

**1.** f (*x*) = *ax*3 + 10*x*2 − 3*ax* − 4

Given that (*x* − 1) is a factor of f (*x*), find the value of the constant *a*.

You must make your method clear.

**(3)**

**(Total for Question 1 is 3 marks)**

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**2.** Given that

f (*x*) = *x*2 − 4*x* + 5 *x* ∈ ℝ

(*a*)express f (*x*) in the form (*x* + *a*)2 + *b* where *a* and *b* are integers to be found.

**(2)**

The curve with equation *y* = f (*x*)

* meets the *y*-axis at the point *P*
* has a minimum turning point at the point *Q*

(*b*)Write down

(i) the coordinates of *P*

(ii) the coordinates of *Q*

**(2)**

**(Total for Question 2 is 4 marks)**

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**3.** The sequence *u*1, *u*2, *u*3, … is defined by

* u*1 = 2

where *k* is an integer.

Given that *u*1 + 2*u*2 + *u*3 = 0

(*a*)show that

3*k*2 − 58*k* + 240 = 0

**(3)**

(*b*)Find the value of *k*, giving a reason for your answer.

**(2)**

(*c*)Find the value of *u*3

**(1)**

**(Total for Question 3 is 6 marks)**

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**4.** The curve with equation *y* = f (*x*) where

f (*x*) = *x*2 + ln (2*x*2 − 4*x* + 5)

has a single turning point at *x* = *α*

(*a*)Show that *α* is a solution of the equation

2*x*3 − 4*x*2 + 7*x* − 2 = 0

**(4)**

The iterative formula



is used to find an approximate value for *α*.

Starting with *x*1 = 0.3

(*b*)calculate, giving each answer to 4 decimal places,

(i) the value of *x*2

(ii) the value of *x*4

**(3)**

Using a suitable interval and a suitable function that should be stated,

(*c*)show that *α* is 0.341 to 3 decimal places.

**(2)**

**(Total for Question 4 is 9 marks)**

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**5. In this question you should show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

A company made a profit of £20 000 in its first year of trading, Year 1

A model for future trading predicts that the yearly profit will increase by 8% each year,

so that the yearly profits will form a geometric sequence.

According to the model,

(*a*)show that the profit for Year 3 will be £23 328

**(1)**

(*b*)find the first year when the yearly profit will exceed £65 000

**(3)**

(*c*)find the total profit for the first 20 years of trading, giving your answer to the

nearest £1000

**(2)**

**(Total for Question 5 is 6 marks)**

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**6.**

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Figure 1 shows a sketch of triangle *ABC*.

Given that

* 
* 

(*a*) find 

**(2)**

(*b*)show that cos *ABC* = 

**(3)**

**(Total for Question 6 is 5 marks)**

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**7.** The circle *C* has equation

*x*2 + *y*2 − 10*x* + 4*y* + 11 = 0

(*a*)Find

(i) the coordinates of the centre of *C*,

(ii) the exact radius of *C*, giving your answer as a simplified surd.

**(4)**

The line *l* has equation *y* = 3*x* + *k* where *k* is a constant.

Given that *l* is a tangent to *C*,

(*b*)find the possible values of *k*, giving your answers as simplified surds.

**(5)**

**(Total for Question 7 is 9 marks)**

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**8.** A scientist is studying the growth of two different populations of bacteria.

The number of bacteria, *N*, in the **first** population is modelled by the equation

*N* = *A*e*kt t* ≥ 0

where *A* and *k* are positive constants and *t* is the time in hours from the start of the study.

Given that

* there were 1000 bacteria in this population at the start of the study
* it took exactly 5 hours from the start of the study for this population to double

(*a*)find a complete equation for the model.

**(4)**

(*b*)Hence find the rate of increase in the number of bacteria in this population exactly

8 hours from the start of the study. Give your answer to 2 significant figures.

**(2)**

The number of bacteria, *M*, in the **second** population is modelled by the equation

*M* = 500e1.4*kt t* ≥ 0

where *k* has the value found in part (*a*)and *t* is the time in hours from the start of the study.

Given that *T* hours after the start of the study, the number of bacteria in the two different

populations was the same,

(*c*)find the value of *T*.

**(3)**

**(Total for Question 8 is 9 marks)**

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**9.**



Given that f(*x*) can be expressed in the form



where *A*, *B* and *C* are constants

(*a*)(i) find the value of *B* and the value of *C*

(ii) show that *A* = 0

**(4)**

(*b*)(i) Use binomial expansions to show that, in ascending powers of *x*

f(*x*) = *p* + *qx* + *rx*2 + …

where *p*, *q* and *r* are simplified fractions to be found.

(ii) Find the range of values of *x* for which this expansion is valid.

**(7)**

**(Total for Question 9 is 11 marks)**

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**10. In this question you should show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**

(*a*)Given that 1 + cos 2*θ* + sin 2*θ* ≠ 0 prove that



**(4)**

(*b*)Hence solve, for 0 < *x* < 180°



giving your answers to one decimal place where appropriate.

**(4)**

**(Total for Question 10 is 8 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**11.**

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Figure 2 shows a sketch of part of the curve with equation

*y* = (ln *x*)2 *x* > 0

The finite region *R*, shown shaded in Figure 2, is bounded by the curve, the line with

equation *x* = 2, the *x*-axis and the line with equation *x* = 4

The table below shows corresponding values of *x* and *y*, with the values of *y* given to 4

decimal places.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *x* | 2 | 2.5 | 3 | 3.5 | 4 |
| *y* | 0.4805 | 0.8396 | 1.2069 | 1.5694 | 1.9218 |

(*a*)Use the trapezium rule, with all the values of *y* in the table, to obtain an estimate for

the area of *R*, giving your answer to 3 significant figures.

**(3)**

(*b*)Use algebraic integration to find the exact area of *R*, giving your answer in the form

*y* = *a* (ln 2)2 + *b* ln 2 + *c*

where *a*, *b* and *c* are integers to be found.

**(5)**

**(Total for Question 11 is 8 marks)**

**\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_**

**12.**

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Figure 3 is a graph of the trajectory of a golf ball after the ball has been hit until it first

hits the ground.

The vertical height, *H* metres, of the ball above the ground has been plotted against the

horizontal distance travelled, *x* metres, measured from where the ball was hit.

The ball is modelled as a particle travelling in a vertical plane above horizontal ground.

Given that the ball

* is hit from a point on the top of a platform of vertical height 3 m above the ground
* reaches its maximum vertical height after travelling a horizontal distance of 90 m
* is at a vertical height of 27 m above the ground after travelling a horizontal  
  distance of 120 m

Given also that *H* is modelled as a **quadratic** function in *x*

(*a*)find *H* in terms of *x*

**(5)**

(*b*)Hence find, according to the model,

(i) the maximum vertical height of the ball above the ground,

(ii) the horizontal distance travelled by the ball, from when it was hit to when it first

hits the ground, giving your answer to the nearest metre.

**(3)**

(*c*)The possible effects of wind or air resistance are two limitations of the model.

Give one other limitation of this model.

**(1)**

**(Total for Question 12 is 9 marks)**

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**13.** A curve *C* has parametric equations

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Show that all points on *C* satisfy

(*x* − 3)2 + *y*2 = 4

**(3)**

**(Total for Question 13 is 3 marks)**

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**14.** Given that



show that



where *A* is a constant to be found.

**(4)**

**(Total for Question 14 is 4 marks)**

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**15.** (i) Use proof by exhaustion to show that for *n* ∈ ℕ, *n* ≤ 4

(*n* + 1)3 > 3*n*

**(2)**

(ii) Given that *m*3 + 5 is odd, use proof by contradiction to show, using algebra, that *m*

is even.

**(4)**

**(Total for Question 15 is 6 marks)**

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**TOTAL FOR PAPER IS 100 MARKS**