

1 a

$$\begin{array}{r}
 3x^2 + 2x - 1 \\
 x - 4 \overline{) 3x^3 - 10x^2 - 9x + 15} \\
 \underline{3x^3 - 12x^2} \phantom{- 9x + 15} \\
 2x^2 - 9x \phantom{+ 15} \\
 \underline{2x^2 - 8x} \phantom{+ 15} \\
 -x + 15 \\
 \underline{-x + 4} \\
 11
 \end{array}$$

quotient:  $3x^2 + 2x - 1$ , remainder: 11

b

$$\begin{array}{r}
 x^2 - 5x - 3 \\
 2x - 1 \overline{) 2x^3 - 11x^2 - x + 3} \\
 \underline{2x^3 - x^2} \phantom{- x + 3} \\
 -10x^2 - x \phantom{+ 3} \\
 \underline{-10x^2 + 5x} \phantom{+ 3} \\
 -6x + 3 \\
 \underline{-6x + 3} \\
 0
 \end{array}$$

quotient:  $x^2 - 5x - 3$ , remainder: 0

c

$$\begin{array}{r}
 2x^2 - x + 6 \\
 2x + 5 \overline{) 4x^3 + 8x^2 + 7x + 32} \\
 \underline{4x^3 + 10x^2} \phantom{+ 7x + 32} \\
 -2x^2 + 7x \phantom{+ 32} \\
 \underline{-2x^2 - 5x} \phantom{+ 32} \\
 12x + 32 \\
 \underline{12x + 30} \\
 2
 \end{array}$$

quotient:  $2x^2 - x + 6$ , remainder: 2

d

$$\begin{array}{r}
 -2x^2 - 6x + 4 \\
 3x + 2 \overline{) -6x^3 - 22x^2 + 0x + 1} \\
 \underline{-6x^3 - 4x^2} \phantom{+ 0x + 1} \\
 -18x^2 + 0x \phantom{+ 1} \\
 \underline{-18x^2 - 12x} \phantom{+ 1} \\
 12x + 1 \\
 \underline{12x + 8} \\
 -7
 \end{array}$$

quotient:  $-2x^2 - 6x + 4$ , remainder -7

2

- a let  $f(x) = x^3 + 4x^2 + x - 6$   
 $f(-2) = -8 + 16 - 2 - 6 = 0$   
 $\therefore (x + 2)$  is a factor

b

$$\begin{array}{r}
 x^2 + 2x - 3 \\
 x + 2 \overline{) x^3 + 4x^2 + x - 6} \\
 \underline{x^3 + 2x^2} \phantom{+ x - 6} \\
 2x^2 + x \phantom{- 6} \\
 \underline{2x^2 + 4x} \phantom{- 6} \\
 -3x - 6 \\
 \underline{-3x - 6} \\
 0
 \end{array}$$

$$\begin{aligned}
 \therefore x^3 + 4x^2 + x - 6 &= (x + 2)(x^2 + 2x - 3) \\
 &= (x + 2)(x + 3)(x - 1)
 \end{aligned}$$

$$\begin{aligned}
 \text{c} &= \frac{(x+2)(x+3)(x-1)}{(x+3)(x-3)} \\
 &= \frac{(x+2)(x-1)}{x-3}
 \end{aligned}$$

3 a let  $f(x) = 2x^3 - 5x^2 + 13x - 15$   
 $f\left(\frac{3}{2}\right) = \frac{27}{4} - \frac{45}{4} + \frac{39}{2} - 15 = 0$   
 $\therefore (2x - 3)$  is a factor

b

$$\begin{array}{r} x^2 - x + 5 \\ 2x - 3 \overline{) 2x^3 - 5x^2 + 13x - 15} \\ \underline{2x^3 - 3x^2} \phantom{+ 13x - 15} \\ - 2x^2 + 13x \phantom{- 15} \\ \underline{- 2x^2 + 3x} \phantom{- 15} \\ 10x - 15 \\ \underline{10x - 15} \\ 0 \end{array}$$

$$\therefore 2x^3 - 5x^2 + 13x - 15 = (2x - 3)(x^2 - x + 5)$$

$$\begin{aligned} \therefore \frac{2x^3 - 5x^2 + 13x - 15}{2x^2 - 7x + 6} &= \frac{(2x - 3)(x^2 - x + 5)}{(2x - 3)(x - 2)} \\ &= \frac{x^2 - x + 5}{x - 2} \end{aligned}$$

4 a  $x - 1$

b

$$\begin{array}{r} x^2 + x + 1 \\ x - 1 \overline{) x^3 + 0x^2 + 0x - 1} \\ \underline{x^3 - x^2} \phantom{+ 0x - 1} \\ x^2 + 0x \phantom{- 1} \\ \underline{x^2 - x} \phantom{- 1} \\ x - 1 \\ \underline{x - 1} \\ 0 \end{array}$$

$$\therefore x^3 - 1 = (x - 1)(x^2 + x + 1)$$

$$\begin{aligned} \therefore \frac{x^3 - 1}{x^2 + x - 2} &= \frac{(x - 1)(x^2 + x + 1)}{(x - 1)(x + 2)} \\ &= \frac{x^2 + x + 1}{x + 2} \end{aligned}$$

5  $\frac{2x+5}{x+3} = \frac{2(x+3)-1}{x+3} = 2 - \frac{1}{x+3}$   
 $\therefore A = 2, B = -1$

6 a  $= \frac{(x+1)+1}{x+1} = 1 + \frac{1}{x+1}$

b  $= \frac{(x-2)+5}{x-2} = 1 + \frac{5}{x-2}$

c  $= \frac{-(1-x)+1}{1-x} = -1 + \frac{1}{1-x}$

d  $= \frac{2(x+2)-3}{x+2} = 2 - \frac{3}{x+2}$

e  $= \frac{\frac{1}{2}(2x-1) - \frac{1}{2}}{2x-1} = \frac{1}{2} - \frac{1}{2(2x-1)}$

f  $= \frac{-2(3+2x)+7}{3+2x} = -2 + \frac{7}{3+2x}$

$$7 \quad \mathbf{a} \quad \begin{array}{r} 1 \\ x^2 + x + 2 \overline{) x^2 + 3x + 5} \\ \underline{x^2 + x + 2} \\ 2x + 3 \end{array}$$

quotient: 1, remainder:  $2x + 3$

$$\mathbf{c} \quad \begin{array}{r} 1 \\ x^2 + 3x - 1 \overline{) x^2 + 0x + 7} \\ \underline{x^2 + 3x - 1} \\ -3x + 8 \end{array}$$

quotient: 1, remainder:  $-3x + 8$

$$\mathbf{e} \quad \begin{array}{r} x - 3 \\ x^2 + x - 2 \overline{) x^3 - 2x^2 - 5x + 8} \\ \underline{x^3 + x^2 - 2x} \\ -3x^2 - 3x + 8 \\ -3x^2 - 3x + 6 \\ \hline 2 \end{array}$$

quotient:  $x - 3$ , remainder: 2

$$\mathbf{g} \quad \begin{array}{r} x + 2 \\ 3x^2 + 4 \overline{) 3x^3 + 6x^2 - 2x + 5} \\ \underline{3x^3 + 0x^2 + 4x} \\ 6x^2 - 6x + 5 \\ \underline{6x^2 + 0x + 8} \\ -6x - 3 \end{array}$$

quotient:  $x + 2$ , remainder:  $-6x - 3$

$$8 \quad \mathbf{a} \quad \begin{array}{r} x + 2 \\ x^2 + 3x - 4 \overline{) x^3 + 5x^2 + 7x - 13} \\ \underline{x^3 + 3x^2 - 4x} \\ 2x^2 + 11x - 13 \\ \underline{2x^2 + 6x - 8} \\ 5x - 5 \end{array}$$

$\therefore = x + 2$  remainder  $5x - 5$

$$\begin{aligned} \mathbf{b} \quad \text{LHS} &= x + 2 + \frac{5x - 5}{x^2 + 3x - 4} \\ &= x + 2 + \frac{5(x - 1)}{(x - 1)(x + 4)} \\ &= x + 2 + \frac{5}{x + 4} \end{aligned}$$

$$\mathbf{b} \quad \begin{array}{r} 2 \\ x^2 - x - 4 \overline{) 2x^2 + 3x - 8} \\ \underline{2x^2 - 2x - 8} \\ 5x \end{array}$$

quotient: 2, remainder:  $5x$

$$\mathbf{d} \quad \begin{array}{r} 3 \\ x^2 + 2 \overline{) 3x^2 - x - 4} \\ \underline{3x^2 + 0x + 6} \\ -x - 10 \end{array}$$

quotient: 3, remainder:  $-x - 10$

$$\mathbf{f} \quad \begin{array}{r} 2x + 3 \\ x^2 - 5x + 1 \overline{) 2x^3 - 7x^2 + 0x + 1} \\ \underline{2x^3 - 10x^2 + 2x} \\ 3x^2 - 2x + 1 \\ \underline{3x^2 - 15x + 3} \\ 13x - 2 \end{array}$$

quotient:  $2x + 3$ , remainder:  $13x - 2$

$$\mathbf{h} \quad \begin{array}{r} 3x + 7 \\ 2x^2 - 5x - 2 \overline{) 6x^3 - x^2 - 44x - 6} \\ \underline{6x^3 - 15x^2 - 6x} \\ 14x^2 - 38x - 6 \\ \underline{14x^2 - 35x - 14} \\ -3x + 8 \end{array}$$

quotient:  $3x + 7$ , remainder:  $-3x + 8$

$$9 \quad \mathbf{a} \quad \begin{array}{r} x - 4 \\ x^2 + 2x - 15 \overline{) x^3 - 2x^2 - 21x + 70} \\ \underline{x^3 + 2x^2 - 15x} \\ -4x^2 - 6x + 70 \\ \underline{-4x^2 - 8x + 60} \\ 2x + 10 \end{array}$$

$$\begin{aligned} \therefore f(x) &= x - 4 + \frac{2x + 10}{x^2 + 2x - 15} \\ &= x - 4 + \frac{2(x + 5)}{(x + 5)(x - 3)} \\ &= x - 4 + \frac{2}{x - 3} \end{aligned}$$

$$\mathbf{b} \quad \frac{3x - 7}{x - 3} = \frac{3(x - 3) + 2}{x - 3} = 3 + \frac{2}{x - 3}$$

$$\begin{aligned} \therefore f(x) &= \frac{3x - 7}{x - 3} \\ \Rightarrow x - 4 + \frac{2}{x - 3} &= 3 + \frac{2}{x - 3} \\ x - 4 &= 3 \\ x &= 7 \end{aligned}$$