1 The domain of each of the following functions is $x \in \mathbb{R}$. For each function, find its inverse $f^{-1}(x)$.

a $f : x \to 10x + 3$ **b** $f : x \to 9 + 2x$ **c** $f : x \to 5 - 6x$

d $f : x \to \frac{x+3}{4}$ **e** $f : x \to \frac{1}{3}(2x - 5)$ **f** $f : x \to 8 - \frac{3}{5}x$

2 For each function, find $f^{-1}(x)$ and state its domain.

a $f(x) \equiv \ln x,\ x \in \mathbb{R},\ x > 0$ **b** $f(x) \equiv \frac{1}{x},\ x \in \mathbb{R},\ x \neq 0$

c $f(x) \equiv \sqrt[4]{x},\ x \in \mathbb{R},\ x > 0$ **d** $f(x) \equiv 3x - 4,\ x \in \mathbb{R},\ 0 \leq x < 3$

e $f(x) \equiv \frac{1}{x-5},\ x \in \mathbb{R},\ x \neq 5$ **f** $f(x) \equiv 2 + \frac{1}{x},\ x \in \mathbb{R},\ x \neq 0$

3 For each of the following functions,

i find, in the form $f^{-1} : x \to \ldots$, the inverse function of f and state its domain,

ii sketch $y = f(x)$ and $y = f^{-1}(x)$ on the same set of axes.

a $f : x \to 2x + 1,\ x \in \mathbb{R}$ **b** $f : x \to \frac{1-x}{5},\ x \in \mathbb{R}$ **c** $f : x \to \frac{10}{x},\ x \in \mathbb{R},\ x \neq 0$

d $f : x \to x^2,\ x \in \mathbb{R},\ x > 0$ **e** $f : x \to e^x,\ x \in \mathbb{R}$ **f** $f : x \to x^3,\ x \in \mathbb{R}$

4 For each of the following, solve the equation $f^{-1}(x) = g(x)$.

a $f : x \to 5x + 1,\ x \in \mathbb{R}$ $g : x \to 2,\ x \in \mathbb{R}$

b $f : x \to \frac{2x-4}{3},\ x \in \mathbb{R}$ $g : x \to 7 - x,\ x \in \mathbb{R}$

c $f : x \to e^x + 2,\ x \in \mathbb{R}$ $g : x \to \ln(3x - 8),\ x \in \mathbb{R},\ x > \frac{8}{3}$

d $f : x \to \sqrt{x+2},\ x \in \mathbb{R},\ x \geq -2$ $g : x \to 3x - 4,\ x \in \mathbb{R}$

e $f : x \to \frac{4}{x+3},\ x \in \mathbb{R},\ x \neq -3$ $g : x \to 5(x + 1),\ x \in \mathbb{R}$

5 The function f is defined by $f : x \to 4 - 2x,\ x \in \mathbb{R}$.

a Sketch $y = f(x)$ and $y = f^{-1}(x)$ on the same set of axes.

b Find the coordinates of the point where the lines $y = f(x)$ and $y = f^{-1}(x)$ intersect.

6 The functions f and g are defined by

$$f : x \to 3 - 2x,\ x \in \mathbb{R} \qquad g : x \to \frac{1}{2x+4},\ x \in \mathbb{R},\ x \neq -2$$

a Find $g^{-1}(x)$ and state its domain and range.

b Express gf in terms of x and state its domain.

c Solve the equation $gf(x) = f^{-1}(x)$.

7 The functions f and g are defined by

$$f : x \to 5x + 2,\ x \in \mathbb{R} \qquad g : x \to \frac{1}{x},\ x \in \mathbb{R},\ x \neq 0$$

a Find the following functions, stating the domain in each case.

i f^{-1} **ii** fg **iii** $(fg)^{-1}$

b Solve the equation $f^{-1}(x) = fg(x)$, giving your answers correct to 2 decimal places.

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8 For each of the following functions, find the inverse function in the form $f^{-1}: x \to \ldots$ and state its domain.

a $f : x \to \frac{1}{2}\ln(4x - 9),\ x \in \mathbb{R},\ x > 2\frac{1}{4}$

b $f : x \to \dfrac{x-2}{x+5},\ x \in \mathbb{R},\ x \neq -5$

c $f : x \to e^{0.4x-2},\ x \in \mathbb{R}$

d $f : x \to \sqrt[3]{x^5 - 3},\ x \in \mathbb{R}$

e $f : x \to \log_{10}(2 - 7x),\ x \in \mathbb{R},\ x < \frac{2}{7}$

f $f : x \to \dfrac{4-x}{3x+2},\ x \in \mathbb{R},\ x \neq -\frac{2}{3}$

9 For each of the following functions,

i find, in the form $f^{-1}: x \to \ldots,$ the inverse function of f and state its domain,

ii sketch $y = f(x)$ and $y = f^{-1}(x)$ on the same set of axes.

a $f : x \to e^{2x},\ x \in \mathbb{R}$

b $f : x \to x^2 + 4,\ x \in \mathbb{R},\ x > 0$

c $f : x \to \ln(x - 3),\ x \in \mathbb{R},\ x > 3$

d $f : x \to x^2 + 6x + 9,\ x \in \mathbb{R},\ x > -3$

10 For each of the following functions,

i find the range of f,

ii find $f^{-1}(x)$, stating its domain.

a $f(x) \equiv x^2 + 6x + 3,\ x \in \mathbb{R},\ x < -3$

b $f(x) \equiv x^2 - 4x + 5,\ x \in \mathbb{R},\ x \geq 2$

c $f(x) \equiv x^2 + 5x - 2,\ x \in \mathbb{R},\ x < -2\frac{1}{2}$

d $f(x) \equiv x^2 - 3x + 5,\ x \in \mathbb{R},\ 2 < x < 4$

e $f(x) \equiv (2 - x)(4 + x),\ x \in \mathbb{R},\ x \geq -1$

f $f(x) \equiv 20x - 5x^2,\ x \in \mathbb{R},\ x > 2$

11 For each of the following, solve the equation $f^{-1}(x) = g(x)$.

a $f : x \to \frac{1}{3}(2x - 5),\ x \in \mathbb{R}$ $\quad g : x \to \dfrac{4}{2-x},\ x \in \mathbb{R},\ x \neq 2$

b $f : x \to \ln\dfrac{x+3}{5},\ x \in \mathbb{R},\ x > -3$ $\quad g : x \to 10 - 6e^{-x},\ x \in \mathbb{R}$

c $f : x \to x^2 - 4,\ x \in \mathbb{R},\ x > 0$ $\quad g : x \to \dfrac{x+6}{3},\ x \in \mathbb{R}$

12 The function f is defined by

$$f : x \to \frac{x+b}{x+a},\ x \in \mathbb{R},\ x \neq 2.$$

a State the value of the constant a.

Given that $f(6) = 4$,

b find the value of the constant b,

c find $f^{-1}(x)$ and state its domain.

13 The functions f and g are defined by

$$f : x \to x^2 - 3x,\ x \in \mathbb{R},\ x \geq 1\tfrac{1}{2},$$

$$g : x \to 2x + 3,\ x \in \mathbb{R}.$$

a Find, in the form $f^{-1}: x \to \ldots,$ the inverse function of f and state its domain.

b On the same set of axes, sketch $y = f(x)$ and $y = f^{-1}(x)$.

Given that $f^{-1}g^{-1}(12) = a(1 + \sqrt{3})$,

c show that $a = 1\frac{1}{2}$.

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