AS Level Maths: Algebraic Methods

1 $$f(x) = 2x^3 - 7x^2 - 17x + 10$$

Use the factor theorem and division to factorise f(x) completely.

(Total for question 1 is 6 marks)

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2 $$g(x) = 4x^3 - 8x^2 - 35x + 75$$

(a) Use the factor theorem to show that $(x + 3)$ is a factor of g(x) **(2)**

(b) Hence show that g(x) can be written in the form $g(x) = (x + 3)(ax + b)^2$, where a and b are constants to be found. **(4)**

(Total for question 2 is 6 marks)

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3 $$f(x) = x^3 + 6x^2 + px + q$$

Given that f(4) = 0 and f(-5) = 36

(a) Find the values of p and q **(3)**

(b) Factorise f(x) completely. **(4)**

(Total for question 3 is 7 marks)

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4 $$f(x) = 2x^3 - x^2 - 13x + 14$$

(a) Use the factor theorem to show that $(x - 2)$ is a factor of f(x) **(2)**

(b) Hence, or otherwise, solve the equation $2x^3 - x^2 - 13x + 14 = 0$ giving your answers to 2 decimal places where appropriate **(5)**

(Total for question 4 is 7 marks)

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5 $$f(x) = x^3 + kx - 2$$

(a) Given that $(x - 2)$ is a factor of f(x) find the value of k **(2)**

(b) Solve the equation f(x) = 0 **(4)**

(Total for question 5 is 6 marks)

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6 $$f(x) = x^3 + 6x^2 + 4x - 15$$

(a) Use the factor theorem to show that $x = -3$ is a solution to f(x) = 0 **(2)**

(b) Find the other solutions to the equation f(x) = 0 giving your answers to 2 decimal places **(5)**

(Total for question 6 is 7 marks)

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7 Prove that $x^2 - 4x + 7$ is positive for all values of x

(Total for question 7 is 3 marks)

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8 Disprove the statement: $n^2 - n + 3$ is a prime number for all values of n

(Total for question 8 is 2 marks)

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9 Prove that the sum of two consecutive odd numbers is a multiple of 4

(Total for question 9 is 3 marks)

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10 Prove that $(x + y)^2 \neq x^2 + y^2$

(Total for question 10 is 3 marks)

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11 (a) Prove that $n^2 + n + 11$ is prime for all integers between 1 and 5. **(3)**

(b) Prove that $n^2 + n + 11$ is not prime for all values of n **(2)**

(Total for question 11 is 6 marks)

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12 Prove by exhaustion that the sum of two even positive integers less than 10 is also even.

(Total for question 12 is 3 marks)

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13 "If I multiply a number by 2 and add 5 the result is always greater than the original number."

State, giving a reason, whether the above statement is always true, sometimes true or never true.

(Total for question 13 is 2 marks)

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14 Prove that $(2n + 3)^2 - (2n - 3)^2$ is a multiple of 6 for all values of n

(Total for question 14 is 4 marks)

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15 Prove that the sum of the squares of two consecutive odd integers is always 2 more than a multiple of 8.

(Total for question 15 is 4 marks)

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16 Prove that $n^2 + 7n + 15 > n + 3$ is true for all values of n

(Total for question 16 is 4 marks)

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17 (a) Prove that for positive values of a and b

$$\frac{9a}{b} + \frac{4b}{a} \geq 12$$

(4)

(b) Prove, by counter example, that this is not true for all values of a and b. **(1)**

(Total for question 17 is 5 marks)

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18 A student is investigating the following statement about natural numbers.

"$n^3 - n$ is a multiple of 4 "

(a) Prove, using algebra, that the statement is true for all odd numbers. **(4)**

(b) Use a counterexample to show that the statement is not always true. **(1)**

(Total for question 18 is 5 marks)

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19 Prove that

n is a prime number greater than 5 $\Rightarrow$ n^4 has final digit 1

(Total for question 19 is 5 marks)

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20 Jacob believes that for all positive integers $n^3 + n + 2$ is a multiple of 4.

Which value of n shown below is a counter example to Dan's belief?

$n = 2$ $n = 3$ $n = 4$ $n = 5$

(Total for question 20 is 1 mark)

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21 Given $(x + 3)$ and $(x - 5)$ are both factors of f(x), where

$$f(x) = px^3 - 7x^2 - 43x + q$$

Find the values of p and q.

(Total for question 21 is 3 marks)

22

$$f(x) = x^3 - 4x^2 + 9x - 10$$

(a) Use the factor theorem to show that $(x - 2)$ is a factor of f(x). **(2)**

(b) Show that f(x) = 0 only has one real root. **(4)**

(Total for question 22 is 6 marks)

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23 (a) Express $n^3 - n$ as a product of three factors. **(1)**

(b) Given that n is a positive integer, prove that $n^3 - n$ is a multiple of 6 **(3)**

(Total for question 23 is 4 marks)

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24 (a) Prove that for all real values of x:

$$x^2 + 4x > 2x - 4$$ **(3)**

(b) "If I add 2 to a number and then square the sum, the result is greater than the square of the original number."

State, giving a reason, if the above statement is always true, sometimes true or never true. **(2)**

(Total for question 24 is 5 marks)

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25 $$f(x) = 4x^3 - 8x^2 - 35x + 75$$

(a) Use the factor theorem to show that $(x + 3)$ is a factor of $f(x)$. **(2)**

(b) Hence, show that $f(x)$ can be written in the form $f(x) = (x + 3)(ax + b)^2$, where a and b are integers to be found. **(4)**

(Total for question 25 is 6 marks)

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26 $$f(x) = 2x^3 - 5x^2 - 4x + 12$$

(a) Use the factor theorem to show that $(x - 2)$ is a factor of $f(x)$. **(2)**

(b) Hence, show that $f(x) = 0$ only has two distinct real roots **(4)**

(c) Sketch the graph of $y = f(x)$ **(3)**

(d) Deduce, giving a reason for your answer, the number of real roots of the equation:

$$2x^3 - 5x^2 - 4x + 10 = 0$$ **(2)**

Given that k is a constant and the curve with equation $y = f(x + k)$ passes through the origin,

(e) find the two possible values of k **(2)**

(Total for question 26 is 13 marks)

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27 Prove that $n^3 + 2$ is not divisible by 8

(Total for question 27 is 4 marks)

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28 Prove that all cube numbers are either a multiple of 9 or 1 more or one less than a multiple of 9.

(Total for question 28 is 4 marks)

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29 $$g(x) = 2x^3 + x^2 - 38x + 35$$

(a) Use the factor theorem to show that $(x + 5)$ is a factor of $g(x)$. **(2)**

(b) Hence, show that $g(x)$ can be written as the product of three linear factors. **(4)**

(Total for question 29 is 6 marks)

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30 N is an integer that is not divisible by 3. Prove that N^2 is of the form $3p + 1$, where p is an integer.

(Total for question 30 is 5 marks)

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31 **In this question you must show detailed reasoning.**

The polynomial f(x) is defined by $f(x) = 2x^3 + 9x^2 - 6x - 5$

(a) Show that $f(1) = 0$ **(1)**

(b) Solve the equation $f(x) = 0$ **(4)**

(Total for question 31 is 5 marks)

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32 (a) Prove that the following statement is not true.

m is an odd number greater than $1 \Rightarrow m^2 + 4$ is prime. **(1)**

(b) By considering separately the case when n is odd and the case when n is even, prove that the following statement is true.

n is a positive integer $\Rightarrow n^2 + 1$ is not a multiple of 4. **(4)**

(Total for question 32 is 5 marks)

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33 The cubic polynomial $6x^3 + kx^2 + 25x - 12$ is denoted by f (x).
It is given that $(2x + 3)$ is a factor of f(x).

(a) Use the factor theorem to show that $k = 31$. **(2)**

(b) Using this value of k, factorise f (x) completely. **(3)**

(Total for question 33 is 5 marks)

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34 $f(x) = 3x^3 - 19x + 24$

(a) Use the factor theorem to show that $(x + 3)$ is a factor of f(x). **(2)**

(b) Solve the equation $f(x) = 0$. **(3)**

(Total for question 34 is 5 marks)

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35 Find a counter example to disprove the statements:

(a) When n is a prime number n^2 is always odd **(2)**

(b) $(x + 2)^2$ is always greater than $x^2 + 2$ **(2)**

(Total for question 35 is 4 marks)
