Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel Level 3 GCE

Tuesday 4 June 2024

Afternoon (Time: 2 hours)

Paper reference **9MA0/01**

Mathematics

Advanced

PAPER 1: Pure Mathematics 1

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 15 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

Pearson

1.
$$g(x) = 3x^3 - 20x^2 + (k+17)x + k$$

where k is a constant.

Given that $(x - 3)$ is a factor of $g(x)$, find the value of k.

(3)

$g(3) = 0$

$3(3)^3 - 20(3)^2 + (k+17)3 + k = 0$

$81 - 180 + 3k + 51 + k = 0$

$4k = 48$

$k = 12$

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Question 1 continued

(Total for Question 1 is 3 marks)

2. (a) Find, in ascending powers of x, the first four terms of the binomial expansion of

$$(1-9x)^{\frac{1}{2}}$$

giving each term in simplest form.

(3)

(b) Give a reason why $x = -\frac{2}{9}$ should **not** be used in the expansion to find an approximation to $\sqrt{3}$

(1)

a/ $1 + \frac{1}{2}(-9x) + \frac{(\frac{1}{2})(-\frac{1}{2})}{2}(-9x)^2 + \frac{(\frac{1}{2})(-\frac{1}{2})(-\frac{3}{2})}{6}(-9x)^3$

$1 - \frac{9}{2}x - \frac{81}{8}x^2 - \frac{729}{16}x^3$

b/ The expansion is valid when $|9x| < 1$

$x < |\frac{1}{9}|$

x has to be between $-\frac{1}{9}$ and $\frac{1}{9}$

$\therefore$ the expansion is not valid for $x = \frac{2}{9}$

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Question 2 continued

(Total for Question 2 is 4 marks)

3. $$\mathrm{f}(x) = x + \tan\left(\frac{1}{2}x\right) \qquad \pi < x < \frac{3\pi}{2}$$

Given that the equation $\mathrm{f}(x) = 0$ has a single root α

(a) show that α lies in the interval [3.6, 3.7]

(2)

(b) Find $\mathrm{f}'(x)$

(2)

(c) Using 3.7 as a first approximation for α, apply the Newton–Raphson method once to obtain a second approximation for α. Give your answer to 3 decimal places.

(2)

a/ $x + \tan(\frac{1}{2}x) = 0$

$3.6 + \tan(\frac{1}{2} \cdot 3.6) = -0.686$

$3.7 + \tan(\frac{1}{2} \cdot 3.7) = 0.212$

change of sign and the function is continuous in the interval ∴ there is a root in the interval [3.6, 3.7]

b/ $f(x) = x + \tan(\frac{1}{2}x)$

$f'(x) = 1 + \frac{1}{2}\sec^2(\frac{1}{2}x)$

c/ $$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$x_1 = 3.7$

$$x_2 = 3.7 - \frac{3.7 + \tan(\frac{1}{2} \cdot 3.7)}{1 + \frac{1}{2}\sec^2(\frac{1}{2} \cdot 3.7)}$$

$$= 3.7 - \frac{0.212}{7.58}$$

$$= \underline{\underline{3.672}}$$

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Question 3 continued

(Total for Question 3 is 6 marks)

4. Given that $y = x^2$, use differentiation from first principles to show that $\frac{dy}{dx} = 2x$

(3)

$f(x) = x^2$

$f(x+h) = (x+h)^2$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{x^2 + 2hx + h^2 - x^2}{h}$$

$$= \lim_{h \to 0} \frac{2hx + h^2}{h}$$

$$= \lim_{h \to 0} (2x + h)$$

$$= \underline{\underline{2x}}$$

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P 7 5 6 9 3 A 0 8 4 4

Question 4 continued

(Total for Question 4 is 3 marks)

5. The function f is defined by

$$f(x) = \frac{2x - 3}{x^2 + 4} \qquad x \in \mathbb{R}$$

(a) Show that

$$f'(x) = \frac{ax^2 + bx + c}{\left(x^2 + 4\right)^2}$$

where a, b and c are constants to be found.

(3)

(b) Hence, using algebra, find the values of x for which f is decreasing.
You must show each step in your working.

(3)

a/ $u = 2x - 3$ $\qquad v = x^2 + 4$

$\frac{du}{dx} = 2$ $\qquad \frac{dv}{dx} = 2x$

$$f'(x) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$$= \frac{2(x^2+4) - 2x(2x-3)}{(x^2+4)^2}$$

$$= \frac{2x^2 + 8 - 4x^2 + 6x}{(x^2+4)^2}$$

$$= \frac{-2x^2 + 6x + 8}{(x^2+4)^2}$$

b/ $\frac{-2x^2 + 6x + 8}{(x^2+4)^2} < 0$

$-2x^2 + 6x + 8 < 0$

$\underline{\underline{x < -1}}$ or $\underline{\underline{x > 4}}$

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Question 5 continued

(Total for Question 5 is 6 marks)

6.

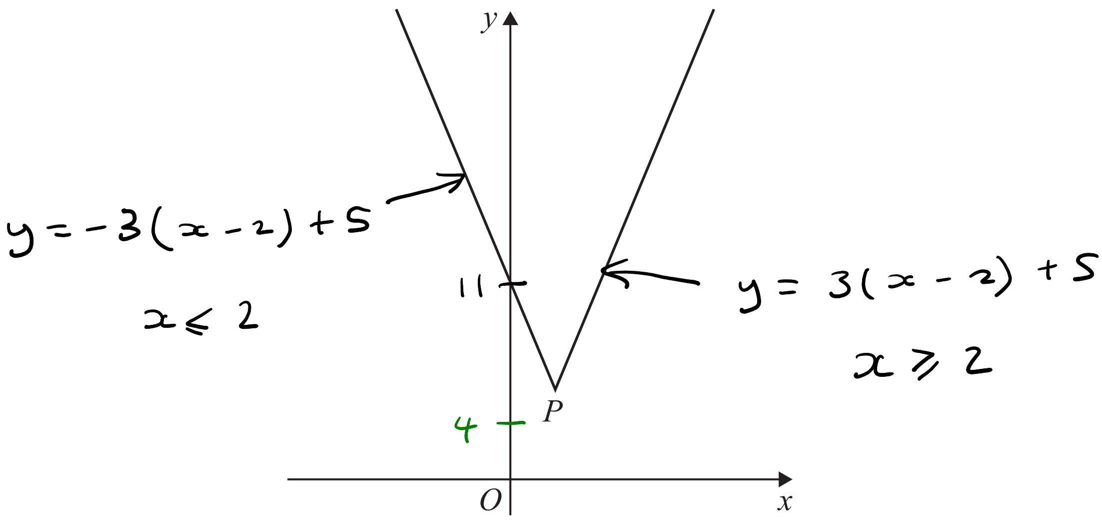

Figure 1

Figure 1 shows a sketch of the graph with equation

$$y = 3|x-2| + 5$$

The vertex of the graph is at the point P, shown in Figure 1.

(a) Find the coordinates of P.

(2)

(b) Solve the equation

$$16 - 4x = 3|x-2| + 5$$

(2)

A line l has equation $y = kx + 4$ where k is a constant.

Given that l intersects $y = 3|x-2| + 5$ at 2 distinct points,

(c) find the range of values of k.

(2)

a/ $(2, 5)$

b/ $16 - 4x = 3(x-2) + 5$ $\qquad$ $16 - 4x = -3(x-2) + 5$

$16 - 4x = 3x - 6 + 5$ $\qquad$ $16 - 4x = -3x + 6 + 5$

$17 = 7x$ $\qquad$ $5 = x$

$\times$

$x = \frac{17}{7}$ $\qquad$ $x \leq 2$ so $x = 5$ is not an intersection

Question 6 continued

c) one intersection if the line passes through (2, 5)

$y = kx + 4$

$5 = 2(k) + 4$

$1 = 2k$

$k = \frac{1}{2}$

only 1 intersection if $k = 3$

$\therefore \quad \frac{1}{2} < k < 3$

(Total for Question 6 is 6 marks)

7.

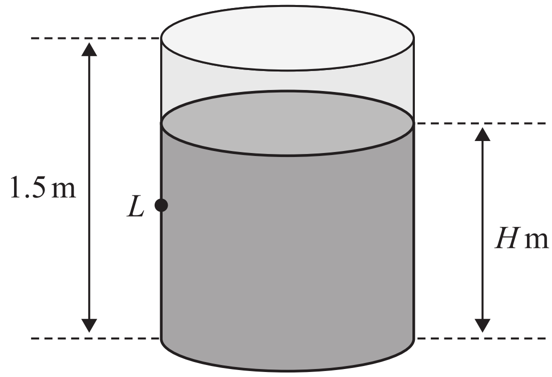

Diagram not drawn to scale.

Figure 2

Figure 2 shows a cylindrical tank of height 1.5 m.

Initially the tank is full of water.

The water starts to leak from a small hole, at a point L, in the side of the tank.

While the tank is leaking, the depth, H metres, of the water in the tank is modelled by the differential equation

$$\frac{\mathrm{d}H}{\mathrm{d}t} = -0.12\mathrm{e}^{-0.2t}$$

where t hours is the time after the leak starts.

Using the model,

(a) show that

$$H = A\mathrm{e}^{-0.2t} + B$$

where A and B are constants to be found,

(3)

(b) find the time taken for the depth of the water to decrease to 1.2 m. Give your answer in hours and minutes, to the nearest minute.

(3)

In the long term, the water level in the tank falls to the same height as the hole.

(c) Find, according to the model, the height of the hole from the bottom of the tank.

(2)

$$\frac{dH}{dt} = -0.12e^{-0.2t}$$

$$\int 1 \, dH = \int -0.12e^{-0.2t} \, dt$$

Question 7 continued

$H = 0.6e^{-0.2t} + c$

when $t = 0$ $H = 1.5$

$1.5 = 0.6 + c$

$c = 0.9$

$H = 0.6e^{-0.2t} + 0.9$

b/ $1.2 = 0.6e^{-0.2t} + 0.9$

$0.3 = 0.6e^{-0.2t}$

$\frac{1}{2} = e^{-0.2t}$

$\ln \frac{1}{2} = -0.2t$

$t = 3.47$ hours

$= \underline{3 \text{ hours } 28 \text{ minutes}}$

c/ As t increases H gets closer to 0.9

$\underline{0.9 \text{ m}}$

Question 7 continued

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Question 7 continued

(Total for Question 7 is 8 marks)

8. The functions f and g are defined by

$$f(x) = 4 - 3x^2 \qquad x \in \mathbb{R}$$

$$g(x) = \frac{5}{2x - 9} \qquad x \in \mathbb{R},\ x \neq \frac{9}{2}$$

(a) Find fg(2)

(2)

(b) Find g^{-1}

(3)

(c) (i) Find gf(x), giving your answer as a simplified fraction.

(ii) Deduce the range of gf(x).

(3)

The function h is defined by

$$h(x) = 2x^2 - 6x + k \qquad x \in \mathbb{R}$$

where k is a constant.

(d) Find the range of values of k for which the equation

$$f(x) = h(x)$$

has no real solutions.

(3)

a/ $g(2) = \frac{5}{2(2)-9}$

$= -1$

$f(-1) = 4 - 3(-1)^2$

$= 1$

b/ $y = \frac{5}{2x-9}$

$y(2x-9) = 5$

$2xy - 9y = 5$

$2xy = 5 + 9y$

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Question 8 continued

$$x = \frac{5 + 9y}{2y}$$

$$g^{-1}(x) = \frac{5 + 9x}{2x} \qquad x \in \mathbb{R} \quad x \neq 0$$

c i) $gf(x) = \dfrac{5}{2(4-3x^2) - 9}$

$$= \frac{5}{8 - 6x^2 - 9}$$

$$= \frac{5}{-6x^2 - 1}$$

$$= -\frac{5}{6x^2 + 1}$$

ii) The smallest the denominator can be is 1

when $x = 0$ $\quad gf(x) = -5$

As x increases $gf(x)$ gets closer to zero

$$\underline{\underline{-5 \leq gf(x) < 0}}$$

d) $4 - 3x^2 = 2x^2 - 6x + k$

$$0 = 5x^2 - 6x + (k-4)$$

No solutions where $b^2 - 4ac < 0$

Question 8 continued

$(-6)^2 - 4(5)(k-4) < 0$

$36 - 20(k-4) < 0$

$36 - 20k + 80 < 0$

$116 < 20k$

$5.8 < k$

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Question 8 continued

(Total for Question 8 is 11 marks)

9. The first 3 terms of a geometric sequence are

$$3^{4k-5} \qquad 9^{7-2k} \qquad 3^{2(k-1)}$$

$$3^{2(7-2k)}$$

where k is a constant.

(a) Using algebra and making your reasoning clear, prove that $k = \frac{5}{2}$

(3)

(b) Hence find the sum to infinity of the geometric sequence.

(3)

$$\frac{3^{2(7-2k)}}{3^{4k-5}} = \frac{3^{2(k-1)}}{3^{2(7-2k)}}$$

$$3^{2(7-2k)-(4k-5)} = 3^{2(k-1)-2(7-2k)}$$

$$14 - 4k - 4k + 5 = 2k - 2 - 14 + 4k$$

$$19 - 8k = 6k - 16$$

$$35 = 14k$$

$$k = \frac{35}{14} = \frac{5}{2}$$

b/ $a = 3^{4(\frac{5}{2})-5}$ $\qquad u_2 = 9^{7-2(\frac{5}{2})}$

$= 81$

$= 243$

$$r = \frac{81}{243} = \frac{1}{3}$$

$$S_\infty = \frac{243}{1-\frac{1}{3}}$$

$$= \frac{729}{2}$$

Question 9 continued

(Total for Question 9 is 6 marks)

10.

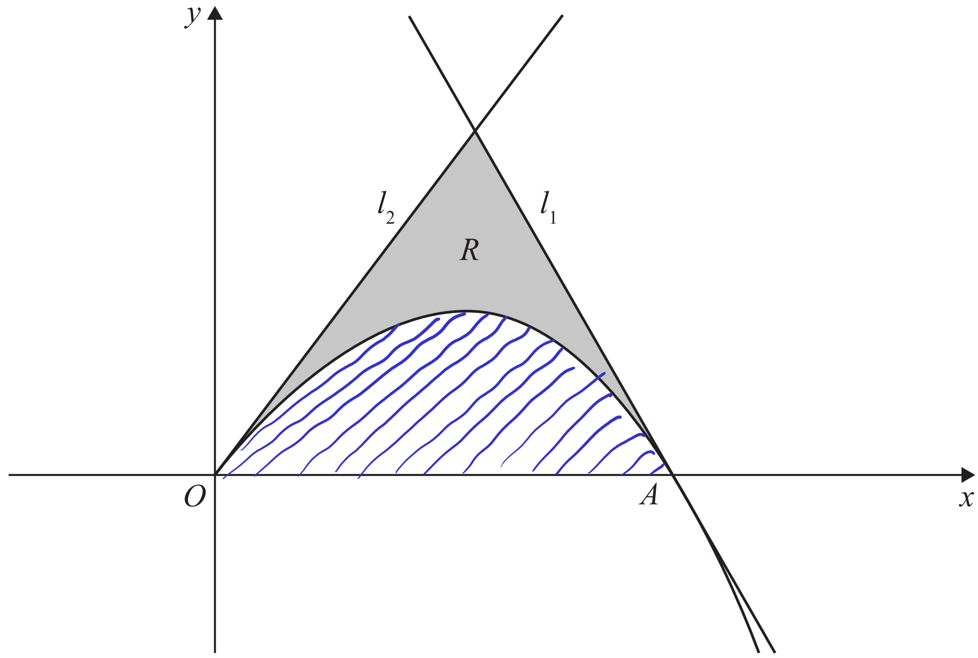

Figure 3

In this question you must show all stages of your working.

Solutions relying entirely on calculator technology are not acceptable.

Figure 3 shows a sketch of part of the curve with equation

$$y = 8x - x^{\frac{5}{2}} \qquad x \geqslant 0$$

The curve crosses the x-axis at the point A.

(a) Verify that the x coordinate of A is 4

(1)

The line l_1 is the tangent to the curve at A.

(b) Use calculus to show that an equation of line l_1 is

$$12x + y = 48$$

(3)

The line l_2 has equation $y = 8x$

The region R, shown shaded in Figure 3, is bounded by the curve, the line l_1 and the line l_2

(c) Use algebraic integration to find the exact area of R.

(5)

a) $0 = 8(4) - 4^{\frac{5}{2}}$

$0 = 0$ ✓

P 7 5 6 9 3 A 0 2 4 4 4

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Question 10 continued

b/ $y = 8x - x^{\frac{5}{2}}$

$\frac{dy}{dx} = 8 - \frac{5}{2}x^{\frac{3}{2}}$

when $x = 4$ $\quad \frac{dy}{dx} = 8 - \frac{5}{2}(4)^{\frac{3}{2}}$

$= -12$

(4, 0)
x y

$y - 0 = -12(x - 4)$

$y = -12x + 48$

$\underline{12x + y = 48}$

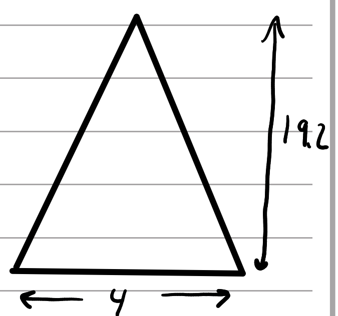
c/ $y = 8x \qquad y = -12x + 48$

l_1 and l_2 intersect where $8x = -12x + 48$

$20x = 48$

$x = 2.4$

when $x = 2.4$ $\quad y = 8(2.4) = 19.2$



Area of triangle $= \frac{1}{2} \cdot 4 \cdot 19.2$

$= 38.4$ units^2

$\int_0^4 8x - x^{\frac{5}{2}} \, dx$

$\left[4x^2 - \frac{2}{7}x^{\frac{7}{2}}\right]_0^4$

Question 10 continued

$$\left(4(4)^2 - \frac{2}{7}(4)^{\frac{7}{2}}\right) - (0) = \frac{192}{7}$$

$$38.4 - \frac{192}{7} = \frac{384}{35} \text{ units}^2$$

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Question 10 continued

(Total for Question 10 is 9 marks)

11.

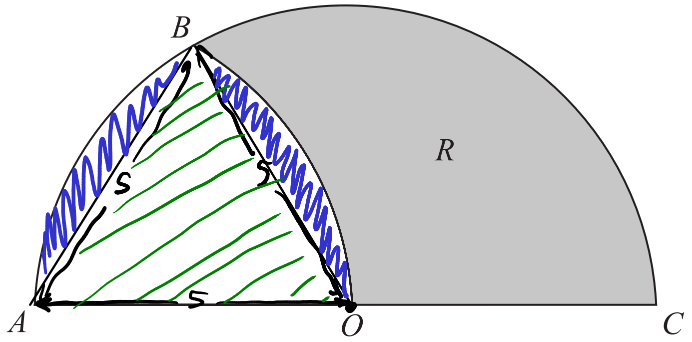

Figure 4

Figure 4 shows the design of a badge.

The shape $ABCOA$ is a semicircle with centre O and diameter 10 cm.

OB is the arc of a circle with centre A and radius 5 cm.

The region R, shown shaded in Figure 4, is bounded by the arc OB, the arc BC and the line OC.

Find the exact area of R.

Give your answer in the form $(a\sqrt{3} + b\pi)\,\text{cm}^2$, where a and b are rational numbers.

(4)

Area of semicircle $= \frac{1}{2} r^2 \theta$

$= \frac{1}{2}(5)^2(\pi)$

$= \frac{25}{2}\pi$

Area of triangle $= \frac{1}{2}(5)(5)\sin\left(\frac{\pi}{3}\right)$

$= \frac{25\sqrt{3}}{4}$

Area of segment $= \frac{1}{2}(5)^2\left(\frac{\pi}{3}\right) - \frac{25\sqrt{3}}{4}$

$= \frac{25\pi}{6} - \frac{25\sqrt{3}}{4}$

$R = \frac{25}{2}\pi - \frac{25\sqrt{3}}{4} - 2\left(\frac{25\pi}{6} - \frac{25\sqrt{3}}{4}\right)$

Question 11 continued

$$= \frac{25}{2}\pi - \frac{25\sqrt{3}}{4} - \frac{25}{3}\pi + \frac{25}{2}\sqrt{3}$$

$$= \frac{25}{6}\pi + \frac{25}{4}\sqrt{3} \quad cm^2$$

(Total for Question 11 is 4 marks)

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12. (a) Express $140\cos\theta - 480\sin\theta$ in the form $K\cos(\theta + \alpha)$

where $K > 0$ and $0 < \alpha < 90°$

State the value of K and give the value of α, in degrees, to 2 decimal places.

(3)

A scientist studies the number of rabbits and the number of foxes in a wood for one year.

The number of rabbits, R, is modelled by the equation

$$R = A + 140\cos(30t)° - 480\sin(30t)°$$

where t months is the time after the start of the year and A is a constant.

Given that, during the year, the maximum number of rabbits in the wood is 1500

(b) (i) find a complete equation for this model.

(ii) Hence write down the minimum number of rabbits in the wood during the year according to the model.

(2)

The actual number of rabbits in the wood is at its minimum value in the middle of April.

(c) Use this information to comment on the model for the number of rabbits.

(2)

The number of foxes, F, in the wood during the same year is modelled by the equation

$$F = 100 + 70\sin(30t + 70)°$$

The number of foxes is at its minimum value after T months.

(d) Find, according to the models, the number of **rabbits** in the wood at time T months.

(4)

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$k\cos(\theta+\alpha) = k\cos\theta\cos\alpha - k\sin\theta\sin\alpha$

$= 140\cos\theta - 480\sin\theta$

$K = \sqrt{140^2 + 480^2}$ $\quad K\sin\alpha = 480$

$= \underline{500}$ $\quad K\cos\alpha = 140$

$\tan\alpha = \frac{480}{140}$

$\alpha = \tan^{-1}\left(\frac{480}{140}\right)$

Question 12 continued

$\alpha = 73.74°$

$\underline{500 \cos(\theta + 73.74)}$

b/ i/ $R = A + 500\cos(30t + 73.74)$

as max $= 1500$ $A = 1000$

$R = 1000 + 500 \cos(30t + 73.74)$

ii/ min $1000 - 500 = \underline{\underline{500}}$

c/ min when $\cos(30t + 73.74) = -1$

$30t + 73.74 = 180$

$t = 3.54$

3.5 months after the start of the year is mid April $\therefore$ the model is suitable.

$F = 100 + 70\sin(30t + 70)$

min when $\sin(30t + 70) = -1$

$30t + 70 = -90, 270$

$30t = -160, 200$

$t = \frac{20}{3}$ months

$R = 1000 + 500\cos\left(30\left(\frac{20}{3}\right) + 73.74\right)$

$= \underline{\underline{1033}}$

Question 12 continued

Question 12 continued

(Total for Question 12 is 11 marks)

13. (a) Given that a is a positive constant, use the substitution $x = a\sin^2\theta$ to show that

$$\int_0^a x^{\frac{1}{2}}\sqrt{a-x}\ \mathrm{d}x = \frac{1}{2}a^2\int_0^{\frac{\pi}{2}}\sin^2 2\theta\ \mathrm{d}\theta$$

(4)

(b) Hence use algebraic integration to show that

$$\int_0^a x^{\frac{1}{2}}\sqrt{a-x}\ \mathrm{d}x = k\pi a^2$$

where k is a constant to be found.

(4)

$x = a\sin^2\theta$ $\qquad \frac{dx}{d\theta} = 2a\sin\theta\cos\theta$

when $x = a$ $\quad a = a\sin^2\theta$

$1 = \sin^2\theta$

$1 = \sin\theta$

$\theta = \frac{\pi}{2}$

$\boxed{\begin{array}{l}\sin^2\theta + \cos^2\theta = 1 \\ a\sin^2\theta + a\cos^2\theta = a\end{array}}$

when $x = 0$ $\quad 0 = a\sin^2\theta$

$0 = \sin\theta$

$\theta = 0$

$$\int_0^{\frac{\pi}{2}} (a\sin^2\theta)^{\frac{1}{2}}\sqrt{a - a\sin^2\theta}\ \frac{dx}{d\theta}\ d\theta$$

$$\int_0^{\frac{\pi}{2}} \sqrt{a}\sin\theta \cdot \sqrt{a\cos^2\theta} \cdot 2a\sin\theta\cos\theta\ d\theta$$

$$\int_0^{\frac{\pi}{2}} \sqrt{a}\sin\theta \cdot \sqrt{a}\cos\theta \cdot 2a\sin\theta\cos\theta\ d\theta$$

$$\int_0^{\frac{\pi}{2}} 2a^2\sin^2\theta\cos^2\theta\ d\theta$$

$$2a^2\int_0^{\frac{\pi}{2}} \sin^2\theta\cos^2\theta\ d\theta$$

Question 13 continued

$$\sin 2A = 2\sin A\cos A$$

$$\frac{1}{2}\sin 2A = \sin A\cos A$$

$$\frac{1}{4}\sin^2 2A = \sin^2 A\cos^2 A$$

$$2a^2\int_0^{\frac{\pi}{2}} \frac{1}{4}\sin^2 2\theta \, d\theta$$

$$\frac{1}{2}a^2\int_0^{\frac{\pi}{2}} \sin^2 2\theta \, d\theta$$

b/

$$\cos 2A = 1 - 2\sin^2 A$$

$$\cos 4\theta = 1 - 2\sin^2 2\theta$$

$$2\sin^2 2\theta = 1 - \cos 4\theta$$

$$\sin^2 2\theta = \frac{1}{2} - \frac{1}{2}\cos 4\theta$$

$$\frac{1}{2}a^2\int_0^{\frac{\pi}{2}} \frac{1}{2} - \frac{1}{2}\cos 4\theta \, d\theta$$

$$\frac{1}{2}a^2 \cdot \left[\frac{1}{2}\theta - \frac{1}{8}\sin 4\theta\right]_0^{\frac{\pi}{2}}$$

$$\frac{1}{2}a^2\left[\left(\frac{1}{4}\pi - 0\right) - (0 - 0)\right.$$

$$\frac{1}{8}\pi a^2$$

Question 13 continued

Question 13 continued

(Total for Question 13 is 8 marks)

14. A balloon is being inflated.

In a simple model,

- the balloon is modelled as a sphere
- the rate of increase of the radius of the balloon is inversely proportional to the square root of the radius of the balloon

At time t seconds, the radius of the balloon is r cm.

(a) Write down a differential equation to model this situation.

(1)

At the instant when $t = 10$

- the radius is 16 cm
- the radius is increasing at a rate of $0.9\,\text{cm s}^{-1}$

(b) Solve the differential equation to show that

$$r^{\frac{3}{2}} = 5.4t + 10$$

(5)

(c) Hence find the radius of the balloon when $t = 20$

Give your answer to the nearest millimetre.

(2)

(d) Suggest a limitation of the model.

(1)

a/ $\frac{dr}{dt} = \frac{k}{\sqrt{r}}$ $\qquad \frac{dr}{dt} = \frac{3.6}{\sqrt{r}}$

b/ $\int \sqrt{r}\, dr = \int 3.6\, dt$

when $t = 10$ $r = 16$

and $\frac{dr}{dt} = 0.9$

$\frac{2}{3} r^{3/2} = 3.6t + c$

$0.9 = \frac{k}{\sqrt{16}}$

$\frac{2}{3}(16)^{\frac{3}{2}} = 3.6(10) + c$

$k = 3.6$

$\frac{128}{3} = 36 + c$

$c = \frac{20}{3}$

Question 14 continued

$$\frac{2}{3} r^{\frac{3}{2}} = 3.6t + \frac{20}{3}$$

$$r^{3/2} = 5.4t + 10$$

c/ when $t = 20$

$$r^{3/2} = 118$$

$$r = (118)^{\frac{2}{3}}$$

$$= 24.1 \text{ cm}$$

d/ The balloon will pop if it keeps expanding.

Question 14 continued

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Question 14 continued

(Total for Question 14 is 9 marks)

15. (i) Show that $k^2 - 4k + 5$ is positive for all real values of k.

(2)

(ii) A student was asked to prove by contradiction that

"There are no positive integers x and y such that $(3x + 2y)(2x - 5y) = 28$"

The start of the student's proof is shown below.

> Assume that positive integers x and y exist such that
> $(3x + 2y)(2x - 5y) = 28$
>
> If $3x + 2y = 14$ and $2x - 5y = 2$
>
> $$\left.\begin{array}{l} 3x + 2y = 14 \\ 2x - 5y = 2 \end{array}\right\} \Rightarrow x = \frac{74}{19},\ y = \frac{22}{19} \quad \text{Not integers}$$

Show the calculations and statements needed to complete the proof.

(4)

a/ $k^2 - 4k + 5$

$(k - 2)^2 - 4 + 5$

$(k - 2)^2 + 1$

The minimum value is 1 $\therefore$ positive for all values of k.

b/ • As x and y are integers $(3x+2y)$ and $(2x - 5y)$ must be integers

• As x and y are positive integers: $3x+2y \geq 5$

$28 = 1 \times 28$
$= 2 \times 14$ (Already checked)
$= 4 \times 7$

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Question 15 continued

$3x + 2y = 28$
$2x - 5y = 1$

$x = \frac{142}{19}$ $y = \frac{53}{19}$ not integers

$3x + 2y = 7$
$2x - 5y = 4$

$x = \frac{43}{19}$ $y = \frac{2}{19}$ not integers

Therefore there are no positive integers x and y such that $(3x+2y)(2x-5y) = 28$

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Turn over ▶

Question 15 continued

(Total for Question 15 is 6 marks)

TOTAL FOR PAPER IS 100 MARKS

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