 7 \\ \hline x-2 & 2x^3 - x^2 - 13x + 14 \\ & 2x^3 - 4x^2 \\ \hline & 3x^2 - 13x \\ & 3x^2 - 6x \\ \hline & -7x + 14 \\ & -7x + 14 \\ \hline & 0 \end{array}$$

$(x-2)(2x^2 + 3x - 7) = 0$

$x = 2 \qquad x = \dfrac{-3 + \sqrt{65}}{4} \qquad x = \dfrac{-3 - \sqrt{65}}{4}$

5 $$f(x) = x^3 + kx - 2$$

(a) Given that $(x - 2)$ is a factor of $f(x)$ find the value of k **(2)**

(b) Solve the equation $f(x) = 0$ **(4)**

a/ $f(2) = 0$

$(2)^3 + 2k - 2 = 0$

$6 + 2k = 0$

$\underline{k = -3}$

b/ $(x - 2)$ is a factor of $x^3 - 3x - 2$

$$
\begin{array}{r|l}
 & x^2 + 2x + 1 \\
\hline
x - 2 & x^3 + 0x^2 - 3x - 2 \\
 & \underline{x^3 - 2x^2} \\
 & \quad 2x^2 - 3x \\
 & \quad \underline{2x^2 - 4x} \\
 & \qquad x - 2 \\
 & \qquad \underline{x - 2} \\
 & \qquad\quad 0
\end{array}
$$

$(x - 2)(x^2 + 2x + 1) = 0$

$(x - 2)(x + 1)^2 = 0$

$\underline{\underline{x = 2}}$ $\underline{\underline{x = -1}}$

6 $$f(x) = x^3 + 6x^2 + 4x - 15$$

(a) Use the factor theorem to show that $x = -3$ is a solution to $f(x) = 0$ **(2)**

(b) Find the other solutions to the equation $f(x) = 0$ giving your answers to 2 decimal places **(5)**

a/ $f(-3) = (-3)^3 + 6(-3)^2 + 4(-3) - 15$

$= 0$

$\therefore$ -3 is a solution

b/ $(x+3)$ is a factor

$$\begin{array}{r|l} & x^2 + 3x - 5 \\ x+3 & x^3 + 6x^2 + 4x - 15 \\ & x^3 + 3x^2 \\ & \quad 3x^2 + 4x \\ & \quad 3x^2 + 9x \\ & \qquad -5x - 15 \\ & \qquad -5x - 15 \\ & \qquad\quad 0 \end{array}$$

$(x+3)(x^2+3x-5) = 0$

$x = \dfrac{-3+\sqrt{29}}{2}$ $\quad$ $x = \dfrac{-3-\sqrt{29}}{2}$

7 Prove that $x^2 - 4x + 7$ is positive for all values of x

$(x-2)^2 - 4 + 7$

$\underbrace{(x-2)^2}_{\text{must be greater than or equal to zero}} + 3$

min value is 3 $\therefore$ positive for all values of x.

---

8 Disprove the statement: $n^2 - n + 3$ is a prime number for all values of n

Find a counter example

$10^2 - 10 + 3 = 93$

$93 = 3 \times 31$

$\therefore$ not prime

---

9 Prove that the sum of two consecutive odd numbers is a multiple of 4

$(2n+1) + (2n+3)$

$2n + 1 + 2n + 3$

$4n + 4$

$4(n+1)$ 4 × something $\therefore$ multiple of 4

10 Prove that $(x + y)^2 \neq x^2 + y^2$

Let $x = 2$ and $y = 3$

$(2+3)^2 = 25$ $2^2 + 3^2 = 13$

$25 \neq 13$

$\therefore (x+y)^2 \neq x^2 + y^2$

11 (a) Prove that $n^2 + n + 11$ is prime for all integers between 1 and 5. **(3)**

(b) Prove that $n^2 + n + 11$ is not prime for all values of n **(2)**

a/ $(1)^2 + (1) + 11 = 13$ ✓

$(2)^2 + (2) + 11 = 17$ ✓

$(3)^2 + (3) + 11 = 23$ ✓

$(4)^2 + (4) + 11 = 31$ ✓

$(5)^2 + (5) + 11 = 41$ ✓ (All prime)

b/ $(10)^2 + (10) + 11 = 121$ $11 \times 11 = 121$

$\therefore$ not prime for all values of n.

---

12 Prove by exhaustion that the sum of two even positive integers less than 10 is also even.

$2 + 2 = 4$ $4 + 4 = 8$ $6 + 6 = 12$

$2 + 4 = 6$ $4 + 6 = 10$ $6 + 8 = 14$

$2 + 6 = 8$ $4 + 8 = 12$

$2 + 8 = 10$ $8 + 8 = 16$

$\therefore$ the sum of two even positive integers less than 10 is even.

---

13 "If I multiply a number by 2 and add 5 the result is always greater than the original number."

State, giving a reason, whether the above statement is always true, sometimes true or never true.

Let $n = 5$ $5 \times 2 + 5 = 15$ $15 > 5$

let $n = -5$ $-5 \times 2 + 5 = -5$ $-5 = -5$

Sometimes true

14 Prove that $(2n + 3)^2 - (2n - 3)^2$ is a multiple of 6 for all values of n

$(2n+3)(2n+3) - (2n-3)(2n-3)$

$4n^2 + 6n + 6n + 9 - (4n^2 - 6n - 6n + 9)$

$24n$

$6(4n)$

$6 \times 4n$ is a multiple of 6 for all values of n.

---

15 Prove that the sum of the squares of two consecutive odd integers is always 2 more than a multiple of 8.

$(2n+1)^2 + (2n+3)^2$

$(4n^2 + 2n + 2n + 1) + (4n^2 + 6n + 6n + 9)$

$8n^2 + 16n + 10$

$8n^2 + 16n + 8 + 2$

$\underbrace{8(n^2 + 2n + 1)}_{\text{Multiple of 8}} + 2$

$\therefore$ always 2 more than a multiple of 8.

---

16 Prove that $n^2 + 7n + 15 > n + 3$ is true for all values of n

$n^2 + 6n + 12 > 0$

$(n+3)^2 - 9 + 12 > 0$

$\underbrace{(n+3)^2}_{\text{must be greater than or equal to zero}} + 3 > 0$

Min value $= 3$ $\therefore$ $n^2 + 6n + 12$ is always greater than zero.

$\therefore$ $n^2 + 7n + 15 > n + 3$

17 (a) Prove that for positive values of a and b

$$\frac{9a}{b} + \frac{4b}{a} \geq 12$$

(4)

(b) Prove, by counter example, that this is not true for all values of a and b. **(1)**

a/ $\frac{9a^2}{b} + 4b \geqslant 12a$

$9a^2 + 4b^2 \geqslant 12ab$

$9a^2 - 12ab + 4b^2 \geqslant 0$

$(3a-2b)(3a-2b) \geqslant 0$

$\underbrace{(3a-2b)^2}$ $\geqslant 0$

any number squared is greater than or equal to zero.

b/ let $a = -2$ and $b = 1$

$$\frac{9(-2)}{1} + \frac{4(1)}{-2} = -20$$

-20 is less than 12 $\therefore$ not true for all values of a and b.

18 A student is investigating the following statement about natural numbers.

"$n^3 - n$ is a multiple of 4"

(a) Prove, using algebra, that the statement is true for all odd numbers. **(4)**

(b) Use a counterexample to show that the statement is not always true. **(1)**

a) $(2n+1)^3 - (2n+1)$

$(2n+1)(2n+1)(2n+1) - (2n+1)$

$(2n+1)(4n^2+4n+1) - (2n+1)$

$8n^3 + 4n^2 + 8n^2 + 4n + 2n + 1 - 2n - 1$

$8n^3 + 12n^2 + 4n$

$4(2n^3 + 3n^2 + 1)$ $\therefore$ $n^3 - n$ is multiple of 4 for all odd numbers

b) let $n = 2$ $2^3 - 2 = 6$ 6 is not a multiple of 4

$\therefore$ not always true

---

19 Prove that

n is a prime number greater than 5 $\Rightarrow n^4$ has final digit 1

all prime numbes greater than 5 are not even

$\therefore$ do not end in 0, 2, 4, 6 or 8

do not end in 5 (or they would be divisible by 5)

$\therefore$ final digit is 3, 7 or 9.

$3^4 = 81$ $7^4 = 2401$ $9^4 = 6561$

$\therefore$ final digit will be 1 for all prime numbers greater than 5.

20 Jacob believes that for all positive integers $n^3 + n + 2$ is a multiple of 4.

Which value of n shown below is a counter example to Dan's belief?

$n = 2$ $n = 3$ $n = 4$ $n = 5$

$4^3 + 4 + 2 = 70$

---

21 Given $(x + 3)$ and $(x - 5)$ are both factors of $f(x)$, where

$$f(x) = px^3 - 7x^2 - 43x + q$$

Find the values of p and q.

$f(-3) = p(-3) - 7(-3)^2 - 43(-3) + q$

$0 = -3p + 66 + q$

$3p - q = 66$ ①

$f(5) = p(5) - 7(5)^2 - 43(5) + q$

$0 = 5p - 390 + q$

$5p + q = 390$ ②

Solve sim. eq. $\underline{\underline{p = 57}}$ $\underline{\underline{q = 105}}$

22 $$f(x) = x^3 - 4x^2 + 9x - 10$$

(a) Use the factor theorem to show that $(x - 2)$ is a factor of $f(x)$. **(2)**

(b) Show that $f(x) = 0$ only has one real root. **(4)**

a/ $f(2) = (2)^3 - 4(2)^2 + 9(2) - 10$
$= 0$

$f(2) = 0 \quad \therefore \quad (x-2)$ is a factor

b/
$$\begin{array}{r|l} & x^2 - 2x + 5 \\ \hline x - 2 & x^3 - 4x^2 + 9x - 10 \\ & \underline{x^3 - 2x^2} \\ & \quad -2x^2 + 9x \\ & \quad \underline{-2x^2 + 4x} \\ & \qquad\qquad 5x - 10 \\ & \qquad\qquad \underline{5x - 10} \\ & \qquad\qquad\quad 0 \end{array}$$

$(x-2)(x^2 - 2x + 5)$
$x = 2$

$x^2 - 2x + 5$ has no real roots as
$(-2)^2 - 4(1)(5) = -16 \quad b^2 - 4ac < 0 \quad \therefore$ no real roots

23 (a) Express $n^3 - n$ as a product of three factors. **(1)**

(b) Given that n is a positive integer, prove that $n^3 - n$ is a multiple of 6 **(3)**

a/ $n(n^2 - 1)$

$n(n+1)(n-1)$

b/ $(n-1) \times n \times (n+1)$ is the product of three consecutive numbers.

Every third number is a multiple of 3.

$\therefore$ $(n-1)$ or n or $(n+1)$ must be a multiple of 3.

either n or $(n+1)$ must be a multiple of 2

Multiple of 2 $\times$ multiple of 3 = multiple of 6.

$\therefore$ $n(n+1)(n-1)$ is a multiple of 6.

24 (a) Prove that for all real values of x:

$$x^2 + 4x > 2x - 4$$ **(3)**

(b) "If I add 2 to a number and then square the sum, the result is greater than the square of the original number."

State, giving a reason, if the above statement is always true, sometimes true or never true. **(2)**

a) $x^2 + 2x + 4 > 0$

$(x+1)^2 - 1 + 4 > 0$

$(x+1)^2 + 3 > 0$ Min value = 3

$(x+1)^2$ is greater than or equal to zero.

$\therefore (x+1)^2 + 3 > 0$

b) $(n+2)^2 > n^2$

True for positive numbers eg let $n = 2$

$(2+2)^2 = 16$ $2^2 = 4$

$16 > 4$

Not true for all negative numbers

eg. $(^-3+2)^2 = 1$ $(-3)^2 = 9$

$\therefore$ Sometimes true

25 $$f(x) = 4x^3 - 8x^2 - 35x + 75$$

(a) Use the factor theorem to show that $(x + 3)$ is a factor of $f(x)$. **(2)**

(b) Hence, show that $f(x)$ can be written in the form $f(x) = (x + 3)(ax + b)^2$, where a and b are integers to be found. **(4)**

a/ $f(-3) = 4(-3)^3 - 8(-3)^2 - 35(-3) + 75$

$= 0$

$\therefore (x+3)$ is a factor.

b/

$$\begin{array}{r|l} & 4x^2 - 20x + 25 \\ \hline x+3 & 4x^3 - 8x^2 - 35x + 75 \\ & 4x^3 + 12x^2 \\ \hline & -20x^2 - 35x \\ & -20x^2 - 60x \\ \hline & 25x + 75 \\ & 25x + 75 \\ \hline & 0 \end{array}$$

$(x+3)(4x^2 - 20x + 25)$

$\underline{\underline{(x+3)(2x-5)^2}}$

26 $f(x) = 2x^3 - 5x^2 - 4x + 12$

(a) Use the factor theorem to show that $(x - 2)$ is a factor of $f(x)$. **(2)**

(b) Hence, show that $f(x) = 0$ only has two distinct real roots **(4)**

(c) Sketch the graph of $y = f(x)$ **(3)**

(d) Deduce, giving a reason for your answer, the number of real roots of the equation:

$$2x^3 - 5x^2 - 4x + 10 = 0$$ **(2)**

Given that k is a constant and the curve with equation $y = f(x + k)$ passes through the origin,

(e) find the two possible values of k **(2)**

a/ $f(2) = 2(2)^3 - 5(2)^2 - 4(2) + 12$

$= 0$

$\therefore$ $(x-2)$ is a factor of $f(x)$.

b/

$$\begin{array}{r|l} & 2x^2 - x - 6 \\ \hline x - 2 & 2x^3 - 5x^2 - 4x + 12 \\ & 2x^3 - 4x^2 \\ \hline & -x^2 - 4x \\ & -x^2 + 2x \\ \hline & -6x + 12 \\ & -6x + 12 \\ \hline & 0 \end{array}$$

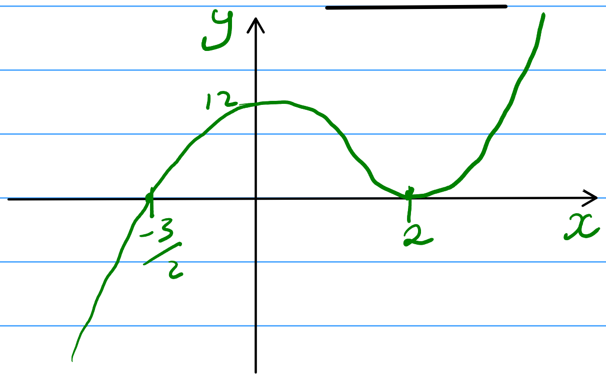
$(x-2)(2x^2 - x - 6)$

$(x-2)(2x+3)(x-2)$

$(2x+3)(x-2)^2$

$x = -\frac{3}{2}$ $x = 2$

c/



d/ 3 real roots

$f(x) - 2$ will cross the x axis 3 times.

e/ $\frac{3}{2}$ or -2

27 Prove that $n^3 + 2$ is not divisible by 8

n can be either even or odd

if n is even $(2n)^3 + 2$

$8n^3 + 2$

$8(n^3) + 2$

2 more than a multiple of 8 is not divisible by 8.

if n is odd $(2n+1)^3 + 2$

$(2n+1)(2n+1)(2n+1) + 2$

$(2n+1)(4n^2+4n+1) + 2$

$8n^3+4n^2+8n^2+4n+2n+1+2$

$8n^3+12n^2+6n+3$

$2(4n^3+6n^2+3n)+3$

even + 3 is odd. Odd numbers are not divisible by 8.

$\therefore$ $n^3 + 2$ is not divisible by 8.

28 Prove that all cube numbers are either a multiple of 9 or 1 more or one less than a multiple of 9.

1, 8, 27, 64, 125, 216

one more, one less, multiple of 9, one more, one less, multiple of 9

All numbers are $3n$, $3n-1$ or $3n+1$

(three times table or one more or one less)

$$(3n)^3 = 27n^3 = 9(3n^3)$$

multiple of 9

$$\begin{aligned}(3n-1)^3 &= (3n-1)(3n-1)(3n-1)\\ &= (3n-1)(9n^2-6n+1)\\ &= 27n^3-9n^2-18n^2+6n+3n-1\\ &= 27n^3-27n^2+9n-1\\ &= 9(3n^3-9n^2+n)-1\end{aligned}$$

one less than a multiple of 9

$$\begin{aligned}(3n+1)^3 &= (3n+1)(3n+1)(3n+1)\\ &= (3n+1)(9n^2+6n+1)\\ &= 27n^3+9n^2+18n^2+6n+3n+1\\ &= 27n^3+27n^2+9n+1\\ &= 9(3n^3+3n^2+1)+1\end{aligned}$$

one more than a multiple of 9.

$\therefore$ all cube numbers are either a multiple of 9, one more than a multiple of 9 or one less than a multiple of 9.

29 $$g(x) = 2x^3 + x^2 - 38x + 35$$

(a) Use the factor theorem to show that $(x + 5)$ is a factor of $g(x)$. **(2)**

(b) Hence, show that $g(x)$ can be written as the product of three linear factors. **(4)**

a/ $g(-5) = 2(-5)^3 + (-5)^2 - 38(-5) + 35$
$= 0$

$\therefore$ $(x+5)$ is a factor.

$$\begin{array}{r|l} & 2x^2 - 9x + 7 \\ \hline x+5 & 2x^3 + x^2 - 38x + 35 \\ & \underline{2x^3 + 10x^2} \\ & -9x^2 - 38x \\ & \underline{-9x^2 - 45x} \\ & 7x + 35 \\ & \underline{7x + 35} \\ & 0 \end{array}$$

$g(x) = (x+5)(2x^2 - 9x + 7)$
$g(x) = \underline{\underline{(x+5)(2x-7)(x-1)}}$

30 N is an integer that is not divisible by 3. Prove that N^2 is of the form $3p + 1$, where p is an integer.

N is one more or one less than a multiple of 3.

$N = 3n+1$ or $N = 3n-1$

$$\begin{aligned} N^2 &= (3n+1)^2 \\ &= (3n+1)(3n+1) \\ &= 9n^2+6n+1 \\ &= (9n^2+6n)+1 \\ &= 3(3n^2+2n)+1 \end{aligned} \qquad \begin{aligned} N^2 &= (3n-1)^2 \\ &= (3n-1)(3n-1) \\ &= 9n^2-6n+1 \\ &= (9n^2-6n)+1 \\ &= 3(3n^2-2n)+1 \end{aligned}$$

$3n^2+2n$ and $3n^2-2n$ are integers $\therefore$ N^2 is in the form $3p+1$.

31 **In this question you must show detailed reasoning.**

The polynomial f(x) is defined by f(x) = $2x^3 + 9x^2 - 6x - 5$

(a) Show that f(1) = 0 **(1)**

(b) Solve the equation f(x) = 0 **(4)**

a) $f(1) = 2(1)^3 + 9(1)^2 - 6(1) - 5$

$= 0$

b) $(x-1)$ is a factor

$$\begin{array}{r|l} & 2x^2 + 11x + 5 \\ \hline x - 1 & 2x^3 + 9x^2 - 6x - 5 \\ & 2x^3 - 2x^2 \\ \hline & \quad 11x^2 - 6x \\ & \quad 11x^2 - 11x \\ \hline & \qquad 5x - 5 \\ & \qquad 5x - 5 \\ \hline & \qquad\quad 0 \end{array}$$

$(x-1)(2x^2 + 11x + 5) = 0$

$(x-1)(2x+1)(x+5) = 0$

$x = 1$ $\quad x = -\frac{1}{2}$ $\quad x = -5$

32 (a) Prove that the following statement is not true.

m is an odd number greater than $1 \Rightarrow m^2 + 4$ is prime. **(1)**

(b) By considering separately the case when n is odd and the case when n is even, prove that the following statement is true.

n is a positive integer $\Rightarrow n^2 + 1$ is not a multiple of 4. **(4)**

a/ Let $m = 9$

$9^2 + 4 = 85 \qquad 85 = 5 \times 17 \therefore$ not prime.

$\therefore m^2 + 4$ is not always prime.

b/ Let $n = 2m$ (even)

$(2m)^2 + 1 = 4m^2 + 1$

one more than a multiple of 4 is not a multiple of 4.

Let $n = 2m+1$ (odd)

$$(2m+1)^2 + 1 = 4m^2 + 4m + 1 + 1$$
$$= 4(m^2 + m) + 2$$

two more than a multiple of 4 is not a multiple of 4

$\therefore n^2 + 1$ is not a multiple of 4.

33 The cubic polynomial $6x^3 + kx^2 + 25x - 12$ is denoted by $f(x)$.
It is given that $(2x + 3)$ is a factor of $f(x)$.

(a) Use the factor theorem to show that $k = 25$. **(2)**

(b) Using this value of k, factorise $f(x)$ completely. **(3)**

a/ $f\left(-\frac{3}{2}\right) = 0$

$$6\left(-\frac{3}{2}\right)^3 + k\left(-\frac{3}{2}\right)^2 + 25\left(-\frac{3}{2}\right) - 12 = 0$$

$$k\left(\frac{9}{4}\right) - \frac{279}{4} = 0$$

$$k = \frac{279}{4} \div \frac{9}{4}$$

$$= 31$$

b/

$$\begin{array}{r|l}
 & 3x^2 + 11x - 4 \\
\hline
2x + 3 & 6x^3 + 31x^2 + 25x - 12 \\
 & \underline{6x^3 + 9x^2} \\
 & \quad 22x^2 + 25x \\
 & \quad \underline{22x^2 + 33x} \\
 & \qquad -8x - 12 \\
 & \qquad \underline{-8x - 12} \\
 & \qquad\quad 0
\end{array}$$

$f(x) = (2x + 3)(3x - 1)(x + 4)$

34 $f(x) = 3x^3 - 19x + 24$

(a) Use the factor theorem to show that $(x + 3)$ is a factor of $f(x)$. **(2)**

(b) Solve the equation $f(x) = 0$. **(3)**

$$f(-3) = 3(-3)^3 - 19(-3) + 24$$
$$= 0$$

$\therefore$ $(x+3)$ is a factor

$$\begin{array}{r|l} & 3x^2 - 9x + 8 \\ \hline x+3 & 3x^3 + 0x^2 - 19x + 24 \\ & \underline{3x^3 + 9x^2} \\ & \quad -9x^2 - 19x \\ & \quad \underline{-9x^2 - 27x} \\ & \qquad 8x + 24 \\ & \qquad \underline{8x + 24} \\ & \qquad\quad 0 \end{array}$$

$$(x+3)\underbrace{(3x^2 - 9x + 8)}_{\text{no real solutions}} = 0$$

$x = -3$

35 Find a counter example to disprove the statements:

(a) When n is a prime number n^2 is always odd **(2)**

(b) $(x + 2)^2$ is always greater than $x^2 + 2$ **(2)**

let $n=2$

a/ $2^2 = 4$ 4 is not odd

$\therefore$ n^2 is not always odd

b/ let $x = -2$

$(-2+2)^2 = 0$ $(-2)^2 + 2 = 6$

0 is not greater than 6 $\therefore$ $(x+2)^2$ is not always greater than $x^2 + 2$