1 **a** $\frac{dy}{dx} = 2x + 6$ **b** $\frac{dy}{dx} = 8x + 2$ **c** $\frac{dy}{dx} = 3x^2 - 12$ **d** $\frac{dy}{dx} = 18x - 3x^2$

$2x + 6 = 0$ $8x + 2 = 0$ $3x^2 - 12 = 0$ $18x - 3x^2 = 0$

$x = -3$ $x = -\frac{1}{4}$ $x^2 = 4$ $3x(6 - x) = 0$

                                           $x = \pm 2$ $x = 0, 6$

e $\frac{dy}{dx} = 3x^2 - 10x + 3$ **f** $\frac{dy}{dx} = 1 - 9x^{-2}$ **g** $y = x^3 - 3x^2 + 3x - 9$ **h** $\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} - 2$

$3x^2 - 10x + 3 = 0$ $1 - 9x^{-2} = 0$ $\frac{dy}{dx} = 3x^2 - 6x + 3$ $\frac{1}{2}x^{-\frac{1}{2}} - 2 = 0$

$(3x - 1)(x - 3) = 0$ $x^2 = 9$ $3x^2 - 6x + 3 = 0$ $x^{-\frac{1}{2}} = 4$

$x = \frac{1}{3}, 3$ $x = \pm 3$ $3(x - 1)^2 = 0$ $x = \frac{1}{16}$

                                           $x = 1$

2 **a** $f'(x) = 4x + 2$ **b** $f'(x) = 6x - 6x^2$ **c** $f'(x) = 9x^2 - 1$

$\therefore 4x + 2 \geq 0$ $\therefore 6x - 6x^2 \geq 0$ $\therefore 9x^2 - 1 \geq 0$

$x \geq -\frac{1}{2}$ $6x(1 - x) \geq 0$ $x^2 \geq \frac{1}{9}$

                            $0 \leq x \leq 1$ $x \leq -\frac{1}{3}$ and $x \geq \frac{1}{3}$

d $f'(x) = 3x^2 + 12x - 15$ **e** $f(x) = x^3 - 12x^2 + 36x$ **f** $f'(x) = 2 - 8x^{-2}$

$\therefore 3x^2 + 12x - 15 \geq 0$ $f'(x) = 3x^2 - 24x + 36$ $\therefore 2 - 8x^{-2} \geq 0$

$3(x + 5)(x - 1) \geq 0$ $\therefore 3x^2 - 24x + 36 \geq 0$ $x^2 \geq 4$

$x \leq -5$ and $x \geq 1$ $3(x - 2)(x - 6) \geq 0$ $x \leq -2$ and $x \geq 2$

                            $x \leq 2$ and $x \geq 6$

3 **a** $f'(x) = 3x^2 + 4x$ **b** $f'(x) = 27 - 3x^2$ **c** $f(x) = 2x^3 - x^2 - 4x + 2$

$\therefore 3x^2 + 4x \leq 0$ $\therefore 27 - 3x^2 \leq 0$ $f'(x) = 6x^2 - 2x - 4$

$x(3x + 4) \leq 0$ $x^2 \geq 9$ $\therefore 6x^2 - 2x - 4 \leq 0$

$-\frac{4}{3} \leq x \leq 0$ $x \leq -3$ and $x \geq 3$ $2(3x + 2)(x - 1) \leq 0$

                                           $-\frac{2}{3} \leq x \leq 1$

4 **a** $(x + 1)$ factor $\therefore f(-1) = 0$

$\therefore -1 + k + 3 = 0$

$k = -2$

b $f'(x) = 3x^2 - 4x$

$\therefore 3x^2 - 4x \geq 0$

$x(3x - 4) \geq 0$

$x \leq 0$ and $x \geq \frac{4}{3}$

5 **a** $\frac{dy}{dx} = 2x + 2$

SP: $2x + 2 = 0$

$x = -1$

$\therefore$ $(-1, -1)$

b $\frac{dy}{dx} = 10x - 4$

SP: $10x - 4 = 0$

$x = \frac{2}{5}$

$\therefore$ $(\frac{2}{5}, \frac{1}{5})$

c $\frac{dy}{dx} = 3x^2 - 3$

SP: $3x^2 - 3 = 0$

$x^2 = 1$

$x = \pm 1$

$\therefore$ $(-1, 6), (1, 2)$

d $\frac{dy}{dx} = 12x^2 + 6x$

SP: $12x^2 + 6x = 0$

$6x(2x + 1) = 0$

$x = -\frac{1}{2}, 0$

$\therefore$ $(-\frac{1}{2}, \frac{9}{4}), (0, 2)$

e $\frac{dy}{dx} = 2 - 8x^{-2}$

SP: $2 - 8x^{-2} = 0$

$x^2 = 4$

$x = \pm 2$

$\therefore$ $(-2, -5), (2, 11)$

f $\frac{dy}{dx} = 3x^2 - 18x - 21$

SP: $3x^2 - 18x - 21 = 0$

$3(x + 1)(x - 7) = 0$

$x = -1, 7$

$\therefore$ $(-1, 22), (7, -234)$

g $\frac{dy}{dx} = -x^{-2} - 8x$

SP: $-x^{-2} - 8x = 0$

$x^3 = -\frac{1}{8}$

$x = -\frac{1}{2}$

$\therefore$ $(-\frac{1}{2}, -3)$

h $\frac{dy}{dx} = 3x^{\frac{1}{2}} - 6$

SP: $3x^{\frac{1}{2}} - 6 = 0$

$x^{\frac{1}{2}} = 2$

$x = 4$

$\therefore$ $(4, -8)$

i $\frac{dy}{dx} = 6x^{-\frac{1}{3}} - 2$

SP: $6x^{-\frac{1}{3}} - 2 = 0$

$x^{-\frac{1}{3}} = 3$

$x = \frac{1}{27}$

$\therefore$ $(\frac{1}{27}, 5\frac{25}{27})$

6 **a** $\frac{dy}{dx} = 4 - 2x$

SP: $4 - 2x = 0$

$x = 2$

$\frac{d^2y}{dx^2} = -2$

$(2, 9)$: max

b $\frac{dy}{dx} = 3x^2 - 3$

SP: $3x^2 - 3 = 0$

$x^2 = 1$

$x = \pm 1$

$\frac{d^2y}{dx^2} = 6x$

$(-1, 2)$: $\frac{d^2y}{dx^2} = -6$, max

$(1, -2)$: $\frac{d^2y}{dx^2} = 6$, min

c $\frac{dy}{dx} = 3x^2 + 18x$

SP: $3x^2 + 18x = 0$

$3x(x + 6) = 0$

$x = -6, 0$

$\frac{d^2y}{dx^2} = 6x + 18$

$(-6, 100)$: $\frac{d^2y}{dx^2} = -18$, max

$(0, -8)$: $\frac{d^2y}{dx^2} = 18$, min

d $\frac{dy}{dx} = 3x^2 - 12x - 36$

SP: $3x^2 - 12x - 36 = 0$

$3(x + 2)(x - 6) = 0$

$x = -2, 6$

$\frac{d^2y}{dx^2} = 6x - 12$

$(-2, 55)$: $\frac{d^2y}{dx^2} = -24$, max

$(6, -201)$: $\frac{d^2y}{dx^2} = 24$, min

e $\frac{dy}{dx} = 4x^3 - 16x$

SP: $4x^3 - 16x = 0$

$4x(x^2 - 4) = 0$

$x = 0, \pm 2$

$\frac{d^2y}{dx^2} = 12x^2 - 16$

$(-2, -18)$: $\frac{d^2y}{dx^2} = 32$, min

$(0, -2)$: $\frac{d^2y}{dx^2} = -16$, max

$(2, -18)$: $\frac{d^2y}{dx^2} = 32$, min

f $\frac{dy}{dx} = 9 - 4x^{-2}$

SP: $9 - 4x^{-2} = 0$

$x^2 = \frac{4}{9}$

$x = \pm \frac{2}{3}$

$\frac{d^2y}{dx^2} = 8x^{-3}$

$(-\frac{2}{3}, -12)$: $\frac{d^2y}{dx^2} = -27$, max

$(\frac{2}{3}, 12)$: $\frac{d^2y}{dx^2} = 27$, min

g $\frac{dy}{dx} = 1 - 3x^{-\frac{1}{2}}$

SP: $1 - 3x^{-\frac{1}{2}} = 0$

$\quad x^{-\frac{1}{2}} = \frac{1}{3}$

$\quad x = 9$

$\frac{d^2y}{dx^2} = \frac{3}{2}x^{-\frac{3}{2}}$

$(9, -9)$: $\frac{d^2y}{dx^2} = \frac{1}{18}$, min

h $\frac{dy}{dx} = -8 + 14x - 6x^2$

SP: $-8 + 14x - 6x^2 = 0$

$\quad -2(3x - 4)(x - 1) = 0$

$\quad x = 1, \frac{4}{3}$

$\frac{d^2y}{dx^2} = 14 - 12x$

$(1, 0)$: $\frac{d^2y}{dx^2} = 2$, min

$(\frac{4}{3}, \frac{1}{27})$: $\frac{d^2y}{dx^2} = -2$, max

i $y = \frac{1}{2}x^2 + 8x^{-2}$

$\frac{dy}{dx} = x - 16x^{-3}$

SP: $x - 16x^{-3} = 0$

$\quad x^4 = 16$

$\quad x = \pm 2$

$\frac{d^2y}{dx^2} = 1 + 48x^{-4}$

$(-2, 4)$: $\frac{d^2y}{dx^2} = 4$, min

$(2, 4)$: $\frac{d^2y}{dx^2} = 4$, min

7

a $\frac{dy}{dx} = 2x - 3x^2$

SP: $2x - 3x^2 = 0$

$\quad x(2 - 3x) = 0$

$\quad x = 0, \frac{2}{3}$

$\frac{d^2y}{dx^2} = 2 - 6x$

$(0, 0)$: $\frac{d^2y}{dx^2} = 2$, min

$(\frac{2}{3}, \frac{4}{27})$: $\frac{d^2y}{dx^2} = -2$, max

b $\frac{dy}{dx} = 3x^2 + 6x + 3$

SP: $3x^2 + 6x + 3 = 0$

$\quad 3(x + 1)^2 = 0$

$\quad x = -1$

$\frac{d^2y}{dx^2} = 6x + 6$

$(-1, -1)$: $\frac{d^2y}{dx^2} = 0$

x	< -1	-1	> -1
$\frac{dy}{dx}$	$+$	0	$+$

$\therefore (-1, -1)$: point of inflexion

c $\frac{dy}{dx} = 4x^3$

SP: $4x^3 = 0$

$\quad x = 0$

$\frac{d^2y}{dx^2} = 12x^2$

$(0, -2)$: $\frac{d^2y}{dx^2} = 0$

x	< 0	0	> 0
$\frac{dy}{dx}$	$-$	0	$+$

$\therefore (0, -2)$: min

d $\frac{dy}{dx} = -12 + 12x - 3x^2$

SP: $-12 + 12x - 3x^2 = 0$

$\quad -3(x - 2)^2 = 0$

$\quad x = 2$

$\frac{d^2y}{dx^2} = 12 - 6x$

$(2, -4)$: $\frac{d^2y}{dx^2} = 0$

x	< 2	2	> 2
$\frac{dy}{dx}$	$-$	0	$-$

$\therefore (2, -4)$: point of inflexion

e $\frac{dy}{dx} = 2x - 16x^{-2}$

SP: $2x - 16x^{-2} = 0$

$\quad x^3 = 8$

$\quad x = 2$

$\frac{d^2y}{dx^2} = 2 + 32x^{-3}$

$(2, 12)$: $\frac{d^2y}{dx^2} = 6$, min

f $\frac{dy}{dx} = 4x^3 + 12x^2$

SP: $4x^3 + 12x^2 = 0$

$\quad 4x^2(x + 3) = 0$

$\quad x = -3, 0$

$\frac{d^2y}{dx^2} = 12x^2 + 24x$

$(-3, -28)$: $\frac{d^2y}{dx^2} = 36$, min

$(0, -1)$: $\frac{d^2y}{dx^2} = 0$

x	$-3 < x < 0$	0	> 0
$\frac{dy}{dx}$	$+$	0	$+$

$\therefore (0, -1)$: point of inflexion

8   **a** $\dfrac{dy}{dx} = 3x^2 + 6x$

SP: $3x^2 + 6x = 0$

$\ 3x(x + 2) = 0$

$\ x = -2,\ 0$

$\dfrac{d^2y}{dx^2} = 6x + 6$

$(-2, 4)$: $\dfrac{d^2y}{dx^2} = -6$, max

$(0, 0)$: $\dfrac{d^2y}{dx^2} = 6$, min
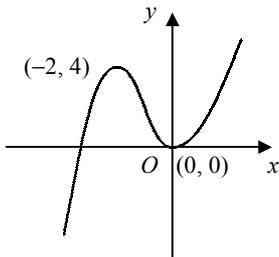
b $\dfrac{dy}{dx} = 1 - x^{-2}$

SP: $1 - x^{-2} = 0$

$\ x^2 = 1$

$\ x = \pm 1$

$\dfrac{d^2y}{dx^2} = 2x^{-3}$

$(-1, -2)$: $\dfrac{d^2y}{dx^2} = -2$, max

$(1, 2)$: $\dfrac{d^2y}{dx^2} = 2$, min
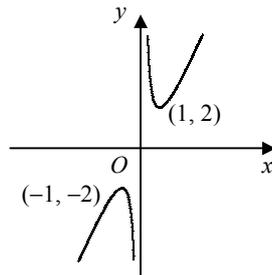

c $\dfrac{dy}{dx} = 3x^2 - 6x + 3$

SP: $3x^2 - 6x + 3 = 0$

$\ 3(x - 1)^2 = 0$

$\ x = 1$

$\dfrac{d^2y}{dx^2} = 6x - 6$

$(1, 0)$: $\dfrac{d^2y}{dx^2} = 0$

x	< 1	1	> 1
$\dfrac{dy}{dx}$	$+$	0	$+$

$\therefore$ $(1, 0)$: point of inflexion
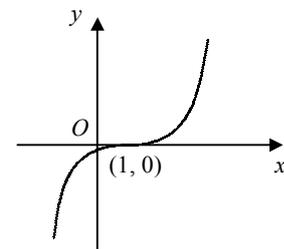
d $\dfrac{dy}{dx} = 3 - 2x^{-\frac{1}{2}}$

SP: $3 - 2x^{-\frac{1}{2}} = 0$

$\ x^{-\frac{1}{2}} = \frac{3}{2}$

$\ x = \frac{4}{9}$

$\dfrac{d^2y}{dx^2} = x^{-\frac{3}{2}}$

$(\frac{4}{9}, -\frac{4}{3})$: $\dfrac{d^2y}{dx^2} = \frac{27}{8}$, min
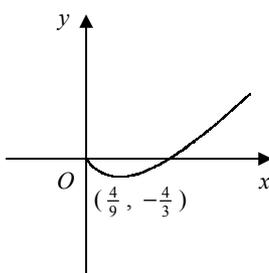
**e** $\dfrac{dy}{dx} = 3x^2 + 8x - 3$

SP: $3x^2 + 8x - 3 = 0$

$\ (3x - 1)(x + 3) = 0$

$\ x = -3,\ \frac{1}{3}$

$\dfrac{d^2y}{dx^2} = 6x + 8$

$(-3, 13)$: $\dfrac{d^2y}{dx^2} = -10$, max

$(\frac{1}{3}, -5\frac{14}{27})$: $\dfrac{d^2y}{dx^2} = 10$, min
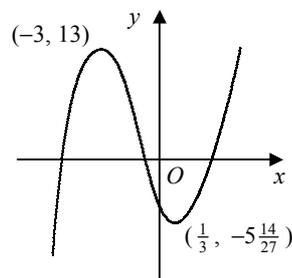
f $y = x^4 - 8x^2 + 12$

$\dfrac{dy}{dx} = 4x^3 - 16x$

SP: $4x^3 - 16x = 0$

$\ 4x(x + 2)(x - 2) = 0$

$\ x = -2,\ 0,\ 2$

$\dfrac{d^2y}{dx^2} = 12x^2 - 16$

$(-2, -4)$: $\dfrac{d^2y}{dx^2} = 32$, min

$(0, 12)$: $\dfrac{d^2y}{dx^2} = -16$, max

$(2, -4)$: $\dfrac{d^2y}{dx^2} = 32$, min