C3 NUMERICAL METHODS — Worksheet A

1 Show in each case that there is a root of the equation $f(x) = 0$ in the given interval.

a $f(x) = x^3 + 3x - 7$ $(1, 2)$ **b** $f(x) = 5\cos x - 3x$ $(0.5, 1)$

c $f(x) = 2e^x + x + 5$ $(-6, -5)$ **d** $f(x) = x^4 - 5x^2 + 1$ $(2.1, 2.2)$

e $f(x) = \ln(4x - 1) + x^2$ $(0.4, 0.5)$ **f** $f(x) = e^{-x} - 9\cos 4x$ $(10, 11)$

2 Given that $|N| \le 5$, find in each case the integer N such that there is a root of the equation $f(x) = 0$ in the interval $(N, N + 1)$.

a $f(x) = x^3 - 3\sqrt{x} - 4$ **b** $f(x) = x\ln x - \frac{12}{x}$ **c** $f(x) = 2x^5 + 4x + 15$

d $f(x) = e^{x-1} + 4x - 2$ **e** $f(x) = e^x - 3\sin x$ **f** $f(x) = \tan(0.1x) + x - 6$

3 Show in each case that there is a root of the given equation in the given interval.

a $x^3 = 12 - \frac{x}{4}$ $[2, 3]$ **b** $12e^x = 9 - 4x$ $[-1, 0]$

c $10\ln 3x = 5 - 7x^2$ $[0.47, 0.48]$ **d** $\sin 4x = 7e^x$ $[-6.5, -6]$

e $4^x = 3x + 10$ $[-4, -3]$ **f** $\tan(\frac{1}{2}x) = 2x - 1$ $[2.6, 2.7]$

4 In each case there is a root of the equation $f(x) = 0$ in the given interval.
Find the integer, a, such that this root lies in the interval $(\frac{a}{10}, \frac{a+1}{10})$.

a $f(x) = x^4 + \frac{3}{x} - 5$ $(1, 2)$ **b** $f(x) = x - \ln(6 + x^2)$ $(2, 3)$

c $f(x) = 5x^3 - 3x^2 + 11$ $(-2, -1)$ **d** $f(x) = \frac{8}{x} - \cos x$ $(11, 12)$

e $f(x) = \operatorname{cosec} x + \sqrt{x}$ $(5, 6)$ **f** $f(x) = x^2 - 7e^{2x+5}$ $(-3, -2)$

5 **a** On the same set of axes, sketch the graphs of $y = x^3$ and $y = 4 - x$.

b Hence, show that the equation $x^3 + x - 4 = 0$ has exactly one real root.

c Show that this root lies in the interval $(1, 1.5)$.

6 $$f : x \to x\ln x - 1,\ x \in \mathbb{R},\ x > 0.$$

a On the same set of axes, sketch the curves $y = \ln x$ and $y = \frac{1}{x}$.

b Hence show that the equation $f(x) = 0$ has exactly one real root.

The real root of $f(x) = 0$ is α.

c Find the integer n such that $n < \alpha < n + 1$.

7 **a** On the same set of axes, sketch the curves $y = e^x$ and $y = 5 - x^2$.

b Hence show that the equation $e^x + x^2 - 5 = 0$ has exactly one negative and one positive real root.

c Show that the negative root lies in the interval $(-3, -2)$.

The positive root, α, is such that $\frac{n}{10} < \alpha < \frac{n+1}{10}$, where n is an integer.

d Find the value of n.

C3 NUMERICAL METHODS Worksheet B

1 For each equation, show that it can be rearranged into the given iterative form. Use this and the given value of x_0 to find x_1, x_2 and x_3. Give your value of x_3 correct to 4 decimal places.

a $9 + 4x - 2x^3 = 0$ $\quad x_{n+1} = \sqrt[3]{2x_n + 4.5}$ $\quad x_0 = 2$

b $e^x - 8x + 5 = 0$ $\quad x_{n+1} = \ln(8x_n - 5)$ $\quad x_0 = 3$

c $\tan x - 5x + 13 = 0$ $\quad x_{n+1} = \arctan(5x_n - 13)$ $\quad x_0 = -1.2$

d $\ln x + \sqrt{x} + 1.4 = 0$ $\quad x_{n+1} = e^{-(\sqrt{x_n} + 1.4)}$ $\quad x_0 = 0.16$

2 For each equation, show that it can be rearranged into the given iterative form and state the values of the constants a and b. Use this and the given value of x_0 to find x_1, x_2 and x_3. Give your value of x_3 correct to 3 decimal places.

a $e^{2x-1} - 6x = 0$ $\quad x_{n+1} = a(\ln bx_n + 1)$ $\quad x_0 = 1.7$

b $\frac{2}{x} + \cos x - 3 = 0$ $\quad x_{n+1} = \frac{a}{b - \cos x_n}$ $\quad x_0 = 0.8$

c $2x^3 - 6x - 11 = 0$ $\quad x_{n+1} = \sqrt{a + \frac{b}{x_n}}$ $\quad x_0 = 2$

d $15\ln(x + 3) - 4x = 0$ $\quad x_{n+1} = e^{ax_n} + b$ $\quad x_0 = -2.5$

3 In each case, use the given iteration formula and value of x_0 to find a root of the equation $f(x) = 0$ to the stated degree of accuracy. Justify the accuracy of your answers.

a $f(x) = 10^x + 3x - 4$ $\quad x_{n+1} = \log_{10}(4 - 3x_n)$ $\quad x_0 = 0.44$ $\quad$ 3 decimal places

b $f(x) = x^2 + \frac{1}{x-5}$ $\quad x_{n+1} = \sqrt{\frac{x_n^3 + 1}{5}}$ $\quad x_0 = 0.5$ $\quad$ 2 significant figures

c $f(x) = 30 - 5x + \sin 2x$ $\quad x_{n+1} = 6 + 0.2\sin 2x_n$ $\quad x_0 = 6$ $\quad$ 3 significant figures

d $f(x) = e^{4-x} - \ln x$ $\quad x_{n+1} = 4 - \ln(\ln x_n)$ $\quad x_0 = 3.7$ $\quad$ 3 decimal places

4 $$f(x) = x^5 - 10x^3 + 4.$$

The equation $f(x) = 0$ has a root in the interval $-4 < x < -3$.

a Use the iteration formula $x_{n+1} = \sqrt[5]{10x_n^3 - 4}$ and the starting value $x_0 = -3.2$ to find the value of this root correct to 2 decimal places.

The equation $f(x) = 0$ can be rearranged into the iterative form $x_{n+1} = \sqrt[3]{\frac{a}{b - x_n^2}}$.

b Find the values of the constants a and b in this formula.

The equation $f(x) = 0$ has another root in the interval $0 < x < 1$.

c Using the iteration formula with your values from part **b** and the starting value $x_0 = 1$, find the value of this root correct to 3 decimal places.

5 $$f : x \to \arcsin 2x - 0.5x - 0.7, \ x \in \mathbb{R}, \ |x| \leq 0.5$$

The equation $f(x) = 0$ can be rearranged into the iterative form $x_{n+1} = a\sin(bx_n + c)$.

a Find the values of the constants a, b and c in this formula.

The equation $f(x) = 0$ has a solution in the interval (0.3, 0.4).

b Using the iterative formula with your values from part **a** and the starting value $x_0 = 0.4$, find this solution correct to 3 decimal places.