1 Given that the point A has position vector $3\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$ and the point B has position vector $-4\mathbf{i} + 7\mathbf{j} + 5\mathbf{k}$

(a) Find the vector $\overrightarrow{AB}$

(b) Find $|\overrightarrow{AB}|$

a/ $\overrightarrow{AB} = \begin{pmatrix} -4 \\ 7 \\ 5 \end{pmatrix} - \begin{pmatrix} 3 \\ 4 \\ -2 \end{pmatrix}$

$= \begin{pmatrix} -7 \\ 3 \\ 7 \end{pmatrix}$ $\quad -7i + 3j + 7k$

b/ $\sqrt{7^2 + 3^2 + 7^2} = \sqrt{107}$

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2 Given that $|3\mathbf{i} + k\mathbf{j} + 2\mathbf{k}| = 7$

Find the two possible values of k

$\sqrt{3^2 + k^2 + 2^2} = 7$

$3^2 + k^2 + 2^2 = 49$

$k^2 = 36$

$k = \pm 6$

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3 Given that the point A has position vector $-5\mathbf{i} + 7\mathbf{j} - 3\mathbf{k}$ and the point B has position vector $-8\mathbf{i} + 2\mathbf{j} - \mathbf{k}$

(a) Find the vector $\overrightarrow{AB}$

(b) Find $|\overrightarrow{AB}|$

a/ $\overrightarrow{AB} = \begin{pmatrix} -8 \\ 2 \\ -1 \end{pmatrix} - \begin{pmatrix} -5 \\ 7 \\ -3 \end{pmatrix} = \begin{pmatrix} -3 \\ -5 \\ 2 \end{pmatrix}$

$-3i - 5j + 2k$

b/ $\sqrt{3^2 + 5^2 + 2^2} = \sqrt{38}$

4 $\mathbf{a} = -5\mathbf{i} + 7\mathbf{j} + 2\mathbf{k}$ and $\mathrm{b} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$
Given that the resultant force of **a** and **b** is $-2\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$ find the values of x, y and z

$$\begin{pmatrix} -5 \\ 7 \\ 2 \end{pmatrix} + \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -2 \\ -3 \\ 6 \end{pmatrix}$$

$-5 + x = -2$ $\quad$ $7 + y = -3$ $\quad$ $2 + z = 6$

$\underline{x = 3}$ $\quad$ $\underline{y = -10}$ $\quad$ $\underline{z = 4}$

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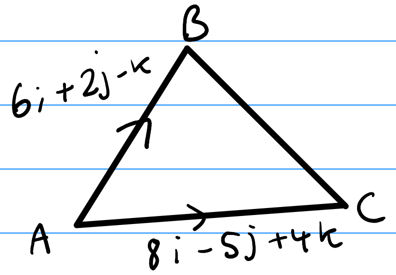
5 In triangle ABC, $\overrightarrow{AB} = 6\mathbf{i} + 2\mathbf{j} - \mathbf{k}$, $\overrightarrow{AC} = 8\mathbf{i} - 5\mathbf{j} + 4\mathbf{k}$

(a) Find the vector $\overrightarrow{BC}$

(b) Find the length of the line AB



$$\overrightarrow{BC} = \begin{pmatrix} -6 \\ -2 \\ 1 \end{pmatrix} + \begin{pmatrix} 8 \\ -5 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 \\ -7 \\ 5 \end{pmatrix}$$

$\underline{2i - 7j + 5k}$

b) $\sqrt{6^2 + 2^2 + 1^2} = \underline{\sqrt{41}}$

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6 Three forces act on an object $\mathbf{F}_1 = -5\mathbf{i} + 7\mathbf{j} + 4\mathbf{k}$, $\mathbf{F}_2 = 4\mathbf{i} + 6\mathbf{j} - 2\mathbf{k}$ and $\mathbf{F}_3 = 3\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}$
Find the resultant force.

$$\begin{pmatrix} -5 \\ 7 \\ 4 \end{pmatrix} + \begin{pmatrix} 4 \\ 6 \\ -2 \end{pmatrix} + \begin{pmatrix} 3 \\ -5 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ 8 \\ 5 \end{pmatrix}$$

$\underline{2i + 8j + 5k}$

7 The points P, Q, R and S have position vectors $\mathbf{p} = -3\mathbf{i} + 10\mathbf{j} - 7\mathbf{k}$, $\mathbf{q} = 5\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$
$\mathbf{r} = -4\mathbf{i} + 6\mathbf{j} - 12\mathbf{k}$ $\mathbf{s} = 8\mathbf{i} - 3\mathbf{j} + 3\mathbf{k}$

Show that $\overrightarrow{PQ}$ is parallel to $\overrightarrow{RS}$

$$\overrightarrow{PQ} = \begin{pmatrix} 5 \\ 4 \\ 3 \end{pmatrix} - \begin{pmatrix} -3 \\ 10 \\ -7 \end{pmatrix} = \begin{pmatrix} 8 \\ -6 \\ 10 \end{pmatrix}$$

$$\overrightarrow{RS} = \begin{pmatrix} 8 \\ -3 \\ 3 \end{pmatrix} - \begin{pmatrix} -4 \\ 6 \\ -12 \end{pmatrix} = \begin{pmatrix} 12 \\ -9 \\ 15 \end{pmatrix}$$

$$\overrightarrow{PQ} = 8i - 6j + 10k$$
$$= 2(4i - 3j + 5k)$$

$$\overrightarrow{RS} = 12i - 9j + 15k$$
$$= 3(4i - 3j + 5k)$$

$\overrightarrow{PQ}$ and $\overrightarrow{RS}$ are both multiples of $(4i - 3j + 5k)$ $\therefore$ parallel

$$\overrightarrow{PQ} = \frac{2}{3}\overrightarrow{RS}$$

8 Relative to a fixed origin O,

the point A has position vector $(3\mathbf{i} + 4\mathbf{j} - 2\mathbf{k})$,

the point B has position vector $(7\mathbf{i} - 5\mathbf{j} + 3\mathbf{k})$,

and the point C has position vector $(a\mathbf{i} + 3\mathbf{j} - 4\mathbf{k})$, where a is a constant and $a > 0$,

D is the point such that $\overrightarrow{AB} = \overrightarrow{BD}$

(a) Find the position vector of D.

Given that $|\overrightarrow{AC}| = 5$

(b) Find the value of a.

a/ $\overrightarrow{AB} = \begin{pmatrix} 7 \\ -5 \\ 3 \end{pmatrix} - \begin{pmatrix} 3 \\ 4 \\ -2 \end{pmatrix} = \begin{pmatrix} 4 \\ -9 \\ 5 \end{pmatrix}$

$\begin{pmatrix} 7 \\ -5 \\ 3 \end{pmatrix} + \begin{pmatrix} 4 \\ -9 \\ 5 \end{pmatrix} = \begin{pmatrix} 11 \\ -14 \\ 8 \end{pmatrix}$

$\underline{\underline{11i - 14j + 8k}}$

b/ $\overrightarrow{AC} = \begin{pmatrix} a \\ 3 \\ -4 \end{pmatrix} - \begin{pmatrix} 3 \\ 4 \\ -2 \end{pmatrix} = \begin{pmatrix} a-3 \\ -1 \\ -2 \end{pmatrix}$

$\sqrt{(a-3)^2 + (1)^2 + (2)^2} = 5$

$(a-3)^2 + 1 + 4 = 25$

$(a-3)^2 = 20$

$a^2 - 6a + 9 = 20$

$a^2 - 6a - 11 = 0$

$a = \underline{\underline{3 + 2\sqrt{5}}}$ a cannot be negative

9 Relative to a fixed origin O,

the point A has position vector $(2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k})$,

the point B has position vector $(5\mathbf{i} - 10\mathbf{j} + 2\mathbf{k})$,

the point C has position vector $(3\mathbf{i} - 7\mathbf{j} - 2\mathbf{k})$,

(a) Find $\overrightarrow{BC}$

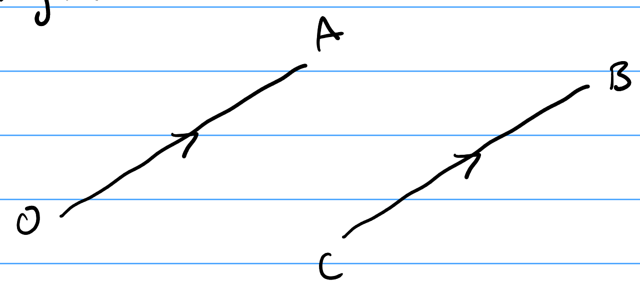
(b) Show that the quadrilateral $OABC$ is a parallelogram, giving reasons for your answer.

a/ $\overrightarrow{BC} = \begin{pmatrix} 3 \\ -7 \\ -2 \end{pmatrix} - \begin{pmatrix} 5 \\ -10 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ 3 \\ -4 \end{pmatrix}$

$\underline{-2i + 3j - 4k}$

b/ $\overrightarrow{OA} = 2i - 3j + 4k$ $\quad$ $\overrightarrow{CB} = 2i - 3j + 4k$

$\overrightarrow{OA}$ and $\overrightarrow{CB}$ are parallel and equal in length.



$\therefore$ $\overrightarrow{OC}$ and $\overrightarrow{AB}$ must also be parallel and equal in length

$\therefore$ parallelogram.

10 Relative to a fixed origin O,

the point A has position vector $(4\mathbf{i} - 2\mathbf{j} - 5\mathbf{k})$,

the point B has position vector $(6\mathbf{i} + \mathbf{j} - 9\mathbf{k})$,

the point C has position vector $(4\mathbf{i} + 6\mathbf{j} - 8\mathbf{k})$,

(a) Find $\overrightarrow{AB}$

(b) Show that the quadrilateral $OABC$ is a trapezium, giving reasons for your answer.

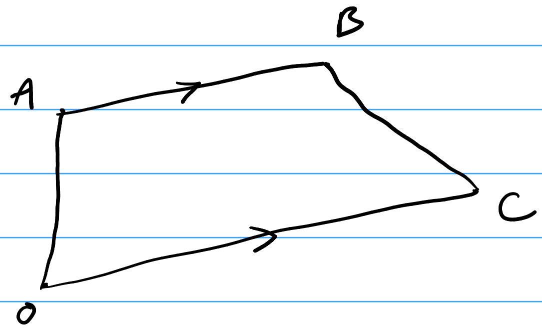
a/ $\overrightarrow{AB} = \begin{pmatrix} 6 \\ 1 \\ -9 \end{pmatrix} - \begin{pmatrix} 4 \\ -2 \\ -5 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix}$

$\underline{2i + 3j - 4k}$

b/ $\overrightarrow{OC} = 4i + 6j - 8k$

$= 2(2i + 3j - 4k)$

$\overrightarrow{OC} = 2\overrightarrow{AB}$



a set of unequal parallel lines $\therefore$ $\underline{\underline{\text{trapezium}}}$

11 The points A and B have position vectors $\begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ -5 \\ 2 \end{pmatrix}$ respectively.

(a) Find the exact length of AB.

(b) Find the position vector of the midpoint of AB.

The points P and Q have position vectors $\begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ 5 \\ 3 \end{pmatrix}$ respectively.

(c) Show that $ABPQ$ is a parallelogram.

a/ $\overrightarrow{AB} = \begin{pmatrix} 3 \\ -5 \\ 2 \end{pmatrix} - \begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \\ -2 \end{pmatrix}$

$|\overrightarrow{AB}| = \sqrt{2^2 + 3^2 + 2^2}$

$= \underline{\underline{\sqrt{17}}}$

b/ $\frac{3+1}{2} = 2$ $\frac{-5+-2}{2} = -\frac{7}{2}$ $\frac{2+4}{2} = 3$

$\underline{\underline{2i - \frac{7}{2}j + 3k}}$

c/ $\overrightarrow{QP} = 2i - 3j - 2k$

$\overrightarrow{QP} = \overrightarrow{AB}$

set of equal parallel lines $\therefore$ parallelogram

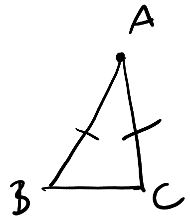
12 The point A has the position vector $\begin{pmatrix} a \\ b \\ 3 \end{pmatrix}$ where a and b are positive constants, point B has the position vector $\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$ and point C has the position vector $\begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$

ABC is an isosceles triangle with $AB = AC$

(a) Show that $a + b = 3$

Given that the area of ABC is $\sqrt{8}$

(b) Find the exact values of a and b.



$\vec{AB} = \begin{pmatrix} 2-a \\ 3-b \\ 1 \end{pmatrix}$ $\qquad \vec{BC} = \begin{pmatrix} -2 \\ -2 \\ -2 \end{pmatrix}$

$\vec{AC} = \begin{pmatrix} 0-a \\ 1-b \\ -1 \end{pmatrix}$

$|\vec{AB}| = |\vec{AC}|$

$(2-a)^2 + (3-b)^2 + 1^2 = (-a)^2 + (1-b)^2 + 1^2$

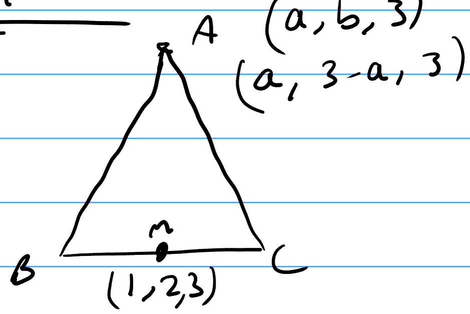
$4 - 4a + \cancel{a^2} + 9 - 6b + \cancel{b^2} + \cancel{1} = \cancel{a^2} + 1 - 2b + \cancel{b^2} + \cancel{1}$

$13 - 4a - 6b = 1 - 2b$

$12 - 4a = 4b$

$12 = 4a + 4b$

$\underline{\underline{3 = a + b}}$



b/ $\frac{1}{2}bh = \sqrt{8}$

Midpoint of BC $(1,2,3)$

$|\vec{BC}| = \sqrt{2^2 + 2^2 + 2^2}$

$= \sqrt{12}$ $\qquad b = 3 - a$

$\vec{AM} = \begin{pmatrix} a \\ 3-a \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} a-1 \\ 1-a \\ 0 \end{pmatrix}$

$|\vec{AM}| = \sqrt{(a-1)^2 + (1-a)^2}$

$$= \sqrt{a^2 - 2a + 1 + 1 - 2a + a^2}$$

$$= \sqrt{2a^2 - 4a + 2}$$

$$\frac{1}{2} \cdot \sqrt{12} \sqrt{2a^2 - 4a + 2} = \sqrt{8}$$

$$\frac{1}{4}(12)(2a^2 - 4a + 2) = 8$$

$$6a^2 - 12a + 6 = 8$$

$$6a^2 - 12a - 2 = 0$$

$$3a^2 - 6a - 1 = 0$$

$$a = \frac{3 + 2\sqrt{3}}{3}$$

a is positive so only one ans.

$$b = 3 - \frac{3 + 2\sqrt{3}}{3}$$

$$= \frac{6 - 2\sqrt{3}}{3}$$

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13 The points A and B have position vectors $\begin{pmatrix} -3 \\ 5 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 4 \\ 7 \end{pmatrix}$ respectively.

Find the exact length of AB.

$$\overrightarrow{AB} = \begin{pmatrix} 1 \\ 4 \\ 7 \end{pmatrix} - \begin{pmatrix} -3 \\ 5 \\ 1 \end{pmatrix} = \begin{pmatrix} 4 \\ -1 \\ 6 \end{pmatrix}$$

$$|\overrightarrow{AB}| = \sqrt{4^2 + 1^2 + 6^2}$$

$$= \sqrt{53}$$