

1 a Express  $\frac{3x+5}{(x+1)(x+3)}$  in partial fractions.

b Hence, find  $\int \frac{3x+5}{(x+1)(x+3)} dx$ .

2 Show that  $\int \frac{3}{(t-2)(t+1)} dt = \ln \left| \frac{t-2}{t+1} \right| + c$ .

3 Integrate with respect to  $x$

a  $\frac{6x-11}{(2x+1)(x-3)}$

b  $\frac{14-x}{x^2+2x-8}$

c  $\frac{6}{(2+x)(1-x)}$

d  $\frac{x+1}{5x^2-14x+8}$

4 a Find the values of the constants  $A$ ,  $B$  and  $C$  such that

$$\frac{x^2-6}{(x+4)(x-1)} \equiv A + \frac{B}{x+4} + \frac{C}{x-1}.$$

b Hence, find  $\int \frac{x^2-6}{(x+4)(x-1)} dx$ .

5 a Express  $\frac{x^2-x-4}{(x+2)(x+1)^2}$  in partial fractions.

b Hence, find  $\int \frac{x^2-x-4}{(x+2)(x+1)^2} dx$ .

6 Integrate with respect to  $x$

a  $\frac{3x^2-5}{x^2-1}$

b  $\frac{x(4x+13)}{(2+x)^2(3-x)}$

c  $\frac{x^2-x+1}{x^2-3x-10}$

d  $\frac{2-6x+5x^2}{x^2(1-2x)}$

7 Show that  $\int_3^4 \frac{3x-5}{(x-1)(x-2)} dx = 2 \ln 3 - \ln 2$ .

8 Find the exact value of

a  $\int_1^3 \frac{x+3}{x(x+1)} dx$

b  $\int_0^2 \frac{3x-2}{x^2+x-12} dx$

c  $\int_1^2 \frac{9}{2x^2-7x-4} dx$

d  $\int_0^2 \frac{2x^2-7x+7}{x^2-2x-3} dx$

e  $\int_0^1 \frac{5x+7}{(x+1)^2(x+3)} dx$

f  $\int_{-1}^1 \frac{2+x}{8-2x-x^2} dx$

9 a Express  $\frac{1}{x^2-a^2}$ , where  $a$  is a positive constant, in partial fractions.

b Hence, show that  $\int \frac{1}{x^2-a^2} dx = \frac{1}{2a} \ln \left| \frac{x-a}{x+a} \right| + c$ .

c Find  $\int \frac{1}{a^2-x^2} dx$ .

10 Evaluate

a  $\int_{-1}^1 \frac{1}{x^2-9} dx$

b  $\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{4}{1-x^2} dx$

c  $\int_0^1 \frac{3}{2x^2-8} dx$