C3 DIFFERENTIATION Worksheet F

1 Given that $f(x) = \frac{x}{x+2}$, find $f'(x)$

a using the product rule, **b** using the quotient rule.

2 Differentiate each of the following with respect to x and simplify your answers.

a $\frac{4x}{1-3x}$ **b** $\frac{e^x}{x-4}$ **c** $\frac{x+1}{2x+3}$ **d** $\frac{\ln x}{2x}$

e $\frac{x}{2-x^2}$ **f** $\frac{\sqrt{x}}{3x+2}$ **g** $\frac{e^{2x}}{1-e^{2x}}$ **h** $\frac{2x+1}{\sqrt{x-3}}$

3 Find $\frac{dy}{dx}$, simplifying your answer in each case.

a $y = \frac{x^2}{x+4}$ **b** $y = \frac{\sqrt{x-4}}{2x^2}$ **c** $y = \frac{2e^x+1}{1-3e^x}$

d $y = \frac{1-x}{x^3+2}$ **e** $y = \frac{\ln(3x-1)}{x+2}$ **f** $y = \sqrt{\frac{x+1}{x+3}}$

4 Find the coordinates of any stationary points on each curve.

a $y = \frac{x^2}{3-x}$ **b** $y = \frac{e^{4x}}{2x-1}$ **c** $y = \frac{x+5}{\sqrt{2x+1}}$

d $y = \frac{\ln 3x}{2x}$ **e** $y = \left(\frac{x+1}{x-2}\right)^2$ **f** $y = \frac{x^2-3}{x+2}$

5 Find an equation for the tangent to each curve at the point on the curve with the given x-coordinate.

a $y = \frac{2x}{3-x}$, $x = 2$ **b** $y = \frac{e^x+3}{e^x+1}$, $x = 0$

c $y = \frac{\sqrt{x}}{5-x}$, $x = 4$ **d** $y = \frac{3x+4}{x^2+1}$, $x = -1$

6 Find an equation for the normal to each curve at the point on the curve with the given x-coordinate. Give your answers in the form $ax + by + c = 0$, where a, b and c are integers.

a $y = \frac{1-x}{3x+1}$, $x = 1$ **b** $y = \frac{4x}{\sqrt{2-x}}$, $x = -2$

c $y = \frac{\ln(2x-5)}{3x-5}$, $x = 3$ **d** $y = \frac{x}{x^3-4}$, $x = 2$

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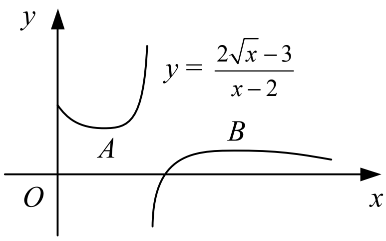

The diagram shows part of the curve $y = \frac{2\sqrt{x}-3}{x-2}$ which is stationary at the points A and B.

a Show that the x-coordinates of A and B satisfy the equation $x - 3\sqrt{x} + 2 = 0$.

b Hence, find the coordinates of A and B.