edexcel

Pearson Edexcel Level 1/Level 2 GCSE (9 – 1) in Mathematics (1MA1)

Two-year Scheme of Work

For first teaching from September 2015

Issue 1 (March 2015)

Contents

Introduction

This scheme of work is based upon a five-term model over two years for both Foundation and Higher tier students. It can be used directly as a scheme of work for the GCSE Mathematics specification (1MA1).

The scheme of work is broken up into two tiers, and then into units and sub-units, so that there is greater flexibility for moving topics around to meet planning needs.

Each unit contains:

- Tier
- Contents, referenced back to the specification
- Prior knowledge
- Keywords.

Each sub-unit contains:

- Recommended teaching time, though of course this is adaptable according to individual teaching needs
- Objectives for students at the end of the sub-unit
- Possible success criteria for students at the end of the sub-unit
- Opportunities for reasoning/problem-solving
- Common misconceptions
- Notes for general mathematical teaching points.

Teachers should be aware that the estimated teaching hours are approximate and should be used as a guideline only. Further information on teaching time for the GCSE Mathematics specification (1MA1) can be found on p.20 of our *Getting Started* document on the Edexcel mathematics website (http://qualifications.pearson.com/en/home.html).

Our free support for the GCSE Mathematics specification (1MA1) can be found on the Edexcel mathematics website (http://qualifications.pearson.com/en/home.html) and on the Emporium (www.edexcelmaths.com).

GCSE Mathematics (1MA1)

Foundation Tier

Scheme of Work

Unit		Title	Estimated hours
1	a	Integers and place value	4
	b	Decimals	3
	c	Indices, powers and roots	5
	d	Factors, multiples and primes	4
2	a	Algebra: the basics	6
	b	Expressions and substitution into formulae	5
3	a	Tables, charts and graphs	11
	b	Pie charts	3
	c	Scatter graphs	4
4	a	Fractions, decimals and percentages	7
	b	Percentages	6
5	a	Equations and inequalities	9
	b	Sequences	5
6	a	Properties of shapes, parallel lines and angle facts	7
	b	Interior and exterior angles of polygons	4
7		Statistics, sampling and the averages	7
8		Perimeter, area and volume	10
9	a	Real-life graphs	8
	b	Straight-line graphs	6
10		Transformations	11
11	a	Ratio	4
	b	Proportion	5
12		Right-angled triangles: Pythagoras and trigonometry	5
13		Probability	12
14		Multiplicative reasoning	7
15	a	Plans and elevations	5
	b	Constructions, loci and bearings	7
16	a	Quadratic equations: expanding and factorising	5
	b	Quadratic equations: graphs	4
17		Circles, cylinders, cones and spheres	6
18	a	Fractions and reciprocals	5
	b	Indices and standard form	5
19	a	Similarity and congruence in 2D	7
	b	Vectors	7
20		Rearranging equations, graphs of cubic and reciprocal functions and simultaneous equations	5

UNIT 1: Number, powers, decimals, HCF and LCM, roots and rounding

Return to Overview

SPECIFICATION REFERENCES

N1 order positive and negative integers, decimals and fractions; use the symbols =, ≠, <, >, ≤, ≥

N2 apply the four operations, including formal written methods, to integers, decimals and simple fractions (proper and improper), and mixed numbers – all both positive and negative; understand and use place value (e.g. when working with very large or very small numbers, and when calculating with decimals)

N3 recognise and use relationships between operations, including inverse operations (e.g. cancellation to simplify calculations and expressions); use conventional notation for priority of operations, including brackets, powers, roots and reciprocals

N4 use the concepts and vocabulary of prime numbers, factors (divisors), multiples, common factors, common multiples, highest common factor, lowest common multiple, prime factorisation, including using product notation and the unique factorisation theorem

N5 apply systematic listing strategies

N6 use positive integer powers and associated real roots (square, cube and higher), recognise powers of 2, 3, 4, 5

N7 calculate with roots and with integer and with integer indices

N13 use standard units of mass, length, time, money and other measures (including standard compound measures) using decimal quantities where appropriate

N14 estimate answers; check calculations using approximation and estimation, including answers obtained using technology

N15 round numbers and measures to an appropriate degree of accuracy (e.g. to a specified number of decimal places or significant figures);

PRIOR KNOWLEDGE

Students will have an appreciation of place value, and recognise even and odd numbers.
Students will have knowledge of using the four operations with whole numbers.
Students should have knowledge of integer complements to 10 and to 100.
Students should have knowledge of strategies for multiplying and dividing whole numbers by 2, 4, 5, and 10.
Students should be able to read and write decimals in figures and words.

KEYWORDS

Integer, number, digit, negative, decimal, addition, subtraction, multiplication, division, remainder, operation, estimate, power, roots, factor, multiple, primes, square, cube, even, odd

1a. Integers and place value	**Teaching time**
(N1, N2, N3, N4, N14, N15)	3-5 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Use and order positive and negative numbers (integers) and decimals; use the symbols <, > and understand the ≠ symbol;
- Add, subtract, multiply and divide positive and negative numbers (integers);
- Recall all multiplication facts to 10 × 10, and use them to derive quickly the corresponding division facts;
- Multiply or divide any number by powers of 10;
- Use brackets and the hierarchy of operations (not including powers);
- Round numbers to a given power of 10;
- Check answers by rounding and using inverse operations.

POSSIBLE SUCCESS CRITERIA

Given 5 digits, what are the largest or smallest answers when subtracting a two-digit number from a three-digit number?
Use inverse operations to justify answers, e.g. 9 x 23 = 207 so 207 ÷ 9 = 23.
Check answers by rounding to nearest 10, 100, or 1000 as appropriate, e.g. 29 × 31 ≈ 30 × 30

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Missing digits in calculations involving the four operations
Questions such as: Phil states 3.44 × 10 = 34.4 and Chris states 3.44 × 10 = 34.40. Who is correct?
Show me another number with 3, 4, 5, 6, 7 digits that includes a 6 with the same value as the "6" in the following number 36, 754

COMMON MISCONCEPTIONS

Stress the importance of knowing the multiplication tables to aid fluency.
Students may write statements such as 150 – 210 = 60.

NOTES

Much of this unit will have been encountered by students in previous Key Stages, meaning that teaching time may focus on application or consolidation of prior learning.
Particular emphasis should be given to the importance of students presenting their work clearly.
Formal written methods of addition, subtraction and multiplication work from right to left, whilst formal division works from left to right.
Any correct method of multiplication will still gain full marks, for example, the grid method, the traditional method, Napier's bones.
Negative numbers in real life can be modelled by interpreting scales on thermometers using F and C.
Encourage the exploration of different calculation methods.
Students should be able to write numbers in words and from words as a real-life skill.

1b. Decimals
(N1, N2, N3, N13, N15)

Teaching time
2-4 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Use decimal notation and place value;
- Identify the value of digits in a decimal or whole number;
- Compare and order decimal numbers using the symbols <, >;
- Understand the ≠ symbol (not equal);
- Write decimal numbers of millions, e.g. 2 300 000 = 2.3 million;
- Add, subtract, multiply and divide decimals;
- Multiply or divide by any number between 0 and 1;
- Round to the nearest integer;
- Round to a given number of decimal places and significant figures;
- Estimate answers to calculations by rounding numbers to 1 significant figure;
- Use one calculation to find the answer to another.

POSSIBLE SUCCESS CRITERIA

Use mental methods for × and ÷, e.g. 5 × 0.6, 1.8 ÷ 3.
Solve a problem involving division by a decimal (up to 2 decimal places).
Given 2.6 × 15.8 = 41.08, what is 26 × 0.158? What is 4108 ÷ 26?
Calculate, e.g. 5.2 million + 4.3 million.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Problems involving shopping for multiple items, such as: Rob purchases a magazine costing £2.10, a newspaper costing 82p and two bars of chocolate. He pays with a £10 note and gets £5.40 change. Work out the cost of one bar of chocolate.
When estimating, students should be able to justify whether the answer will be an overestimate or underestimate.

COMMON MISCONCEPTIONS

Significant figures and decimal place rounding are often confused.
Some students may think 35 877 = 36 to two significant figures.

NOTES

Practise long multiplication and division, use mental maths problems with decimals such as 0.1, 0.001.
Amounts of money should always be rounded to the nearest penny.

1c. Indices, powers and roots

(N6, N7)

Teaching time
4-6 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Find squares and cubes:
  - recall integer squares up to 10 x 10 and the corresponding square roots;
  - understand the difference between positive and negative square roots;
  - recall the cubes of 1, 2, 3, 4, 5 and 10;
- Use index notation for squares and cubes;
- Recognise powers of 2, 3, 4, 5;
- Evaluate expressions involving squares, cubes and roots:
  - add, subtract, multiply and divide numbers in index form;
  - cancel to simplify a calculation;
- Use index notation for powers of 10, including negative powers;
- Use the laws of indices to multiply and divide numbers written in index notation;
- Use brackets and the hierarchy of operations with powers inside the brackets, or raising brackets to powers;
- Use calculators for all calculations: positive and negative numbers, brackets, square, cube, powers and roots, and all four operations.

POSSIBLE SUCCESS CRITERIA

What is the value of 2^3?

Evaluate $(2^3 \times 2^5) \div 2^4$.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Problems such as: What two digit number is special because adding the sum of its digits to the product of its digits gives me my original number?

COMMON MISCONCEPTIONS

The order of operations is often not applied correctly when squaring negative numbers, and many calculators will reinforce this misconception.

10^3, for example, is interpreted as 10×3.

NOTES

Pupils need to know how to enter negative numbers into their calculator.

Use the language of ‘negative’ number and not minus number to avoid confusion with calculations.

Note that the students need to understand the term ‘surd’ as there will be occasions when their calculator displays an answer in surd form, for example, $4\sqrt{2}$.

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1d. Factors, multiples and primes — Teaching time

(N4, N5) — 3-5 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- List all three-digit numbers that can be made from three given integers;
- Recognise odd, even and prime (two digit) numbers;
- Identify factors and multiples and list all factors and multiples of a number systematically;
- Find the prime factor decomposition of positive integers and write as a product using index notation;
- Find common factors and common multiples of two numbers;
- Find the LCM and HCF of two numbers, by listing, Venn diagrams and using prime factors: include finding LCM and HCF given the prime factorisation of two numbers;
- Understand that the prime factor decomposition of a positive integer is unique – whichever factor pair you start with – and that every number can be written as a product of two factors;
- Solve simple problems using HCF, LCM and prime numbers.

POSSIBLE SUCCESS CRITERIA

Given the digits 1, 2 and 3, find how many numbers can be made using all the digits.
Convince me that 8 is not prime.
Understand that every number can be written as a unique product of its prime factors.
Recall prime numbers up to 100.
Understand the meaning of prime factor.
Write a number as a product of its prime factors.
Use a Venn diagram to sort information.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Students should be able to provide convincing counter-arguments to statements concerning properties of stated numbers, i.e. Sharon says 108 is a prime number. Is she correct?
Questions that require multiple layers of operations such as:

> Pam writes down one multiple of 9 and two different factors of 40. She then adds together her three numbers. Her answer is greater than 20 but less than 30. Find three numbers that Jan could have written down.

COMMON MISCONCEPTIONS

1 is a prime number.
Particular emphasis should be made on the definition of 'product' as multiplication as many students get confused and think it relates to addition.

NOTES

Use a number square to find primes (Eratosthenes sieve).
Using a calculator to check factors of large numbers can be useful.
Students need to be encouraged to learn squares from 2 × 2 to 15 × 15 and cubes of 2, 3, 4, 5 and 10 and corresponding square and cube roots.

UNIT 2: Expressions, substituting into simple formulae, expanding and factorising

Return to Overview

SPECIFICATION REFERENCES

N1 order positive and negative integers, decimals and fractions; use the symbols =, ≠, <, >, ≤, ≥

N3 recognise and use relationships between operations, including inverse operations (e.g. cancellation to simplify calculations and expressions); use conventional notation for priority of operations, including brackets, powers, roots and reciprocals

A1 use and interpret algebraic notation, including:

- ab in place of $a \times b$
- $3y$ in place of $y + y + y$ and $3 \times y$
- a^2 in place of $a \times a$, a^3 in place of $a \times a \times a$, a^2b in place of $a \times a \times b$
- $\frac{a}{b}$ in place of $a \div b$
- coefficients written as fractions rather than as decimals
- brackets

A2 substitute numerical values into formulae and expressions, including scientific formulae

A3 understand and use the concepts and vocabulary of expressions, equations, formulae, identities, inequalities, terms and factors

A4 simplify and manipulate algebraic expressions … by:

- collecting like terms
- multiplying a single term over a bracket
- taking out common factors …
- simplifying expressions involving sums, products and powers, including the laws of indices

A5 understand and use standard mathematical formulae; rearrange formulae to change the subject

A6 know the difference between an equation and an identity; argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments

A7 where appropriate, interpret simple expressions as functions with inputs and outputs

A21 translate simple situations or procedures into algebraic expressions or formulae; derive an equation, solve the equation and interpret the solution

PRIOR KNOWLEDGE

Students should have prior knowledge of some of these topics, as they are encountered at Key Stage 3:

- the ability to use negative numbers with the four operations and recall and use hierarchy of operations and understand inverse operations;
- dealing with decimals and negatives on a calculator;
- using index laws numerically.

KEYWORDS

Expression, identity, equation, formula, substitute, term, 'like' terms, index, power, collect, substitute, expand, bracket, factor, factorise, linear, simplify

2a. Algebra: the basics
(N1, N3, A1, A3, A4)

Teaching time
5-7 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Use notation and symbols correctly;
- Write an expression;
- Select an expression/equation/formula/identity from a list;
- Manipulate and simplify algebraic expressions by collecting 'like' terms;
- Multiply together two simple algebraic expressions, e.g. $2a \times 3b$;
- Simplify expressions by cancelling, e.g. $\frac{4x}{2} = 2x$;
- Use index notation and the index laws when multiplying or dividing algebraic terms;
- Understand the ≠ symbol and introduce the identity ≡ sign;

POSSIBLE SUCCESS CRITERIA

Simplify $4p - 2q + 3p + 5q$.
Simplify $z^4 \times z^3$, $y^3 \div y^2$, $(a^7)^2$.
Simplify $x^{-4} \times x^2$, $w^2 \div w^{-1}$.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Forming expressions and equations using area and perimeter of 2D shapes.

COMMON MISCONCEPTIONS

Any poor number skills involving negatives and times tables will become evident.

NOTES

Some of this will be a reminder from Key Stage 3.
Emphasise correct use of symbolic notation, i.e. $3 \times y = 3y$ and not $y3$ and $a \times b = ab$.
Use lots of concrete examples when writing expressions, e.g. 'B' boys + 'G' girls.
Plenty of practice should be given and reinforce the message that making mistakes with negatives and times tables is a different skill to that being developed.

2b. Expressions and substitution into formula (A2, A4, A5, A6, A7, A21)

Teaching time
4-6 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Multiply a single number term over a bracket;
- Write and simplify expressions using squares and cubes;
- Simplify expressions involving brackets, i.e. expand the brackets, then add/subtract;
- Argue mathematically to show algebraic expressions are equivalent;
- Recognise factors of algebraic terms involving single brackets;
- Factorise algebraic expressions by taking out common factors;
- Write expressions to solve problems representing a situation;
- Substitute numbers into simple algebraic expressions;
- Substitute numbers into expressions involving brackets and powers;
- Substitute positive and negative numbers into expressions;
- Derive a simple formula, including those with squares, cubes and roots;
- Substitute numbers into a (word) formula;

POSSIBLE SUCCESS CRITERIA

Expand and simplify $3(t - 1)$.

Understand $6x + 4 \neq 3(x + 2)$.

Argue mathematically that $2(x + 5) = 2x + 10$.

Evaluate the expressions for different values of x: $3x^2 + 4$ or $2x^3$.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Forming and solving equations involving algebra and other areas of mathematics such as area and perimeter.

COMMON MISCONCEPTIONS

$3(x + 4) = 3x + 4$.

The convention of not writing a coefficient with a single value, i.e. x instead of $1x$, may cause confusion.

Some students may think that it is always true that $a = 1$, $b = 2$, $c = 3$.

If $a = 2$ sometimes students interpret $3a$ as 32.

Making mistakes with negatives, including the squaring of negative numbers.

NOTES

Provide students with lots of practice.

This topic lends itself to regular reinforcement through starters in lessons.

Use formulae from mathematics and other subjects, expressed initially in words and then using letters and symbols.

Include substitution into the kinematics formulae given on the formula sheet, i.e. $v = u + at$, $v^2 - u^2 = 2as$, and $s = ut + \frac{1}{2}at^2$.

UNIT 3: Drawing and interpreting graphs, tables and charts

Return to Overview

SPECIFICATION REFERENCES

G2 use the standard ruler and compass constructions (perpendicular bisector of a line segment, constructing a perpendicular to a given line from/at a given point, bisecting a given angle); use these to construct given figures and solve loci problems; know that the perpendicular distance from a point to a line is the shortest distance to the line

G14 use standard units of measure and related concepts (length, area, volume/capacity, mass, time, money, etc.)

G15 measure line segments and angles in geometric figures …

S2 interpret and construct tables, charts and diagrams, including frequency tables, bar charts, pie charts and pictograms for categorical data, vertical line charts for ungrouped discrete numerical data, tables and line graphs for time series data and know their appropriate use

S4 interpret, analyse and compare the distributions of data sets from univariate empirical distributions through:

- appropriate graphical representation involving discrete, continuous and grouped data
- appropriate measures of central tendency (… mode and modal class) and spread (range, including consideration of outliers)

S5 apply statistics to describe a population

S6 use and interpret scatter graphs of bivariate data; recognise correlation and know that it does not indicate causation; draw estimated lines of best fit; make predictions; interpolate and extrapolate apparent trends whilst knowing the dangers of so doing

PRIOR KNOWLEDGE

Students should be able to read scales on graphs, draw circles, measure angles and plot coordinates in the first quadrant, and know that there are 360 degrees in a full turn and 180 degrees at a point on a straight line.
Students should have experience of tally charts.
Students will have used inequality notation.
Students must be able to find the midpoint of two numbers.
Students should be able to use the correct notation for time using 12- and 24-hour clocks.

KEYWORDS

Mean, median, mode, range, average, discrete, continuous, qualitative, quantitative, data, scatter graph, line of best fit, correlation, positive, negative, sample, population, stem and leaf, frequency, table, sort, pie chart, estimate

3a. Tables, charts and graphs
(G14, S2, S4, S5)

Teaching time
10-12 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Use suitable data collection techniques (data to be integer and decimal values);
- Design and use data-collection sheets for grouped, discrete and continuous data, use inequalities for grouped data, and introduce ≤ and ≥ signs; Sort, classify and tabulate data, both discrete and continuous quantitative data, and qualitative data; Extract data from lists and tables;
- Use correct notation for time, 12- and 24-hour clock and work out time taken for a journey from a timetable;
- Construct tables for time–series data;
- Design, complete and use two-way tables for discrete and grouped data;
- Calculate the total frequency from a frequency table;
- Read off frequency values from a table;
- Read off frequency values from a frequency table;
- Find greatest and least values from a frequency table;
- Identify the mode from a frequency table;
- Identify the modal class from a grouped frequency table;
- Plotting coordinates in first quadrant and read graph scales in multiples;
- Produce and interpret:
    - pictograms;
    - composite bar charts;
    - dual/comparative bar charts for categorical and ungrouped discrete data;
    - bar-line charts;
    - vertical line charts;
    - line graphs;
    - line graphs for time–series data;
    - histograms with equal class intervals;
    - stem and leaf (including back-to-back);
- Calculate total population from a bar chart or table;
- Find greatest and least values from a bar chart or table;
- Find the mode from a stem and leaf diagram;
- Identify the mode from a bar chart;
- Recognise simple patterns, characteristic and relationships in bar charts and line graphs;
- Interpret and discuss any data.

POSSIBLE SUCCESS CRITERIA

Construct a frequency table for a continuous data set, deciding on appropriate intervals using inequalities
Plan a journey using timetables.
Decide the most appropriate chart or table given a data set.
State the mode, smallest value or largest value from a stem and leaf diagram.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Misleading graphs, charts or tables can provide an opportunity for students to critically evaluate the way information is presented.
Students should be able to decide what the scales on any axis should be to be able to present information.

COMMON MISCONCEPTIONS

Students struggle to make the link between what the data in a frequency table represents, so for example may state the 'frequency' rather than the interval when asked for the modal group.

NOTES

Other averages are covered in unit 5, but you may choose to cover them in this unit.
Ensure that students are given the opportunity to draw and complete two-way tables from words.
Ensure that you include a variety of scales, including decimal numbers of millions and thousands, time scales in hours, minutes, seconds.
Misleading graphs are a useful life skill.

3b. Pie charts (G2, G15, S2, S4)	Teaching time 2-4 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Interpret tables; represent data in tables and charts;
- Know which charts to use for different types of data sets;
- Draw circles and arcs to a given radius;
- Know there are 360 degrees in a full turn, 180 degrees in a half turn, and 90 degrees in a quarter turn;
- Measure and draw angles, to the nearest degree; Construct pie charts for categorical data and discrete/continuous numerical data;
- Interpret simple pie charts using simple fractions and percentages; $\frac{1}{2}$, $\frac{1}{4}$ and multiples of 10% sections;
- From a pie chart:
  - find the mode;
  - find the total frequency;
- Understand that the frequency represented by corresponding sectors in two pie charts is dependent upon the total populations represented by each of the pie charts.

POSSIBLE SUCCESS CRITERIA

From a simple pie chart identify the frequency represented by $\frac{1}{4}$ and $\frac{1}{2}$ sections.
From a simple pie chart identify the mode.
Find the angle for one item.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

From inspection of a pie chart, students should be able to identify the fraction of the total represented and know when that total can be calculated and compared with another pie chart.

COMMON MISCONCEPTIONS

Same size sectors for different sized data sets represent the same number rather than the same proportion.

NOTES

Relate $\frac{1}{4}$, $\frac{1}{2}$, etc to percentages.
Practise dividing by 20, 30, 40, 60, etc.
Compare pie charts to identify similarities and differences.
Angles when drawing pie charts should be accurate to 2°.

3c. Scatter graphs (S4, S6)

Teaching time
3-5 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Draw scatter graphs;
- Interpret points on a scatter graph;
- Identify outliers and ignore them on scatter graphs;
- Draw the line of best fit on a scatter diagram by eye, and understand what it represents;
- Use the line of best fit make predictions; interpolate and extrapolate apparent trends whilst knowing the dangers of so doing;
- Distinguish between positive, negative and no correlation using lines of best fit;
- Use a line of best fit to predict values of a variable given values of the other variable;
- Interpret scatter graphs in terms of the relationship between two variables;
- Interpret correlation in terms of the problem;
- Understand that correlation does not imply causality;
- State how reliable their predictions are, i.e. not reliable if extrapolated.

POSSIBLE SUCCESS CRITERIA

Justify an estimate they have made using a line of best fit.
Identify outliers and explain why they may occur.
Given two sets of data in a table, model the relationship and make predictions.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Many real-life situations that give rise to two variables provide opportunities for students to extrapolate and interpret the resulting relationship (if any) between the variables.

COMMON MISCONCEPTIONS

Lines of best fit are often forgotten, but correct answers still obtained by sight.
Interpreting scales of different measurements and confusion between x and y axes when plotting points.

NOTES

Students need to be constantly reminded of the importance of drawing a line of best fit.
Support with copy and complete statements, e.g. as the ___ increases, the ___ decreases.
Statistically the line of best fit should pass through the coordinate representing the mean of the data.
Students should label the axes clearly, and use a ruler for all straight lines and a pencil for all drawing.
Remind students that the line of best fit does not necessarily go through the origin of the graph.

UNIT 4: Fractions and percentages

Return to Overview

SPECIFICATION REFERENCES

N1 order positive and negative integers, decimals and fractions; use the symbols =, ≠, <, >, ≤, ≥

N2 apply the four operations, including formal written methods, to integers, decimals and simple fractions (proper and improper), and mixed numbers – all both positive and negative; understand and use place value (e.g. when working with very large or very small numbers, and when calculating with decimals)

N3 recognise and use relationships between operations, including inverse operations (e.g. cancellation to simplify calculations and expressions); use conventional notation for priority of operations, including brackets, powers, roots and reciprocals

N8 calculate exactly with fractions …

N10 work interchangeably with terminating decimals and their corresponding fractions (such as 3.5 and $\frac{7}{2}$ or 0.375 and $\frac{3}{8}$)

N12 interpret fractions and percentages as operators

R3 express one quantity as a fraction of another, where the fraction is less than 1 or greater than 1

R9 define percentage as 'number of parts per hundred'; interpret percentages and percentage changes as a fraction or a decimal, and interpret these multiplicatively; express one quantity as a percentage of another; compare two quantities using percentages; work with percentages greater than 100%; solve problems involving percentage change, including percentage increase/decrease, and original value problems and simple interest including in financial mathematics

S2 interpret and construct tables, charts and diagrams, including frequency tables, bar charts, pie charts and pictograms for categorical data, vertical line charts for ungrouped discrete numerical data, tables and line graphs for time series data and know their appropriate use

PRIOR KNOWLEDGE

Students should be able to use the four operations of number.
Students should be able to find common factors.
Students have a basic understanding of fractions as being 'parts of a whole'.
Students should be able to define percentage as 'number of parts per hundred'.
Students should know number complements to 10 and multiplication tables.

KEYWORDS

Decimal, percentage, inverse, addition, subtraction, multiplication, division, fractions, mixed, improper, recurring, integer, decimal, terminating, percentage, VAT, increase, decrease, multiplier, profit, loss

4a. Fractions, decimals and percentages (N1, N2, N3, N8, N10, N12, R3, R9 S2)

Teaching time
6–8 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Use diagrams to find equivalent fractions or compare fractions;
- Write fractions to describe shaded parts of diagrams;
- Express a given number as a fraction of another, using very simple numbers, some cancelling, and where the fraction is both < 1 and > 1;
- Write a fraction in its simplest form and find equivalent fractions;
- Order fractions, by using a common denominator;
- Compare fractions, use inequality signs, compare unit fractions;
- Convert between mixed numbers and improper fractions;
- Add and subtract fractions;
- Add fractions and write the answer as a mixed number;
- Multiply and divide an integer by a fraction;
- Multiply and divide a fraction by an integer, including finding fractions of quantities or measurements, and apply this by finding the size of each category from a pie chart using fractions;
- Understand and use unit fractions as multiplicative inverses;
- Multiply fractions: simplify calculations by cancelling first;
- Divide a fraction by a whole number and another fraction;
- Recall the fraction-to-decimal conversion and convert fractions to decimals;
- Convert a fraction to a decimal to make a calculation easier,

  e.g. $0.25 \times 8 = \frac{1}{4} \times 8$, or $\frac{3}{8} \times 10 = 0.375 \times 10$;
- Recognise recurring decimals and convert fractions such as $\frac{3}{7}$, $\frac{1}{3}$ and $\frac{2}{3}$ into recurring decimals;
- Compare and order fractions, decimals and integers, using inequality signs;
- Understand that a percentage is a fraction in hundredths;
- Express a given number as a percentage of another number;
- Convert between fractions, decimals and percentages;
- Order fractions, decimals and percentages, including use of inequality signs.

POSSIBLE SUCCESS CRITERIA

Express a given number as a fraction of another, including where the fraction > 1.

Simplify $\frac{120}{100}$.

$\frac{3}{5} \times 15$, $20 \times \frac{3}{4}$.

$\frac{1}{2}$ of 36 m, $\frac{1}{4}$ of £20.

Find the size of each category from a pie chart using fractions.

Calculate: $\frac{1}{2} \times \frac{6}{7}$, $\frac{3}{5} \div 3$.

Write terminating decimals (up to 3 d.p.) as fractions.

Convert between fractions, decimals and percentages, common ones such as $\frac{1}{2}$, $\frac{1}{10}$, $\frac{1}{4}$, $\frac{3}{4}$ and $\frac{n}{10}$.
Order integers, decimals and fractions.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Questions that involve rates of overtime pay including simple calculations involving fractional (>1, e.g. 1.5) and hourly pay. These can be extended into calculating rates of pay given the final payment and number of hours worked.
Working out the number of people/things where the number of people/things in different categories is given as a fraction, decimal or percentage.

COMMON MISCONCEPTIONS

The larger the denominator the larger the fraction.
Incorrect links between fractions and decimals, such as thinking that $\frac{1}{5}$ = 0.15, 5% = 0.5, 4% = 0.4, etc.
It is not possible to have a percentage greater than 100%.

NOTES

Emphasise the importance of being able to convert between fractions, decimals and percentages to make calculations easier.
When expressing a given number as a fraction of another, start with very simple numbers < 1, and include some cancelling before fractions using numbers > 1.
Students should be reminded of basic percentages and fraction conversions.
When adding and subtracting fractions, start with same denominator, then where one denominator is a multiple of the other (answers ≤ 1), and finally where both denominators have to be changed (answers ≤ 1).
Regular revision of fractions is essential.
Demonstrate how to the use the fraction button on the calculator.
Use real-life examples where possible.
Use long division to illustrate recurring decimals.

4b. Percentages	**Teaching time**
(N12, R9)	5-7 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:
- Express a given number as a percentage of another number;
- Find a percentage of a quantity without a calculator: 50%, 25% and multiples of 10% and 5%;
- Find a percentage of a quantity or measurement (use measurements they should know from Key Stage 3 only);
- Calculate amount of increase/decrease;

- Use percentages to solve problems, including comparisons of two quantities using percentages;
- Percentages over 100%;
- Use percentages in real-life situations, including percentages greater than 100%:
  - Price after VAT (not price before VAT);
  - Value of profit or loss;
  - Simple interest;
  - Income tax calculations;
- Use decimals to find quantities;
- Find a percentage of a quantity, including using a multiplier;
- Use a multiplier to increase or decrease by a percentage in any scenario where percentages are used;
- Understand the multiplicative nature of percentages as operators.

POSSIBLE SUCCESS CRITERIA

What is 10%, 15%, 17.5% of £30?

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Sale prices offer an ideal opportunity for solving problems allowing students the opportunity to investigate the most effective way to work out the "sale" price.
Problems that involve consecutive reductions such as: Sale Prices are 10% off the previous day's price. If a jacket is £90 on Monday, what is the price on Wednesday?

COMMON MISCONCEPTIONS

It is not possible to have a percentage greater than 100%.

NOTES

When finding a percentage of a quantity or measurement, use only measurements they should know from Key Stage 3.
Amounts of money should always be rounded to the nearest penny.
Use real-life examples where possible.
Emphasise the importance of being able to convert between decimals and percentages and the use of decimal multipliers to make calculations easier.

UNIT 5: Equations, inequalities and sequences

Return to Overview

SPECIFICATION REFERENCES

N1 order positive and negative integers, decimals and fractions; use the symbols =, ≠, <, >, ≤, ≥

N15 round numbers and measures to an appropriate degree of accuracy (e.g. to a specified number of decimal places or significant figures); use inequality notation to specify simple error intervals due to truncation or rounding

N16 apply and interpret limits of accuracy

A3 understand and use the concepts and vocabulary of expressions, equations, formulae, identities, inequalities, terms and factors

A5 understand and use standard mathematical formulae; rearrange formulae to change the subject

A7 where appropriate, interpret simple expressions as functions with inputs and outputs

A17 solve linear equations in one unknown algebraically (including those with the unknown on both sides of the equation); find approximate solutions using a graph

A21 translate simple situations or procedures into algebraic expressions or formulae; derive an equation, solve the equation and interpret the solution

A22 solve linear inequalities in one variable; represent the solution set on a number line

A23 generate terms of a sequence from either a term-to-term or a position-to-term rule

A24 recognise and use sequences of triangular, square and cube numbers, simple arithmetic progressions; Fibonacci type sequences and simple geometric progressions (r^n where n is an integer, and r is a rational number > 0)

A25 deduce expressions to calculate the nth term of linear sequences.

PRIOR KNOWLEDGE

Students should be able to use inequality signs between numbers.
Students should be able to use negative numbers with the four operations, recall and use the hierarchy of operations and understand inverse operations.
Students should be able to deal with decimals and negatives on a calculator.
Students should be able to use index laws numerically.
Students should be able to draw a number line.

KEYWORDS

Arithmetic, geometric, function, sequence, nth term, derive, quadratic, triangular, cube, square, odd, even, solve, change, subject, inequality, represent, substitute, bracket, expand, linear, equation, balance, accuracy

5a. Equations and inequalities (N1, N15, N16, A3, A5, A7, A17, A21, A22)

Teaching time 8-10 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Select an expression/equation/formula/identity from a list;
- Write expressions and set up simple equations including forming an equation from a word problem;
- Use function machines;
- Solve simple equations including those:
  - with integer coefficients, in which the unknown appears on either side or on both sides of the equation;
  - which contain brackets, including those that have negative signs occurring anywhere in the equation, and those with a negative solution;
  - with one unknown, with integer or fractional coefficients;
- Rearrange simple equations;
- Substitute into a formula, and solve the resulting equation;
- Find an approximate solution to a linear equation using a graph;
- Solve angle or perimeter problems using algebra.
- Show inequalities on number lines;
- Write down whole number values that satisfy an inequality;
- Solve an inequality such as $-3 < 2x + 1 < 7$ and show the solution set on a number line;
- Solve two inequalities in x, find the solution sets and compare them to see which value of x satisfies both;
- Use the correct notation to show inclusive and exclusive inequalities;
- Construct inequalities to represent a set shown on a number line;
- Solve simple linear inequalities in one variable, and represent the solution set on a number line;
- Round answers to a given degree of accuracy.

POSSIBLE SUCCESS CRITERIA

Solve: $x + 5 = 12$, $x - 6 = 3$, $\frac{x}{2} = 5$, $2x - 5 = 19$, $2x + 5 = 8x - 7$

Given expressions for the angles on a line or in a triangle in terms of a, find the value of a.

Given expressions for the sides of a rectangle and the perimeter, form and solve an equation to find missing values.

Solve $-3 < 2x + 1$ and show the solution set on a number line.

State the whole numbers that satisfy a given inequality.

Recognise that measurements given to the nearest whole unit may be inaccurate by up to one half in either direction.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Problems that:

- could be solved by forming equations such as: Pat and Paul have a combined salary of £800 per week. Pat earns £200 per week more than Paul. How much does Paul earn?
- involve the application of a formula with conflicting results such as: Pat and Paul are using the formula $y = 8n + 4$ When $n = 2$, Pat states that $y = 86$ and Paul states $y = 20$. Who is correct?

COMMON MISCONCEPTIONS

Rules of adding and subtracting negatives.
Inverse operations can be misapplied.
When solving inequalities, students often state their final answer as a number quantity and either exclude the inequality or change it to =.

NOTES

Emphasise good use of notation.
Students need to realise that not all linear equations can be solved by observation or trial and improvement, and hence the use of a formal method is important.
Students can leave their answer in fraction form where appropriate.
Emphasise the importance of leaving their answer as an inequality (and not change to =).

5b. Sequences (A7, A23, A24, A25)	Teaching time 4-6 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Recognise sequences of odd and even numbers, and other sequences including Fibonacci sequences;
- Use function machines to find terms of a sequence;
- Write the term-to-term definition of a sequence in words;
- Find a specific term in the sequence using position-to-term or term-to-term rules;
- Generate arithmetic sequences of numbers, triangular number, square and cube integers and sequences derived from diagrams;
- Recognise such sequences from diagrams and draw the next term in a pattern sequence;
- Find the next term in a sequence, including negative values;
- Find the nth term
    - for a pattern sequence;
    - a linear sequence;
    - of an arithmetic sequence;
- Use the nth term of an arithmetic sequence to
    - generate terms;
    - decide if a given number is a term in the sequence, or find the first term over a certain number;
    - find the first term greater/less than a certain number;
- Continue a geometric progression and find the term-to-term rule, including negatives, fraction and decimal terms;
- Continue a quadratic sequence and use the nth term to generate terms;
- Distinguish between arithmetic and geometric sequences.

POSSIBLE SUCCESS CRITERIA

Given a sequence, 'Which is the 1st term greater than 50?'
What is the amount of money after x months saving the same amount or the height of tree that grows 6 m per year?
What are the next terms in the following sequences?
1, 3, 9, …		100, 50, 25, …		2, 4, 8, 16, …
Write down an expression for the nth term of the arithmetic sequence 2, 5, 8, 11, …
Is 67 a term in the sequence 4, 7, 10, 13, …?

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Evaluating statements about whether or not specific numbers or patterns are in a sequence and justifying the reasons.

NOTES

Emphasise use of $3n$ meaning $3 \times n$.
Students need to be clear on the description of the pattern in words, the difference between the terms and the algebraic description of the nth term.
Students are not expected to find the nth term of a quadratic sequence.

UNIT 6: Angles, polygons and parallel lines

Return to Overview

SPECIFICATION REFERENCES

G1 use conventional terms and notation: points, lines, vertices, edges, planes, parallel lines, perpendicular lines, right angles, polygons, regular polygons and polygons with reflection and/or rotation symmetries; use the standard conventions for labelling and referring to the sides and angles of triangles; draw diagrams from written description

G3 apply the properties of angles at a point, angles at a point on a straight line, vertically opposite angles; understand and use alternate and corresponding angles on parallel lines; derive and use the sum of angles in a triangle (e.g. to deduce and use the angle sum in any polygon, and to derive properties of regular polygons)

G4 derive and apply the properties and definitions of special types of quadrilaterals, including square, rectangle, parallelogram, trapezium, kite and rhombus; and triangles and other plane figures using appropriate language

G5 use the basic congruence criteria for triangles (SSS, SAS, ASA, RHS)

G6 apply angle facts, triangle congruence, similarity and properties of quadrilaterals to conjecture and derive results about angles and sides, including … the fact that the base angles of an isosceles triangle are equal, and use known results to obtain simple proofs

G11 solve geometrical problems on coordinate axes

PRIOR KNOWLEDGE

Students should be able to use a ruler and protractor.
Students should have an understanding of angles as a measure of turning.
Students should be able to name angles and distinguish between acute, obtuse, reflex and right angles.
Students should recognise reflection symmetry, be able to identify and draw lines of symmetry, and complete diagrams with given number of lines of symmetry.
Students should recognise rotation symmetry and be able to identify orders of rotational symmetry, and complete diagrams with given order of rotational symmetry.

KEYWORDS

Quadrilateral, angle, polygon, interior, exterior, proof, tessellation, rotational symmetry, parallel, corresponding, alternate, co-interior, vertices, edge, face, sides, triangle, perpendicular, isosceles, scalene, clockwise, anticlockwise, hexagons, heptagons, octagons, decagons, obtuse, acute, reflex, quadrilateral, triangle, regular, irregular, two-dimensional, three-dimensional, measure, line, angle, order, intersecting

6a. Properties of shapes, parallel lines and angle facts (G1, G3, G4, G6, G11)

Teaching time
6-8 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Estimate sizes of angles;
- Measure angles using a protractor;
- Use geometric language appropriately;
- Use letters to identify points, lines and angles;
- Use two-letter notation for a line and three-letter notation for an angle;
- Describe angles as turns and in degrees and understand clockwise and anticlockwise;
- Know that there are 360° in a full turn, 180° in a half turn and 90° in a quarter turn;
- Identify a line perpendicular to a given line on a diagram and use their properties;
- Identify parallel lines on a diagram and use their properties;
- Find missing angles using properties of corresponding and alternate angles;
- Understand and use the angle properties of parallel lines.
- Recall the properties and definitions of special types of quadrilaterals, including symmetry properties;
- List the properties of each special type of quadrilateral, or identify (name) a given shape;
- Draw sketches of shapes;
- Classify quadrilaterals by their geometric properties and name all quadrilaterals that have a specific property;
- Identify quadrilaterals from everyday usage;
- Given some information about a shape on coordinate axes, complete the shape; Understand and use the angle properties of quadrilaterals;
- Use the fact that angle sum of a quadrilateral is 360°;
- Recall and use properties of angles at a point, angles at a point on a straight line, right angles, and vertically opposite angles;
- Distinguish between scalene, equilateral, isosceles and right-angled triangles;
- Derive and use the sum of angles in a triangle;
- Find a missing angle in a triangle, using the angle sum of a triangle is 180°;
- Understand and use the angle properties of triangles, use the symmetry property of isosceles triangle to show that base angles are equal;
- Use the side/angle properties of isosceles and equilateral triangles;
- Understand and use the angle properties of intersecting lines;
- Understand a proof that the exterior angle of a triangle is equal to the sum of the interior angles at the other two vertices; Use geometrical language appropriately, give reasons for angle calculations and show step-by-step deduction when solving problems.

POSSIBLE SUCCESS CRITERIA

Name all quadrilaterals that have a specific property.
Use geometric reasoning to answer problems giving detailed reasons.
Find the size of missing angles at a point or at a point on a straight line.
Convince me that a parallelogram is a rhombus.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Multi-step "angle chasing" style problems that involve justifying how students have found a specific angle.
Geometrical problems involving algebra whereby equations can be formed and solved allow students the opportunity to make and use connections with different parts of mathematics.
What is the same, and what is different between families of polygons?

COMMON MISCONCEPTIONS

Pupils may believe, incorrectly, that perpendicular lines have to be horizontal/vertical or all triangles have rotational symmetry of order 3.
Some students will think that all trapezia are isosceles, or a square is only square if 'horizontal', or a 'non-horizontal' square is called a diamond.
Some students may think that the equal angles in an isosceles triangle are the 'base angles'.
Incorrectly identifying the 'base angles' (i.e. the equal angles) of an isosceles triangle when not drawn horizontally.

NOTES

Emphasise that diagrams in examinations are seldom drawn accurately.
Make sure drawings are neat, labelled and accurate.
Give students lots of practice.
Angles should be accurate to within 2°.
Investigate Rangoli patterns.
Use tracing paper to assist with symmetry questions.
Ask students to find their own examples of symmetry in real life.
Emphasise that diagrams in examinations are seldom drawn accurately.
Make sure drawings are neat, labelled and accurate.
Students should have plenty of practice drawing examples to illustrate the properties and encourage them to check their drawings.
Emphasise the need to give geometric reasons when required.

6b. Interior and exterior angles of polygons (G1, G3, G5, G11)

Teaching time
3-5 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Recognise and name pentagons, hexagons, heptagons, octagons and decagons;
- Understand 'regular' and 'irregular' as applied to polygons;
- Use the sum of angles of irregular polygons;
- Calculate and use the sums of the interior angles of polygons;
- Calculate and use the angles of regular polygons;
- Use the sum of the interior angles of an n-sided polygon;
- Use the sum of the exterior angles of any polygon is 360°;
- Use the sum of the interior angle and the exterior angle is 180°;
- Identify shapes which are congruent (by eye);
- Explain why some polygons fit together and others do not;

POSSIBLE SUCCESS CRITERIA

Deduce and use the angle sum in any polygon.
Derive the angle properties of regular polygons.
Given the size of its exterior angle, how many sides does the polygon have?

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Problems whereby students have to justify the number of sides that a regular polygon has given an interior or exterior angle.

COMMON MISCONCEPTIONS

Pupils may believe, incorrectly, that all polygons are regular.

NOTES

Study Escher drawings.
Use examples of tiling patterns with simple shapes to help students investigate if shapes 'fit together'.

UNIT 7: Statistics, sampling and the averages

Teaching time
6-8 hours

Return to Overview

SPECIFICATION REFERENCES

S1 infer properties of populations or distributions from a sample, while knowing the limitations of sampling

S2 interpret and construct tables, charts and diagrams, including frequency tables, bar charts, pie charts and pictograms for categorical data, vertical line charts for ungrouped discrete numerical data, tables and line graphs for time–series data and know their appropriate use

S4 interpret, analyse and compare the distributions of data sets from univariate empirical distributions through: …

- appropriate measures of central tendency (median, mean, mode and modal class) and spread (range, including consideration of outliers)

PRIOR KNOWLEDGE

Students should be able to calculate the midpoint of two numbers.
Students will have drawn the statistical diagrams in unit 3.
Students will have used inequality notation.

KEYWORDS

Mean, median, mode, range, average, discrete, continuous, qualitative, quantitative, data, sample, population, stem and leaf, frequency, table, sort, pie chart, estimate, primary, secondary, interval, midpoint, survey

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Specify the problem and:
  - plan an investigation;
  - decide what data to collect and what statistical analysis is needed;
  - consider fairness;
- Recognise types of data: primary secondary, quantitative and qualitative;
- Identify which primary data they need to collect and in what format, including grouped data;
- Collect data from a variety of suitable primary and secondary sources;
- Understand how sources of data may be biased and explain why a sample may not be representative of a whole population;
- Understand sample and population.
- Calculate the mean, mode, median and range for discrete data;
- Interpret and find a range of averages as follows:
  - median, mean and range from a (discrete) frequency table;
  - range, modal class, interval containing the median, and estimate of the mean from a grouped data frequency table;
  - mode and range from a bar chart;
  - median, mode and range from stem and leaf diagrams;
  - mean from a bar chart;

- Understand that the expression 'estimate' will be used where appropriate, when finding the mean of grouped data using mid-interval values;
- Compare the mean, median, mode and range (as appropriate) of two distributions using bar charts, dual bar charts, pictograms and back-to-back stem and leaf;
- Recognise the advantages and disadvantages between measures of average.

POSSIBLE SUCCESS CRITERIA

Explain why a sample may not be representative of a whole population.
Carry out a statistical investigation of their own and justify how sources of bias have been eliminated.
Show me an example of a situation in which biased data would result.
State the median, mode, mean and range from a small data set.
Extract the averages from a stem and leaf diagram.
Estimate the mean from a table.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

When using a sample of a population to solve contextual problem, students should be able to justify why the sample may not be representative of the whole population.
Students should be able to provide a correct solution as a counter-argument to statements involving the "averages", e.g. Susan states that the median is 15, she is wrong. Explain why.
Given the mean, median and mode of five positive whole numbers, can you find the numbers?

COMMON MISCONCEPTIONS

The concept of an unbiased sample is difficult for some students to understand.
Often the $\sum(m \times f)$ is divided by the number of classes rather than $\sum f$ when estimating the mean.

NOTES

Emphasise the difference between primary and secondary sources and remind students about the different between discrete and continuous data.
Discuss sample size and mention that a census is the whole population (the UK census takes place every 10 years in a year ending with a 1 – the next one is due in 2021).
Specify the problem and planning for data collection is not included in the programme of study but is a perquisite to understand the context of the topic.
Writing a questionnaire is not part of the new specification, but is a good topic to demonstrate bias and ways to reduce bias in terms of timing, location and question types that can introduce bias.
Encourage students to cross out the midpoints of each group once they have used these numbers to in $m \times f$. This helps students to avoid summing m instead of f.
Remind students how to find the midpoint of two numbers.
Emphasise that continuous data is measured, i.e. length, weight, and discrete data can be counted, i.e. number of shoes.
When comparing the mean and range of two distributions support with 'copy and complete' sentences, or suggested wording.

SPECIFICATION REFERENCES

N14 estimate answers; check calculations using approximation and estimation, including answers obtained using technology
R1 change freely between related standard units (e.g. time, length, area, volume/capacity, mass) and compound units (e.g. speed, rates of pay, prices, density, pressure) in numerical and algebraic contexts
G11 solve geometrical problems on coordinate axes
G12 identify properties of the faces, surfaces, edges and vertices of: cubes, cuboids, prisms, cylinders, pyramids, cones and spheres
G14 use standard units of measure and related concepts (length, area, volume/capacity, mass, time, money, etc.)
G16 know and apply formulae to calculate: area of triangles, parallelograms, trapezia; volume of cuboids and other right prisms (including cylinders)
G17 … calculate: perimeters of 2D shapes, including … composite shapes

PRIOR KNOWLEDGE

Students should be able to measure lines and recall the names of 2D shapes.
Students should be able to use strategies for multiplying and dividing by powers of 10.
Students should be able to find areas by counting squares and volumes by counting cubes.
Students should be able to interpret scales on a range of measuring instruments.

KEYWORDS

Triangle, rectangle, parallelogram, trapezium, area, perimeter, formula, length, width, prism, compound, measurement, polygon, cuboid, volume, symmetry, vertices, edge, face, units, conversion

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Indicate given values on a scale, including decimal value;
- Know that measurements using real numbers depend upon the choice of unit;
- Convert between units of measure within one system, including time and metric units to metric units of length, area and volume and capacity e.g. 1ml = 1cm^3;
- Make sensible estimates of a range of measures in everyday settings;
- Measure shapes to find perimeters and areas using a range of scales;
- Find the perimeter of
    - rectangles and triangles;
    - parallelograms and trapezia;
    - compound shapes;
- Recall and use the formulae for the area of a triangle and rectangle;
- Find the area of a trapezium and recall the formula;
- Find the area of a parallelogram;
- Calculate areas and perimeters of compound shapes made from triangles and rectangles;
- Estimate surface areas by rounding measurements to 1 significant figure;
- Find the surface area of a prism;

- Find surface area using rectangles and triangles;
- Identify and name common solids: cube, cuboid, cylinder, prism, pyramid, sphere and cone;
- Sketch nets of cuboids and prisms;
- Recall and use the formula for the volume of a cuboid;
- Find the volume of a prism, including a triangular prism, cube and cuboid;
- Calculate volumes of right prisms and shapes made from cubes and cuboids;
- Estimate volumes etc by rounding measurements to 1 significant figure;

POSSIBLE SUCCESS CRITERIA

Find the area/perimeter of a given shape, stating the correct units.
Justify whether a certain number of small boxes fit inside a larger box.
Calculate the volume of a triangular prism with correct units.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Given two 2D that shapes have equal areas, work out all the dimensions of the sides of the shapes.
Problems involving straight-forward and compound shapes in a real-life context should be explored to reinforce the concept of area. For example, the floor plan of a garden linked to the purchase of grass seed.

COMMON MISCONCEPTIONS

Shapes involving missing lengths of sides often result in incorrect answers.
Students often confuse perimeter and area.
Volume often gets confused with surface area.

NOTES

Use questions that involve different metric measures that need converting.
Measurement is essentially a practical activity: use a range of everyday shapes to bring reality to lessons.
Ensure that students are clear about the difference between perimeter and area.
Practical examples help to clarify the concepts, i.e. floor tiles, skirting board, etc.
Discuss the correct use of units.
Drawings should be done in pencil.
Consider 'how many small boxes fit in a larger box'-type questions.
Practical examples should be used to enable students to understand the difference between perimeter, area and volume.

UNIT 9: Real-life and algebraic linear graphs

Return to Overview

SPECIFICATION REFERENCES

N13 use standard units of mass, length, time, money and other measures (including standard compound measures) using decimal quantities where appropriate
A7 where appropriate, interpret simple expressions as functions with inputs and outputs
A8 work with coordinates in all four quadrants
A9 plot graphs of equations that correspond to straight-line graphs in the coordinate plane; …
A10 identify and interpret gradients and intercepts of linear functions graphically and algebraically
A12 Recognise, sketch and interpret graphs of linear functions …
A14 plot and interpret … graphs of non-standard functions in real contexts, to find approximate solutions to problems such as simple kinematic problems involving distance, speed and acceleration
A17 solve linear equations in one unknown algebraically (including those with the unknown on both sides of the equation); find approximate solutions using a graph
R1 change freely between related standard units (e.g. time, length, area, volume/capacity, mass) and compound units (e.g. speed, rates of pay, prices, density, pressure) in numerical and algebraic contexts
R11 use compound units such as speed, … unit pricing, …
R14 interpret the gradient of a straight line graph as a rate of change; recognise and interpret graphs that illustrate direct and inverse proportion
G11 solve geometrical problems on coordinate axes
G14 use standard units of measure and related concepts (length, area, volume/capacity, mass, time, money, etc.)

PRIOR KNOWLEDGE

Students should be able to plot coordinates and read scales
Students should be able to substitute into a formula.

KEYWORDS

Linear, graph, distance, time, coordinate, quadrant, real-life graph, gradient, intercept, function, solution, parallel

9a. Real-life graphs (N13, A7, A8, A9, A10, A14, R1, R11, R14, G11, G14)

Teaching time
7-9 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Use input/output diagrams;
- Draw, label and scale axes;
- Use axes and coordinates to specify points in all four quadrants in 2D;
- Identify points with given coordinates and coordinates of a given point in all four quadrants;
- Find the coordinates of points identified by geometrical information in 2D (all four quadrants);
- Find the coordinates of the midpoint of a line segment; Read values from straight-line graphs for real-life situations;
- Draw straight line graphs for real-life situations, including ready reckoner graphs, conversion graphs, fuel bills graphs, fixed charge and cost per unit;
- Draw distance–time graphs and velocity–time graphs;
- Work out time intervals for graph scales;
- Interpret distance–time graphs, and calculate: the speed of individual sections, total distance and total time;
- Interpret information presented in a range of linear and non-linear graphs;
- Interpret graphs with negative values on axes;
- Interpret gradient as the rate of change in distance–time and speed–time graphs, graphs of containers filling and emptying, and unit price graphs.

POSSIBLE SUCCESS CRITERIA

Interpret a description of a journey into a distance–time or speed–time graph.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Students should be able to decide what the scales on any axis should be to be able to draw a correct graph.
Conversion graphs can be used to provide opportunities for students to justify which distance is further, or whether or not certain items can be purchase in different currencies.

COMMON MISCONCEPTIONS

With distance–time graphs, students struggle to understand that the perpendicular distance from the x-axis represents distance.

NOTES

Clear presentation of axes is important.
Ensure that you include questions that include axes with negative values to represent, for example, time before present time, temperature or depth below sea level.
Careful annotation should be encouraged: it is good practice to get the students to check that they understand the increments on the axes.
Use standard units of measurement to draw conversion graphs.
Use various measures in distance–time and velocity–time graphs, including miles, kilometres, seconds, and hours.

9b. Straight-line graphs (A7, A9, A10, A12, A17)	**Teaching time** 5–7 hours

OBJECTIVES

- By the end of the sub-unit, students should be able to:
- Use function machines to find coordinates (i.e. given the input x, find the output y);
- Plot and draw graphs of $y = a$, $x = a$, $y = x$ and $y = -x$;
- Recognise straight-line graphs parallel to the axes;
- Recognise that equations of the form $y = mx + c$ correspond to straight-line graphs in the coordinate plane;
- Plot and draw graphs of straight lines of the form $y = mx + c$ using a table of values;
- Sketch a graph of a linear function, using the gradient and y-intercept;
- Identify and interpret gradient from an equation $y = mx + c$;
- Identify parallel lines from their equations;
- Plot and draw graphs of straight lines in the form $ax + by = c$;
- Find the equation of a straight line from a graph;
- Find the equation of the line through one point with a given gradient;
- Find approximate solutions to a linear equation from a graph;
- Find the gradient of a straight line from real-life graphs too.

POSSIBLE SUCCESS CRITERIA

Plot and draw the graph for $y = 2x - 4$.
Which of these lines are parallel: $y = 2x + 3$, $y = 5x + 3$, $y = 2x - 9$, $2y = 4x - 8$

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Students should be able to decide what the scales on any axis should in order to draw a correct graph.

COMMON MISCONCEPTIONS

When not given a table of values, students rarely see the relationship between the coordinate axes.

NOTES

Emphasise the importance of drawing a table of values when not given one.
Values for a table should be taken from the x-axis.

UNIT 10: Transformations

Teaching Time
10-12 hours

Return to Overview

SPECIFICATION REFERENCES

R6 express a multiplicative relationship between two quantities as a ratio or a fraction

G1 use conventional terms and notations: points, lines, vertices, edges, planes, parallel lines, perpendicular lines, right angles, polygons, regular polygons and polygons with reflection and/or rotation symmetries; …

G7 identify, describe and construct congruent and similar shapes, including on coordinate axes, by considering rotation, reflection, translation and enlargement (including fractional scale factors)

G24 describe translations as 2D vectors

PRIOR KNOWLEDGE

Students should recall basic shapes.
Students should be able to plot points in all four quadrants.
Students should have an understanding of the concept of rotation.
Students should be able to draw and recognise lines parallel to axes and $y = x$, $y = -x$.
Students will have encountered the terms clockwise and anticlockwise previously.

KEYWORDS

Transformation, rotation, reflection, enlargement, translation, single, combination, scale factor, mirror line, centre of rotation, centre of enlargement, column vector, vector, similarity, congruent, angle, direction, coordinate, describe

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Identify congruent shapes by eye;
- Understand that rotations are specified by a centre, an angle and a direction of rotation;
- Find the centre of rotation, angle and direction of rotation and describe rotations fully using the angle, direction of turn, and centre;
- Rotate and draw the position of a shape after rotation about the origin or any other point including rotations on a coordinate grid;
- Identify correct rotations from a choice of diagrams;
- Understand that translations are specified by a distance and direction using a vector;
- Translate a given shape by a vector;
- Use column vectors to describe and transform 2D shapes using single translations on a coordinate grid;
- Understand that distances and angles are preserved under rotations and translations, so that any figure is congruent under either of these transformations;
- Understand that reflections are specified by a mirror line;
- Identify correct reflections from a choice of diagrams;
- Identify the equation of a line of symmetry;
- Transform 2D shapes using single reflections (including those not on coordinate grids) with vertical, horizontal and diagonal mirror lines;
- Describe reflections on a coordinate grid;
- Scale a shape on a grid (without a centre specified);

- Understand that an enlargement is specified by a centre and a scale factor;
- Enlarge a given shape using (0, 0) as the centre of enlargement, and enlarge shapes with a centre other than (0, 0);
- Find the centre of enlargement by drawing;
- Describe and transform 2D shapes using enlargements by:
  - a positive integer scale factor;
  - a fractional scale factor;
- Identify the scale factor of an enlargement of a shape as the ratio of the lengths of two corresponding sides, simple integer scale factors, or simple fractions;
- Understand that distances and angles are preserved under reflections, so that any figure is congruent under this transformation;
- Understand that similar shapes are enlargements of each other and angles are preserved – define similar in this unit;
- Describe and transform 2D shapes using combined rotations, reflections, translations, or enlargements.

POSSIBLE SUCCESS CRITERIA

Understand that translations are specified by a distance and direction (using a vector).
Describe and transform a given shape by either a rotation or a translation.
Describe and transform a given shape by a reflection.
Convince me the scale factor is, for example, 2.5.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Students should be given the opportunity to explore the effect of reflecting in two parallel mirror lines and combining transformations.

COMMON MISCONCEPTIONS

The directions on a column vector often get mixed up.
Student need to understand that the 'units of movement' are those on the axes, and care needs to be taken to check the scale.
Correct language must be used: students often use 'turn' rather than 'rotate'.

NOTES

Emphasise the need to describe the transformations fully, and if asked to describe a 'single' transformation they should not include two types.
Include rotations with the centre of rotation inside the shape.
Use trial and error with tracing paper to find the centre of rotation.
It is essential that the students check the increments on the coordinate grid when translating shapes.
Students may need reminding about how to find the equations of straight lines, including those parallel to the axes.
When reflecting shapes, the students must include mirror lines on or through original shapes.
As an extension, consider reflections with the mirror line through the shape and enlargements with the centre of enlargement inside the shape.
NB enlargement using negative scale factors is not included.

UNIT 11: Ratio and Proportion

Return to Overview

SPECIFICATION REFERENCES

N11 identify and work with fractions in ratio problems
N13 use standard units of mass, length, time, money and other measures (including standard compound measures) using decimal quantities where appropriate
R1 change freely between related standard units (e.g. time, length, area, volume/capacity, mass) and compound units (e.g. speed, rates of pay, prices, density, pressure) in numerical and algebraic contexts
R4 use ratio notation, including reduction to simplest form
R5 divide a given quantity into two parts in a given part : part or part : whole ratio; express the division of a quantity into two parts as a ratio; apply ratio to real contexts and problems (such as those involving conversion, comparison, scaling, mixing, concentrations)
R6 express a multiplicative relationship between two quantities as a ratio or a fraction
R7 understand and use proportion as equality of ratios
R8 relate ratios to fractions and to linear functions
R10 solve problems involving direct and inverse proportion, including graphical and algebraic representations
R12 compare lengths, areas and volumes using ratio notation; make links to similarity (including trigonometric ratios) and scale factors
R14 interpret the gradient of a straight line graph as a rate of change; recognise and interpret graphs that illustrate direct and inverse proportion

PRIOR KNOWLEDGE

Students should know the four operations of number.
Students should have a basic understanding of fractions as being 'parts of a whole'.

KEYWORDS

Ratio, proportion, share, parts, fraction, function, direct proportion, inverse proportion, graphical, linear, compare

11a. Ratio (N11, N13, R1, R4, R5, R6, R8, R12)

Teaching time 3-5 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Understand and express the division of a quantity into a of number parts as a ratio;
- Write ratios in their simplest form;
- Write/interpret a ratio to describe a situation;
- Share a quantity in a given ratio including three-part ratios;
- Solve a ratio problem in context:
  - use a ratio to find one quantity when the other is known;
  - use a ratio to compare a scale model to a real-life object;
  - use a ratio to convert between measures and currencies;
  - problems involving mixing, e.g. paint colours, cement and drawn conclusions;
- Compare ratios;
- Write ratios in form 1 : m or m : 1;
- Write a ratio as a fraction;
- Write a ratio as a linear function;
- Write lengths, areas and volumes of two shapes as ratios in simplest form;
- Express a multiplicative relationship between two quantities as a ratio or a fraction.

POSSIBLE SUCCESS CRITERIA

Write a ratio to describe a situation such as 1 blue for every 2 red, or 3 adults for every 10 children.
Recognise that two paints mixed red to yellow 5 : 4 and 20 : 16 are the same colour.
Express the statement 'There are twice as many girls as boys' as the ratio 2 : 1 or the linear function $y = 2x$, where x is the number of boys and y is the number of girls.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Problems involving sharing in a ratio that include percentages rather than specific numbers, such as: In a youth club the ratio of the number of boys to the number of girls is 3 : 2. 30% of the boys are under the age of 14, and 60% of the girls are under the age of 14. What percentage of the youth club is under the age of 14?

COMMON MISCONCEPTIONS

Students find three-part ratios difficult.
Using a ratio to find one quantity when the other is known often results in students 'sharing' the known amount.

NOTES

Emphasise the importance of reading the question carefully.
Include ratios with decimals 0.2 : 1.
Converting imperial units to imperial units aren't specifically in the programme of study, but still useful and provide a good context for multiplicative reasoning.
It is also useful generally for students to know rough metric equivalents of commonly used imperial measures, such as pounds, feet, miles and pints.

11b. Proportion (R1, R5, R7, R10, R14)	**Teaching time** 4-6 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Understand and use proportion as equality of ratios;
- Solve word problems involving direct and indirect proportion;
- Work out which product is the better buy;
- Scale up recipes;
- Convert between currencies;
- Find amounts for 3 people when amount for 1 given;
- Solve proportion problems using the unitary method;
- Recognise when values are in direct proportion by reference to the graph form;
- Understand inverse proportion: as x increases, y decreases (inverse graphs done in later unit);
- Recognise when values are in direct proportion by reference to the graph form;
- Understand direct proportion ---> relationship $y = kx$.

POSSIBLE SUCCESS CRITERIA

Recognise that two paints mixed red to yellow 5 : 4 and 20 : 16 are the same colour.
If it takes 2 builders 10 days to build a wall, how long will it take 3 builders?
Scale up recipes and decide if there is enough of each ingredient.
Given two sets of data in a table, are they in direct proportion?

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Problems in context, such as scaling a recipe, or diluting lemonade or chemical solutions, will show how proportional reasoning is used in real-life contexts.

NOTES

Find out/prove whether two variables are in direct proportion by plotting the graph and using it as a model to read off other values.
Possible link with scatter graphs.

UNIT 12: Right-angled triangles: Pythagoras and trigonometry

Teaching Time
4-6 hours

Return to Overview

SPECIFICATION REFERENCES

N7 calculate with roots, and with integer indices
N15 round numbers and measures to an appropriate degree of accuracy (e.g. to a specified number of decimal places or significant figures); …
A4 simplify and manipulate algebraic expressions (including those involving surds) by: collecting like terms, multiplying a single term over a bracket, …
G6 apply angle facts, triangle congruence, similarity and properties of quadrilaterals to conjecture and derive results about angles and sides, including Pythagoras' Theorem and the fact that the base angles of an isosceles triangle are equal, and use known results to obtain simple proofs
G20 know the formulae for: Pythagoras' Theorem $a^2 + b^2 = c^2$ and the trigonometric ratios, sine, cosine and tan; apply them to find angles and lengths in right-angled triangles in two dimensional figures
G21 know the exact values of sin θ and cos θ for θ = 0°, 30°, 45°, 60° and 90°; know the exact value of tan θ for θ = 0°, 30°, 45° and 60°

PRIOR KNOWLEDGE

Students should be able to rearrange simple formulae and equations, as preparation for rearranging trigonometric formulae.
Students should recall basic angle facts.
Students should understand when to leave an answer in surd form.
Students can plot coordinates in all four quadrants and draw axes.

KEYWORDS

Triangle, right angle, angle, Pythagoras' Theorem, sine, cosine, tan, trigonometry, opposite, hypotenuse, adjacent, ratio, elevation, depression, length, accuracy

OBJECTIVES

By the end of the unit, students should be able to:

- Understand, recall and use Pythagoras' Theorem in 2D, including leaving answers in surd form and being able to justify if a triangle is right-angled or not;
- Calculate the length of the hypotenuse and of a shorter side in a right-angled triangle, including decimal lengths and a range of units;
- Apply Pythagoras' Theorem with a triangle drawn on a coordinate grid;
- Calculate the length of a line segment AB given pairs of points;
- Understand, use and recall the trigonometric ratios sine, cosine and tan, and apply them to find angles and lengths in general triangles in 2D figures;
- Use the trigonometric ratios to solve 2D problems including angles of elevation and depression;
- Round answers to appropriate degree of accuracy, either to a given number of significant figures or decimal places, or make a sensible decision on rounding in context of question;
- Know the exact values of sin θ and cos θ for θ = 0°, 30°, 45°, 60° and 90°; know the exact value of tan θ for θ = 0°, 30°, 45° and 60°.

POSSIBLE SUCCESS CRITERIA

Does 2, 3, 6 give a right angled triangle?
Justify when to use Pythagoras' Theorem and when to use trigonometry.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Combined triangle problems that involve consecutive application of Pythagoras' Theorem or a combination of Pythagoras' Theorem and the trigonometric ratios.
In addition to abstract problems, students should be encouraged to apply Pythagoras' Theorem and/or the trigonometric ratios to real-life scenarios that require them to evaluate whether their answer fulfils certain criteria, e.g. the angle of elevation of 6.5 m ladder cannot exceed 65°. What is the greatest height it can reach?

COMMON MISCONCEPTIONS

Answers may be displayed on a calculator in surd form.
Students forget to square root their final answer or round their answer prematurely.

NOTES

Students may need reminding about surds.
Drawing the squares on the 3 sides will help to illustrate the theorem.
Include examples with triangles drawn in all four quadrants.
Scale drawings are not acceptable.
Calculators need to be in degree mode.
To find in right-angled triangles the exact values of sin θ and cos θ for θ = 0°, 30°, 45°, 60° and 90°, use triangles with angles of 30°, 45° and 60°.
Use a suitable mnemonic to remember SOHCAHTOA.
Use Pythagoras' Theorem and trigonometry together.

UNIT 13: Probability	Teaching Time 11-13 hours

Return to Overview

SPECIFICATION REFERENCES

N5 apply systematic listing strategies
P1 record, describe and analyse the frequency of outcomes of probability experiments using tables and frequency trees
P2 apply ideas of randomness, fairness and equally likely events to calculate expected outcomes of multiple future experiments
P3 relate relative expected frequencies to theoretical probability, using appropriate language and the 0–1 probability scale
P4 apply the property that the probabilities of an exhaustive set of outcomes sum to one; apply the property that the probabilities of an exhaustive set of mutually exclusive events sum to one
P5 <u>understand that empirical unbiased samples tend towards theoretical probability distributions, with increasing sample size</u>
P6 enumerate sets and combinations of sets systematically, using tables, grids, Venn diagrams <u>and tree diagrams</u>
P7 construct theoretical possibility spaces for single and combined experiments with equally likely outcomes and use these to calculate theoretical probabilities
P8 <u>calculate the probability of independent and dependent combined events, including using tree diagrams and other representations, and know the underlying assumptions</u>

PRIOR KNOWLEDGE

Students should know how to add and multiply fractions and decimals.
Students should have experience of expressing one number as a fraction of another number.

KEYWORDS

Probability, dependent, independent, conditional, tree diagrams, sample space, outcomes, theoretical, relative frequency, fairness, experimental

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Distinguish between events which are impossible, unlikely, even chance, likely, and certain to occur;
- Mark events and/or probabilities on a probability scale of 0 to 1;
- Write probabilities in words or fractions, decimals and percentages;
- Find the probability of an event happening using theoretical probability;
- Use theoretical models to include outcomes using dice, spinners, coins;
- List all outcomes for single events systematically;
- Work out probabilities from frequency tables, frequency trees, and two way tables;
- Record outcomes of probability experiments in tables;
- Add simple probabilities;
- Identify different mutually exclusive outcomes and know that the sum of the probabilities of all outcomes is 1;
- Using $1 - p$ as the probability of an event not occurring where p is the probability of the event occurring;

- Find a missing probability from a list or table including algebraic terms;
- Find the probability of an event happening using relative frequency;
- Estimate the number of times an event will occur, given the probability and the number of trials – for both experimental and theoretical probabilities;
- List all outcomes for combined events systematically;
- Use and draw sample space diagrams;
- Work out probabilities from Venn diagrams to represent real-life situations and also 'abstract' sets of numbers/values;
- Use union and intersection notation;
- Compare experimental data and theoretical probabilities;
- Compare relative frequencies from samples of different sizes;
- Find the probability of successive events, such as several throws of a single dice;
- Use tree diagrams to calculate the probability of two independent events;
- Use tree diagrams to calculate the probability of two dependent events.

POSSIBLE SUCCESS CRITERIA

Mark events on a probability scale and use the language of probability.
If the probability of outcomes are x, $2x$, $4x$, $3x$ calculate x.
Calculate the probability of an event from a two-way table or frequency table.
Decide if a coin, spinner or game is fair.
Understand the use of the 0–1 scale to measure probability.
List all the outcomes for an experiment.
Know and apply the fact that the sum of probabilities for all outcomes is 1.
Draw a Venn diagram of students studying French, German or both, and then calculate the probability that a student studies French given that they also study German

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Lotteries provides a real life link to probability. Work out the probabilities of winning on different lotteries.
Students should be given the opportunity to justify the probability of events happening or not happening.

COMMON MISCONCEPTIONS

Not using fractions or decimals when working with probability trees.

NOTES

Use this as an opportunity for practical work.
Probabilities written in fraction form should be cancelled to their simplest form.
Probability without replacement is best illustrated visually and by initially working out probability 'with' replacement.
Encourage students to work 'across' the branches working out the probability of each successive event. The probability of the combinations of outcomes should = 1.
Emphasise that were an experiment repeated it will usually lead to different outcomes, and that increasing sample size generally leads to better estimates of probability and population characteristics.
Probabilities written in fraction form should be cancelled to their simplest form.

UNIT 14: Multiplicative reasoning: more percentages, rates of change, compound measures

Teaching time
6–8 hours

Return to Overview

SPECIFICATION REFERENCES

N13 use standard units of mass, length, time, money and other measures (including standard compound measures) using decimal quantities where appropriate
R1 change freely between related standard units (e.g. time, length, area, volume/capacity, mass) and compound units (e.g. speed, rates of pay, prices, density, pressure) in numerical and algebraic contexts
R9 … express one quantity as a percentage of another; … solve problems involving percentage change, … and original value problems … including in financial mathematics
R11 use compound units such as speed, rates of pay, unit pricing, density and pressure
R13 understand that X is inversely proportional to Y is equivalent to X is proportional to $\frac{1}{Y}$; interpret equations that describe direct and inverse proportion
R16 set up, solve and interpret the answers in growth and decay problems, including compound interest
G14 use standard units of measure and related concepts (length, area, volume/capacity, mass, time, money, etc)

PRIOR KNOWLEDGE

Students should be able to interpret scales on a range of measuring instruments.
Students should be able to find a percentage of an amount and relate percentages to decimals.
Students should be able to rearrange equations and use these to solve problems.
Students should know speed = distance/time, density = mass/volume.

KEYWORDS

Ratio, proportion, best value, proportional change, compound measure, density, mass, volume, speed, distance, time, density, mass, volume, pressure, acceleration, velocity, inverse, direct

OBJECTIVES

By the end of the unit, students should be able to:

- Understand and use compound measures:
  - density;
  - pressure;
  - speed:
    - convert between metric speed measures;
    - read values in km/h and mph from a speedometer;
    - calculate average speed, distance, time – in miles per hour as well as metric measures;
    - use kinematics formulae from the formulae sheet to calculate speed, acceleration (with variables defined in the question);
    - change d/t in m/s to a formula in km/h, i.e. d/t × (60 × 60)/1000 – with support;
- Express a given number as a percentage of another number in more complex situations;
- Calculate percentage profit or loss;
- Make calculations involving repeated percentage change, not using the formula;

- Find the original amount given the final amount after a percentage increase or decrease;
- Use compound interest;
- Use a variety of measures in ratio and proportion problems:
  - currency conversion;
  - rates of pay;
  - best value;
- Set up, solve and interpret the answers in growth and decay problems;
- Understand that X is inversely proportional to Y is equivalent to X is proportional to $\frac{1}{Y}$;
- Interpret equations that describe direct and inverse proportion.

POSSIBLE SUCCESS CRITERIA

Know that measurements using real numbers depend upon the choice of unit, with speedometers and rates of change.
Change m/s to km/h.
Understand direct proportion as: as x increase, y increases.
Understand inverse proportion as: as x increases, y decreases.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Speed/distance type problems that involve students justifying their reasons why one vehicle is faster than another.
Calculations involving value for money are a good reasoning opportunity that utilise different skills.
Working out best value of items using different currencies given an exchange rate.

COMMON MISCONCEPTIONS

Some students may think that compound interest and simple interest are the same method of calculating interest.
Incomplete methods when using multipliers, i.e. reduce £80 by 15% = 80 × 0.15.

NOTES

Encourage students to use a single multiplier.
Include simple fractional percentages of amounts with compound interest and encourage use of single multipliers.
Amounts of money should be rounded to the nearest penny, but emphasise the importance of not rounding until the end of the calculation if doing in stages.
Use a formula triangle to help students see the relationship for compound measures – this will help them evaluate which inverse operations to use.
Help students to recognise the problem they are trying to solve by the unit measurement given, e.g. km/h is a unit of speed as it is speed divided by a time.

UNIT 15: Constructions: triangles, nets, plan and elevation, loci, scale drawings and bearings

Return to Overview

SPECIFICATION REFERENCES

R2 use scale factors, scale diagrams and maps

G1 use conventional terms and notation: points, lines, vertices, edges, planes, parallel lines, perpendicular lines, right angles, polygons, regular polygons and polygons with reflection and/or rotation symmetries; use the standard conventions for labelling and referring to the sides and angles of triangles; draw diagrams from written description;

G2 use the standard ruler and compass constructions (perpendicular bisector of a line segment, constructing a perpendicular to a given line from/at a given point, bisecting a given angle); use these to construct given figures and solve loci problems; know that the perpendicular distance from a point to a line is the shortest distance to the line

G5 use the basic congruence criteria for triangles (SSS, SAS, ASA, RHS)

G9 identify and apply circle definitions and properties, including: centre, radius, chord, diameter, circumference, tangent, arc, sector and segment

G12 identify properties of the faces, surfaces, edges and vertices of: cubes, cuboids, prisms, cylinders, pyramids, cones and spheres

G13 construct and interpret plans and elevations of 3D shapes

G15 measure line segments and angles in geometric figures, including interpreting maps and scale drawings and use of bearings

PRIOR KNOWLEDGE

Students should be able to measure and draw lines.

KEYWORDS

Construct, circle, arc, sector, face, edge, vertex, two-dimensional, three-dimensional, solid, elevations, congruent, angles, regular, irregular, bearing, degree, bisect, perpendicular, loci, map, scale, plan, region

15a. Plans and elevations (G1, G2, G9, G12, G13, G15)	**Teaching time** 4-6 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Understand clockwise and anticlockwise;
- Draw circles and arcs to a given radius or given the diameter;
- Measure and draw lines, to the nearest mm;
- Measure and draw angles, to the nearest degree;
- Know and use compass directions;
- Draw sketches of 3D solids;
- Know the terms face, edge and vertex;
- Identify and sketch planes of symmetry of 3D solids;
- Use isometric grids to draw 2D representations of 3D solids;
- Make accurate drawings of triangles and other 2D shapes using a ruler and a protractor;
- Construct diagrams of everyday 2D situations involving rectangles, triangles, perpendicular and parallel lines;
- Understand and draw front and side elevations and plans of shapes made from simple solids;
- Given the front and side elevations and the plan of a solid, draw a sketch of the 3D solid.

POSSIBLE SUCCESS CRITERIA

Be able to estimate the size of given angles.
Convert fluently between metric units of length.
Use bearings in a real-life context to describe the bearing between two towns on a map.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Interpreting scale drawings and maps involving lengths that need to be measured (rather than given in the problem).

COMMON MISCONCEPTIONS

Some pupils may use the wrong scale of a protractor. For example, they measure an obtuse angle as 60° rather than as 120°.
Often 5 sides only are drawn for a cuboid.

NOTES

This is a very practical topic, and provides opportunities for some hands-on activities.
Whilst not an explicit objective, it is useful for students to draw and construct nets and show how they fold to make 3D solids, allowing students to make the link between 3D shapes and their nets. This will enable students to understand that there is often more than one net that can form a 3D shape.

15b. Constructions, loci and bearings
(R2, G2, G5, G15)

Teaching time
6-8 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Understand congruence, as two shapes that are the same size and shape;
- Visually identify shapes which are congruent;
- Use straight edge and a pair of compasses to do standard constructions:
    - understand, from the experience of constructing them, that triangles satisfying SSS, SAS, ASA and RHS are unique, but SSA triangles are not;
    - construct the perpendicular bisector of a given line;
    - construct the perpendicular from a point to a line;
    - construct the bisector of a given angle;
    - construct angles of 90°, 45°;
- Draw and construct diagrams from given instructions, including the following:
    - a region bounded by a circle and an intersecting line;
    - a given distance from a point and a given distance from a line;
    - equal distances from two points or two line segments;
    - regions may be defined by 'nearer to' or 'greater than';
- Find and describe regions satisfying a combination of loci;
- Use constructions to solve loci problems (2D only);
- Use and interpret maps and scale drawings;
- Estimate lengths using a scale diagram;
- Make an accurate scale drawing from a diagram;
- Use three-figure bearings to specify direction;
- Mark on a diagram the position of point B given its bearing from point A;
- Give a bearing between the points on a map or scaled plan;
- Given the bearing of a point A from point B, work out the bearing of B from A;
- Use accurate drawing to solve bearings problems;
- Solve locus problems including bearings.

POSSIBLE SUCCESS CRITERIA

Sketch the locus of point on a vertex of a rotating shape as it moves along a line, i.e. a point on the circumference or at the centre of a wheel.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Link problems with other areas of mathematics, such as the trigonometric ratios and Pythagoras' Theorem.

COMMON MISCONCEPTIONS

Correct use of a protractor may be an issue.

NOTES

Drawings should be done in pencil.
Relate loci problems to real-life scenarios, including mobile phone masts and coverage.
Construction lines should not be erased.

UNIT 16: Algebra: quadratic equations and graphs

Return to Overview

SPECIFICATION REFERENCES

A4 simplify and manipulate algebraic expressions by: … expanding products of two binomials; factorising freturn to overview expressions of the form $x^2 + bx + c$, including the difference of two squares; …
A11 identify and interpret roots, intercepts, turning points of quadratic functions graphically; deduce roots algebraically
A12 recognise, sketch and interpret graphs of … quadratic functions; …
A18 solve quadratic equations algebraically by factorising; find approximate solutions using a graph

PRIOR KNOWLEDGE

Students should be able to square negative numbers.
Students should be able to substitute into formulae.
Students should be able to plot points on a coordinate grid.
Students should be able to expand single brackets and collect 'like' terms.

KEYWORDS

Quadratic, function, solve, expand, factorise, simplify, expression, graph, curve, factor, coefficient, bracket

16a. Quadratic equations: expanding and factorising (A4, A11, A18)

Teaching time
4–6 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Define a 'quadratic' expression;
- Multiply together two algebraic expressions with brackets;
- Square a linear expression, e.g. $(x + 1)^2$;
- Factorise quadratic expressions of the form $x^2 + bx + c$;
- Factorise a quadratic expression $x^2 - a^2$ using the difference of two squares;
- Solve quadratic equations by factorising;
- Find the roots of a quadratic function algebraically.

POSSIBLE SUCCESS CRITERIA

Solve $3x^2 + 4 = 100$.

Expand $(x + 2)(x + 6)$.

Factorise $x^2 + 7x + 10$.

Solve $x^2 + 7x + 10 = 0$.

Solve $(x - 3)(x + 4)= 0$.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Visual proof of the difference of two squares.

COMMON MISCONCEPTIONS

x terms can sometimes be 'collected' with x^2.

NOTES

This unit can be extended by including quadratics where $a \neq 1$.

Emphasise the fact that x^2 and x are different 'types' of term – illustrate this with numbers.

16b. Quadratic equations: graphs (A11, A12, A18)	Teaching time 3–5 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Generate points and plot graphs of simple quadratic functions, then more general quadratic functions;
- Identify the line of symmetry of a quadratic graph;
- Find approximate solutions to quadratic equations using a graph;
- Interpret graphs of quadratic functions from real-life problems;
- Identify and interpret roots, intercepts and turning points of quadratic graphs.

POSSIBLE SUCCESS CRITERIA

Recognise a quadratic graph from its shape.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Matching graphs with their respective functions.

COMMON MISCONCEPTIONS

Squaring negative numbers can be a problem.

NOTES

The graphs should be drawn freehand and in pencil, joining points using a smooth curve.
Encourage efficient use of the calculator.
Extension work can be through plotting cubic and reciprocal graphs, solving simultaneous equations graphically.

UNIT 17: Perimeter, area and volume 2: circles, cylinders, cones and spheres

Teaching time
5-7 hours

Return to Overview

SPECIFICATION REFERENCES

N8 calculate exactly with multiples of π
N14 estimate answers; check calculations using approximation and estimation, including answers obtained using technology
G9 identify and apply circle definitions and properties, including: centre, radius, chord, diameter, circumference, tangent, arc, sector and segment
G16 know and apply formulae to calculate: area of triangles, parallelograms, trapezia; volume of cuboids and other right prisms (including cylinders)
G17 know the formulae: circumference of a circle = $2\pi r$ = πd, area of a circle = πr^2; calculate: perimeters of 2D shapes, including circles; areas of circles and composite shapes; surface area and volume of spheres, pyramids, cones and composite solids
G18 calculate arc lengths, angles and areas of sectors of circles

PRIOR KNOWLEDGE

Students should know the formula for calculating the area of a rectangle.
Students should know how to use the four operations on a calculator.

KEYWORDS

Area, perimeter, formula, length, width, measurement, volume, circle, segment, arc, sector, cylinder, circumference, radius, diameter, pi, sphere, cone, hemisphere, segment, accuracy, surface area

OBJECTIVES

By the end of the unit, students should be able to:

- Recall the definition of a circle and identify, name and draw parts of a circle including tangent, chord and segment;
- Recall and use formulae for the circumference of a circle and the area enclosed by a circle circumference of a circle = $2\pi r$ = πd, area of a circle = πr^2;
- Use $\pi \approx 3.142$ or use the π button on a calculator;
- Give an answer to a question involving the circumference or area of a circle in terms of π;
- Find radius or diameter, given area or perimeter of a circles;
- Find the perimeters and areas of semicircles and quarter-circles;
- Calculate perimeters and areas of composite shapes made from circles and parts of circles;
- Calculate arc lengths, angles and areas of sectors of circles;
- Find the surface area and volume of a cylinder;
- Find the surface area and volume of spheres, pyramids, cones and composite solids;
- Round answers to a given degree of accuracy.

POSSIBLE SUCCESS CRITERIA

Recall terms related to a circle.
Understand that answers in terms of pi are more accurate.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Calculate the radius/diameter given the area/circumference type questions could be explored, including questions that require evaluation of statements, such as Andy states "Diameter = 2 × Radius" and Bob states "'Radius = 2 × Diameter". Who is correct?

COMMON MISCONCEPTIONS

Diameter and radius are often confused and recollection which formula to use for area and circumference of circles is often poor.

NOTES

Emphasise the need to learn the circle formula: 'Cherry Pie's Delicious' and 'Apple Pies are too' are good ways to remember them.
Formulae for curved surface area and volume of a sphere, and surface area and volume of a cone, will be given on the formulae sheet in the examination.
Ensure that students know it is more accurate to leave answers in terms of π but only when asked to do so.

UNIT 18: More fractions, reciprocals, standard form, zero and negative indices

Return to Overview

SPECIFICATION REFERENCES

N2 apply the four operations, including formal written methods, to integers, decimals and simple fractions (proper and improper), and mixed numbers – all both positive and negative; understand and use place value (e.g. when working with very large or very small numbers, and when calculating with decimals)

N3 recognise and use relationships between operations, including inverse operations (e.g. cancellation to simplify calculations and expressions); use conventional notation for priority of operations, including brackets, powers, roots and reciprocals

N7 calculate with roots, and with integer indices

N9 calculate with and interpret standard form $A \times 10^n$, where $1 \leq A < 10$ and n is an integer.

PRIOR KNOWLEDGE

Students should know how to do the four operations with fractions.
Students should be able to write powers of 10 in index form and recognise and recall powers of 10, i.e. $10^2 = 100$.
Students should recall the index laws.

KEYWORDS

Add, subtract, multiply, divide, mixed, improper, fraction, decimal, indices, standard form, power, reciprocal, index

18a. Fractions (N2, N3) | **Teaching time** 4–6 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Add and subtract mixed number fractions;
- Multiply mixed number fractions;
- Divide mixed numbers by whole numbers and vice versa;
- Find the reciprocal of an integer, decimal or fraction;
- Understand 'reciprocal' as multiplicative inverse, knowing that any non-zero number multiplied by its reciprocal is 1 (and that zero has no reciprocal because division by zero is not defined).

POSSIBLE SUCCESS CRITERIA

What is the reciprocal of 4, $\frac{1}{2}$, −2, $-\frac{1}{2}$?

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Students should be able to justify when fractions are equal and provide correct answers as a counter-argument.
Links with other areas of mathematics should be used where appropriate to embed the notion that fractions are not just used in isolation, e.g. use 6 ½ cm instead of 6.5 cm.

COMMON MISCONCEPTIONS

The larger the denominator the larger the fraction.

NOTES

Regular revision of fractions is essential.
Demonstrate how to the use the fraction button on the calculator.
Use real-life examples where possible.

18b. Indices and standard form (N7, N9)

Teaching time
4-6 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Use index laws to simplify and calculate the value of numerical expressions involving multiplication and division of integer powers, fractions and powers of a power;
- Use numbers raised to the power zero, including the zero power of 10;
- Convert large and small numbers into standard form and vice versa;
- Add, subtract, multiply and divide numbers in standard form;
- Interpret a calculator display using standard form and know how to enter numbers in standard form.

POSSIBLE SUCCESS CRITERIA

Write 51 080 in standard form.
Write 3.74×10^{-6} as an ordinary number.
What is 9^0?

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Link with other areas of mathematics, such as compound measures, by using speed of light in standard form.

COMMON MISCONCEPTIONS

Some students may think that any number multiplied by a power of ten qualifies as a number written in standard form.
When rounding to significant figures some students may think, for example, that 6729 rounded to one significant figure is 7.

NOTES

Negative fractional indices are not included at Foundation tier, but you may wish to extend the work to include these.
Standard form is used in science and there are lots of cross curricular opportunities.
Students need to be provided with plenty of practice in using standard form with calculators.

UNIT 19: Congruence, similarity and vectors

Return to Overview

SPECIFICATION REFERENCES

R6 express a multiplicative relationship between two quantities as a ratio or a fraction

R12 compare lengths, areas and volumes using ratio notation; make links to similarity (including trigonometric ratios) and scale factors

G5 use the basic congruence criteria for triangles (SSS, SAS, ASA, RHS)

G7 identify, describe and construct congruent and similar shapes, including on coordinate axes, by considering rotation, reflection, translation and enlargement (including fractional scale factors)

G19 apply the concepts of congruence and similarity, including the relationships between lengths in similar figures

G24 describe translations as 2D vectors

G25 apply addition and subtraction of vectors, multiplication by vectors by a scalar, and diagrammatic and column representations of vectors

PRIOR KNOWLEDGE

Students will have used column vectors when dealing with translations.
Students can recall and apply Pythagoras' Theorem on a coordinate grid.
Students should be able to recognise and enlarge shapes and calculate scale factors.
Students know how to calculate area and volume in various metric measures.
Students should be able to measure lines and angles and using compasses, ruler and protractor, and construct standard constructions.

KEYWORDS

Vector, direction, magnitude, scalar, multiple, parallel, collinear, ratio, column vector, congruence, side, angle, compass, construction, shape, volume, length, area, volume, scale factor, enlargement, similar, perimeter,

19a. Similarity and congruence in 2D (R6, R12, G5, G7, G19)

Teaching time
6–8 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Use the basic congruence criteria for triangles (SSS, SAS, ASA and RHS);
- Solve angle problems involving congruence;
- Identify shapes which are similar; including all circles or all regular polygons with equal number of sides;
- Understand similarity of triangles and of other plane shapes, use this to make geometric inferences, and solve angle problems using similarity;
- Identify the scale factor of an enlargement of a shape as the ratio of the lengths of two corresponding sides;
- Understand the effect of enlargement on perimeter of shapes;
- Solve problems to find missing lengths in similar shapes;
- Know that scale diagrams, including bearings and maps are 'similar' to the real-life examples.

POSSIBLE SUCCESS CRITERIA

Understand similarity as one shape being an enlargement of the other.
Recognise that all corresponding angles in similar shapes are equal in size when the corresponding lengths of sides are not equal in size.
Use *AB* notation for describing lengths and $\angle ABC$ notation for describing angles.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Using scale diagrams, including bearings and maps, provides a rich source of real-life examples and links to other areas of mathematics.

COMMON MISCONCEPTIONS

Students may incorrectly believe that all polygons are regular or that all triangles have a rotational symmetry of order 3.
Often students think that when a shape is enlarged the angles also get bigger.

NOTES

Use simple scale factors that are easily calculated mentally to introduce similar shapes.
Reinforce the fact that the sizes of angles are maintained when a shape is enlarged.
Make links between similarity and trigonometric ratios.

19b. Vectors (G24, G25)

Teaching time
6–8 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Understand and use column notation in relation to vectors;
- Be able to represent information graphically given column vectors;
- Identify two column vectors which are parallel;
- Calculate using column vectors, and represent graphically, the sum of two vectors, the difference of two vectors and a scalar multiple of a vector.

POSSIBLE SUCCESS CRITERIA

Know that if one vector is a multiple of the other, they are parallel.
Add and subtract vectors using column vectors.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Investigations involving vectors around 2D shapes such as a square can be extended to include considering the area enclosed in the same shapes.

COMMON MISCONCEPTIONS

Students find it difficult to understand that two vectors can be parallel and equal as they can be in different locations in the plane.

NOTES

Students find manipulation of column vectors relatively easy compared to the pictorial and algebraic manipulation methods – encourage them to draw any vectors that they calculate on the picture.

UNIT 20: Rearranging equations, graphs of cubic and reciprocal functions and simultaneous equations

Teaching time
4–6 hours

Return to Overview

SPECIFICATION REFERENCES

N1 order positive and negative integers, decimals and fractions; use the symbols =, ≠, <, >, ≤, ≥

A3 understand and use the concepts and vocabulary of expressions, equations, formulae, identities, inequalities, terms and factors

A5 understand and use standard mathematical formulae; rearrange formulae to change the subject

A6 … argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments

A9 … use the form $y = mx + c$ to identify parallel lines; find the equation of the line through two given points, or through one point with a given gradient

A12 recognise, sketch and interpret graphs of … the reciprocal function $y = \frac{1}{x}$ with $x \neq 0$

A14 plot and interpret … reciprocal graphs …

A19 solve two simultaneous equations in two variables (linear/linear) algebraically; find approximate solutions using a graph

A21 translate simple situations or procedures into algebraic expressions or formulae; derive an equation (or two simultaneous equations), solve the equation(s) and interpret the solution.

A22 solve linear inequalities in one or two variable(s) represent the solution set on a number line

R10 solve problems involving direct and inverse proportion, including graphical and algebraic representations

R14 … recognise and interpret graphs that illustrate direct and inverse proportion

PRIOR KNOWLEDGE

Students should be able to draw linear graphs.
Students should be able to plot coordinates and sketch simple functions with a table of values.
Students should be able to substitute into and solve equations.
Students should have experience of using formulae.
Students should recall and use the hierarchy of operations and use of inequality symbols.

KEYWORDS

Reciprocal, linear, gradient, functions, direct, indirect, estimate, cubic, subject, rearrange, simultaneous, substitution, elimination, proof

OBJECTIVES

By the end of the unit, students should be able to:

- Know the difference between an equation and an identity and use and understand the $\neq$ symbol;
- Change the subject of a formula involving the use of square roots and squares;
- Answer 'show that' questions using consecutive integers (n, $n + 1$), squares a^2, b^2, even numbers $2n$, and odd numbers $2n + 1$;
- Solve problems involving inverse proportion using graphs, and read values from graphs;
- Find the equation of the line through two given points;
- Recognise, sketch and interpret graphs of simple cubic functions;
- Recognise, sketch and interpret graphs of the reciprocal function $y = \frac{1}{x}$ with $x \neq 0$;
- Use graphical representations of indirect proportion to solve problems in context;
- identify and interpret the gradient from an equation $ax + by = c$;
- Write simultaneous equations to represent a situation;
- Solve simultaneous equations (linear/linear) algebraically and graphically;
- Solve simultaneous equations representing a real-life situation, graphically and algebraically, and interpret the solution in the context of the problem;

POSSIBLE SUCCESS CRITERIA

Solve two simultaneous equations in two variables (linear/linear) algebraically and find approximate solutions using a graph.
Identify expressions, equations, formulae and identities from a list.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Simple simultaneous equations can be formed and solved from real life scenarios, such as 2 adult and 2 child tickets cost £18, and 1 adult and 3 child tickets costs £17. What is the cost of 1 adult ticket?

COMMON MISCONCEPTIONS

The effects of transforming functions are often confused.

NOTES

Emphasise the need for good algebraic notation.

GCSE Mathematics (1MA1)

Higher Tier

Scheme of Work

Unit		Title	Estimated hours
1	a	Calculations, checking and rounding	4
	b	Indices, roots, reciprocals and hierarchy of operations	4
	c	Factors, multiples, primes, standard form and surds	7
2	a	Algebra: the basics, setting up, rearranging and solving equations	10
	b	Sequences	4
3	a	Averages and range	4
	b	Representing and interpreting data and scatter graphs	5
4	a	Fractions and percentages	12
	b	Ratio and proportion	6
5	a	Polygons, angles and parallel lines	6
	b	Pythagoras' Theorem and trigonometry	6
6	a	Graphs: the basics and real-life graphs	6
	b	Linear graphs and coordinate geometry	8
	c	Quadratic, cubic and other graphs	6
7	a	Perimeter, area and circles	5
	b	3D forms and volume, cylinders, cones and spheres	7
	c	Accuracy and bounds	5
8	a	Transformations	6
	b	Constructions, loci and bearings	7
9	a	Solving quadratic and simultaneous equations	7
	b	Inequalities	6
10		Probability	8
11		Multiplicative reasoning	8
12		Similarity and congruence in 2D and 3D	6
13	a	Graphs of trigonometric functions	6
	b	Further trigonometry	9
14	a	Collecting data	4
	b	Cumulative frequency, box plots and histograms	6
15		Quadratics, expanding more than two brackets, sketching graphs, graphs of circles, cubes and quadratics	7
16	a	Circle theorems	5
	b	Circle geometry	5
17		Changing the subject of formulae (more complex), algebraic fractions, solving equations arising from algebraic fractions, rationalising surds, proof	7
18		Vectors and geometric proof	9
19	a	Reciprocal and exponential graphs; Gradient and area under graphs	7
	b	Direct and inverse proportion	7

UNIT 1: Powers, decimals, HCF and LCM, positive and negative, roots, rounding, reciprocals, standard form, indices and surds

Return to Overview

SPECIFICATION REFERENCES

N2 apply the four operations, including formal written methods, to integers, decimals … both positive and negative; understand and use place value (e.g. working with very large or very small numbers, and when calculating with decimals)

N3 recognise and use relationships between operations, including inverse operations (e.g. cancellation to simplify calculations and expressions); use conventional notation for priority of operations, including brackets, powers, roots and reciprocals

N4 use the concepts and vocabulary of prime numbers, factors (divisors), multiples, common factors, common multiples, highest common factor, lowest common multiple, prime factorisation, including using product notation and the unique factorisation theorem

N5 apply systematic listing strategies **including use of the product rule for counting (i.e. if there are m ways of doing one task and for each of these, there are n ways of doing another task, then the total number of ways the two tasks can be done is $m \times n$ ways)**

N6 use positive integer powers and associated real roots (square, cube and higher), recognise powers of 2, 3, 4, 5; **estimate powers and roots of any given positive number**

N7 calculate with roots and with integer **and fractional** indices

N8 calculate exactly with … **surds**; … **simplify surd expressions involving squares (e.g. $\sqrt{12} = \sqrt{(4 \times 3)} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$)**

N9 calculate with and interpret standard form $A \times 10^n$, where $1 \le A < 10$ and n is an integer.

N14 estimate answers; check calculations using approximation and estimation, including answers obtained using technology

N15 round numbers and measures to an appropriate degree of accuracy (e.g. to a specified number of decimal places or significant figures); …

PRIOR KNOWLEDGE

It is essential that students have a firm grasp of place value and be able to order integers and decimals and use the four operations.
Students should have knowledge of integer complements to 10 and to 100, multiplication facts to 10 × 10, strategies for multiplying and dividing by 10, 100 and 1000.
Students will have encountered squares, square roots, cubes and cube roots and have knowledge of classifying integers.

KEYWORDS

Integer, number, digit, negative, decimal, addition, subtraction, multiplication, division, remainder, operation, estimate, power, roots, factor, multiple, primes, square, cube, even, odd, surd, rational, irrational standard form, simplify

1a. Calculations, checking and rounding (N2, N3, N5, N14, N15)

Teaching time
3-5 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Add, subtract, multiply and divide decimals, whole numbers including any number between 0 and 1;
- Put digits in the correct place in a decimal calculation and use one calculation to find the answer to another;
- Use the product rule for counting (i.e. if there are m ways of doing one task and for each of these, there are n ways of doing another task, then the total number of ways the two tasks can be done is $m \times n$ ways);
- Round numbers to the nearest 10, 100, 1000, the nearest integer, to a given number of decimal places and to a given number of significant figures;
- Estimate answers to one- or two-step calculations, including use of rounding numbers and formal estimation to 1 significant figure: mainly whole numbers and then decimals.

POSSIBLE SUCCESS CRITERIA

Given 5 digits, what is the largest even number, largest odd number, or largest or smallest answers when subtracting a two-digit number from a three-digit number?
Given 2.6 × 15.8 = 41.08 what is 26 × 0.158? What is 4108 ÷ 26?

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Problems that include providing reasons as to whether an answer is an overestimate or underestimate.
Missing digits in calculations involving the four operations.
Questions such as: Phil states 3.44 × 10 = 34.4, and Chris states 3.44 × 10 = 34.40. Who is correct?
Show me another number with 3, 4, 5, 6, 7 digits that includes a 6 with the same value as the "6" in the following number 36 754.

COMMON MISCONCEPTIONS

Significant figure and decimal place rounding are often confused.
Some pupils may think 35 934 = 36 to two significant figures.

NOTES

The expectation for Higher tier is that much of this work will be reinforced throughout the course.
Particular emphasis should be given to the importance of clear presentation of work.
Formal written methods of addition, subtraction and multiplication work from right to left, whilst formal division works from left to right.
Any correct method of multiplication will still gain full marks, for example, the grid method, the traditional method, Napier's bones.
Encourage the exploration of different calculation methods.
Amounts of money should always be rounded to the nearest penny.
Make sure students are absolutely clear about the difference between significant figures and decimal places.

1b. Indices, roots, reciprocals and hierarchy of operations
(N3, N6, N7)

Teaching time
3-5 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Use index notation for integer powers of 10, including negative powers;
- Recognise powers of 2, 3, 4, 5;
- Use the square, cube and power keys on a calculator and estimate powers and roots of any given positive number, by considering the values it must lie between, e.g. the square root of 42 must be between 6 and 7;
- Find the value of calculations using indices including positive, fractional and negative indices;
- Recall that $n^0 = 1$ and $n^{-1} = \frac{1}{n}$ for positive integers n as well as, $n^{\frac{1}{2}} = \sqrt{n}$ and $n^{\frac{1}{3}} = \sqrt[3]{n}$ for any positive number n;
- Understand that the inverse operation of raising a positive number to a power n is raising the result of this operation to the power $\frac{1}{n}$;
- Use index laws to simplify and calculate the value of numerical expressions involving multiplication and division of integer powers, fractional and negative powers, and powers of a power;
- Solve problems using index laws;
- Use brackets and the hierarchy of operations up to and including with powers and roots inside the brackets, or raising brackets to powers or taking roots of brackets;
- Use an extended range of calculator functions, including +, −, ×, ÷, x^2, $\sqrt{x}$, memory, x^y, $x^{\frac{1}{y}}$, brackets;
- Use calculators for all calculations: positive and negative numbers, brackets, powers and roots, four operations.

POSSIBLE SUCCESS CRITERIA

What is the value of 2^5?
Prove that the square root of 45 lies between 6 and 7.
Evaluate $(2^3 \times 2^5) \div 2^4$, 4^0, $8^{-\frac{2}{3}}$.
Work out the value of n in $40 = 5 \times 2^n$.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Problems that use indices instead of integers will provide rich opportunities to apply the knowledge in this unit in other areas of Mathematics.

COMMON MISCONCEPTIONS

The order of operations is often not applied correctly when squaring negative numbers, and many calculators will reinforce this misconception.

NOTES

Students need to know how to enter negative numbers into their calculator.
Use negative number and not minus number to avoid confusion with calculations.

1c. Factors, multiples, primes, standard form and surds
(N3, N4, N8, N9)

Teaching time
6-8 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Identify factors, multiples and prime numbers;
- Find the prime factor decomposition of positive integers – write as a product using index notation;
- Find common factors and common multiples of two numbers;
- Find the LCM and HCF of two numbers, by listing, Venn diagrams and using prime factors – include finding LCM and HCF given the prime factorisation of two numbers;
- Solve problems using HCF and LCM, and prime numbers;
- Understand that the prime factor decomposition of a positive integer is unique, whichever factor pair you start with, and that every number can be written as a product of prime factors;
- Convert large and small numbers into standard form and vice versa;
- Add, subtract, multiply and divide numbers in standard form;
- Interpret a calculator display using standard form and know how to enter numbers in standard form;
- Understand surd notation, e.g. calculator gives answer to sq rt 8 as 4 rt 2;
- Simplify surd expressions involving squares (e.g. $\sqrt{12} = \sqrt{(4 \times 3)} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$).

POSSIBLE SUCCESS CRITERIA

Know how to test if a number up to 120 is prime.
Understand that every number can be written as a unique product of its prime factors.
Recall prime numbers up to 100.
Understand the meaning of prime factor.
Write a number as a product of its prime factors.
Use a Venn diagram to sort information.
Write 51080 in standard form.
Write 3.74×10^{-6} as an ordinary number.
Simplify $\sqrt{8}$.
Convert a 'near miss', or any number, into standard form; e.g. 23×10^{7}.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Evaluate statements and justify which answer is correct by providing a counter-argument by way of a correct solution.
Links with other areas of Mathematics can be made by using surds in Pythagoras and when using trigonometric ratios.

COMMON MISCONCEPTIONS

1 is a prime number.
Particular emphasis should be made on the definition of "product" as multiplication, as many students get confused and think it relates to addition.
Some students may think that any number multiplied by a power of ten qualifies as a number written in standard form.

Higher tier

When rounding to significant figures some students may think, for example, that 6729 rounded to one significant figure is 7.

NOTES

Use a number square to find primes (Eratosthenes sieve).
Using a calculator to check the factors of large numbers can be useful.
Students need to be encouraged to learn squares from 2×2 to 15×15 and cubes of 2, 3, 4, 5 and 10, and corresponding square and cube roots.
Standard form is used in science and there are lots of cross-curricular opportunities.
Students need to be provided with plenty of practice in using standard form with calculators.
Rationalising the denominator is covered later in unit 17.

UNIT 2: Expressions, substituting into simple formulae, expanding and factorising, equations, sequences and inequalities, simple proof

Return to Overview

SPECIFICATION REFERENCES

N1 … use the symbols =, ≠, <, >, ≤, ≥

N3 recognise and use relationships between operations, including inverse operations (e.g. cancellation to simplify calculations and expressions); use conventional notation for priority of operations, including brackets, powers, roots and reciprocals

N8 calculate exactly with fractions …

N9 calculate with and interpret standard form $A \times 10^n$, where $1 \leq A < 10$ and n is an integer.

A1 use and interpret algebraic notation, including:

- ab in place of $a \times b$
- $3y$ in place of $y + y + y$ and $3 \times y$
- a^2 in place of $a \times a$, a^3 in place of $a \times a \times a$, a^2b in place of $a \times a \times b$
- $\frac{a}{b}$ in place of $a \div b$
- coefficients written as fractions rather than as decimals
- brackets

A2 substitute numerical values into formulae and expressions, including scientific formulae

A3 understand and use the concepts and vocabulary of expressions, equations, formulae, identities, inequalities, terms and factors

A4 simplify and manipulate algebraic expressions … by:

- collecting like terms
- multiplying a single term over a bracket
- taking out common factors
- expanding products of two … binomials
- factorising quadratic expressions of the form $x^2 + bx + c$, including the difference of two squares; …
- simplifying expressions involving sums, products and powers, including the laws of indices

A5 understand and use standard mathematical formulae; rearrange formulae to change the subject

A6 know the difference between an equation and an identity; argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments **and proofs**

A7 where appropriate, interpret simple expressions as functions with inputs and outputs; …

A17 solve linear equations in one unknown algebraically …;

A20 **find approximate solutions to equations numerically using iteration**

A21 translate simple situations or procedures into algebraic expressions or formulae; derive an equation …, solve the equation and interpret the solution

A23 generate terms of a sequence from either a term-to-term or a position-to-term rule

A24 recognise and use sequences of triangular, square and cube numbers, simple arithmetic progressions, Fibonacci type sequences and simple geometric progressions (r^n where n is an integer, and r is a rational number > 0), recognise and use other sequences **or a surd)**

A25 deduce expressions to calculate the nth term of linear sequences.

PRIOR KNOWLEDGE

Students should have prior knowledge of some of these topics, as they are encountered at Key Stage 3:

- the ability to use negative numbers with the four operations and recall and use hierarchy of operations and understand inverse operations;
- dealing with decimals and negatives on a calculator;
- using index laws numerically.

KEYWORDS

Expression, identity, equation, formula, substitute, term, 'like' terms, index, power, negative and fractional indices, collect, substitute, expand, bracket, factor, factorise, quadratic, linear, simplify, approximate, arithmetic, geometric, function, sequence, nth term, derive

2a. Algebra: the basics, setting up, rearranging and solving equations (N1, N3, N8, A1, A2, A3, A4, A5, A6, A7, A17, A20, A21)

Teaching time 9-11 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Use algebraic notation and symbols correctly;
- Know the difference between a term, expression, equation, formula and an identity;
- Write and manipulate an expression by collecting like terms;
- Substitute positive and negative numbers into expressions such as $3x + 4$ and $2x^3$ and then into expressions involving brackets and powers;
- Substitute numbers into formulae from mathematics and other subject using simple linear formulae, e.g. $l \times w$, $v = u + at$;
- Simplify expressions by cancelling, e.g. $\frac{4x}{2} = 2x$;
- Use instances of index laws for positive integer powers including when multiplying or dividing algebraic terms;
- Use instances of index laws, including use of zero, fractional and negative powers;
- Multiply a single term over a bracket and recognise factors of algebraic terms involving single brackets and simplify expressions by factorising, including subsequently collecting like terms;
- Expand the product of two linear expressions, i.e. double brackets working up to negatives in both brackets and also similar to $(2x + 3y)(3x - y)$;
- Know that squaring a linear expression is the same as expanding double brackets;
- Factorise quadratic expressions of the form $ax^2 + bx + c$;
- Factorise quadratic expressions using the difference of two squares;
- Set up simple equations from word problems and derive simple formulae;
- Understand the ≠ symbol (not equal), e.g. $6x + 4 \neq 3(x + 2)$, and introduce identity ≡ sign;
- Solve linear equations, with integer coefficients, in which the unknown appears on either side or on both sides of the equation;
- Solve linear equations which contain brackets, including those that have negative signs occurring anywhere in the equation, and those with a negative solution;
- Solve linear equations in one unknown, with integer or fractional coefficients;
- Set up and solve linear equations to solve to solve a problem;
- Derive a formula and set up simple equations from word problems, then solve these equations, interpreting the solution in the context of the problem;
- Substitute positive and negative numbers into a formula, solve the resulting equation including brackets, powers or standard form;
- Use and substitute formulae from mathematics and other subjects, including the kinematics formulae $v = u + at$, $v^2 - u^2 = 2as$, and $s = ut + \frac{1}{2}at^2$;
- Change the subject of a simple formula, i.e. linear one-step, such as $x = 4y$;
- Change the subject of a formula, including cases where the subject is on both sides of the original formula, or involving fractions and small powers of the subject;
- Simple proofs and use of ≡ in "show that" style questions; know the difference between an equation and an identity;

- Use iteration to find approximate solutions to equations, for simple equations in the first instance, then quadratic and cubic equations.

POSSIBLE SUCCESS CRITERIA

Simplify $4p - 2q^2 + 1 - 3p + 5q^2$.
Evaluate $4x^2 - 2x$ when $x = -5$.
Simplify $z^4 \times z^3$, $y^3 \div y^2$, $(a^7)^2$, $\left(8x^6y^4\right)^{\frac{1}{3}}$.
Expand and simplify $3(t - 1) + 57$.
Factorise $15x^2y - 35x^2y^2$.
Expand and simplify $(3x + 2)(4x - 1)$.
Factorise $6x^2 - 7x + 1$.
A room is 2 m longer than it is wide. If its area is 30 m^2 what is its perimeter?
Use fractions when working in algebraic situations.
Substitute positive and negative numbers into formulae.
Be aware of common scientific formulae.
Know the meaning of the 'subject' of a formula.
Change the subject of a formula when one step is required.
Change the subject of a formula when two steps are required.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Forming and solving equations involving algebra and other areas of mathematics such as area and perimeter.
Evaluate statements and justify which answer is correct by providing a counter-argument by way of a correct solution.

COMMON MISCONCEPTIONS

When expanding two linear expressions, poor number skills involving negatives and times tables will become evident.
Hierarchy of operations applied in the wrong order when changing the subject of a formula.
$a^0 = 0$.
$3xy$ and $5yx$ are different "types of term" and cannot be "collected" when simplifying expressions.
The square and cube operations on a calculator may not be similar on all makes.
Not using brackets with negative numbers on a calculator.
Not writing down all the digits on the display.

NOTES

Some of this will be a reminder from Key Stage 3 and could be introduced through investigative material such as handshake, frogs etc.
Practise factorisation where more than one variable is involved. NB More complex quadratics are covered in a later unit.
Plenty of practice should be given for factorising, and reinforce the message that making mistakes with negatives and times tables is a different skill to that being developed. Encourage students to expand linear sequences prior to simplifying when dealing with "double brackets".
Emphasise good use of notation.

Students need to realise that not all linear equations can be solved by observation or trial and improvement, and hence the use of a formal method is important.
Students can leave their answer in fraction form where appropriate. Emphasise that fractions are more accurate in calculations than rounded percentage or decimal equivalents.
Use examples involving formulae for circles, spheres, cones and kinematics when changing the subject of a formula.
For substitution use the distance–time–speed formula, and include speed of light given in standard form.
Students should be encouraged to use their calculator effectively by using the replay and ANS/EXE functions; reinforce the use of brackets and only rounding their final answer with trial and improvement.

2b. Sequences (N9, A23, A24, A25)

Teaching time 3-5 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Recognise simple sequences including at the most basic level odd, even, triangular, square and cube numbers and Fibonacci-type sequences;
- Generate sequences of numbers, squared integers and sequences derived from diagrams;
- Describe in words a term-to-term sequence and identify which terms cannot be in a sequence;
- Generate specific terms in a sequence using the position-to-term rule and term-to-term rule;
- Find and use (to generate terms) the nth term of an arithmetic sequence;
- Use the nth term of an arithmetic sequence to decide if a given number is a term in the sequence, or find the first term above or below a given number;
- Identify which terms cannot be in a sequence by finding the nth term;
- Continue a quadratic sequence and use the nth term to generate terms;
- Find the nth term of quadratic sequences;
- Distinguish between arithmetic and geometric sequences;
- Use finite/infinite and ascending/descending to describe sequences;
- Recognise and use simple geometric progressions (rn where n is an integer, and r is a rational number > 0 or a surd);
- Continue geometric progression and find term to term rule, including negative, fraction and decimal terms;
- Solve problems involving sequences from real life situations.

POSSIBLE SUCCESS CRITERIA

Given a sequence, 'which is the 1st term greater than 50?'

Be able to solve problems involving sequences from real-life situations, such as:

- 1 grain of rice on first square, 2 grains on second, 4 grains on third, etc (geometric progression), or person saves £10 one week, £20 the next, £30 the next, etc;
- What is the amount of money after x months saving the same amount, or the height of tree that grows 6 m per year;
- Compare two pocket money options, e.g. same number of £ per week as your age from 5 until 21, or starting with £5 a week aged 5 and increasing by 15% a year until 21.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Evaluate statements about whether or not specific numbers or patterns are in a sequence and justify the reasons.

COMMON MISCONCEPTIONS

Students struggle to relate the position of the term to "n".

NOTES

Emphasise use of $3n$ meaning 3 x n.

Students need to be clear on the description of the pattern in words, the difference between the terms and the algebraic description of the nth term.

UNIT 3: Averages and range, collecting data, representing data

Return to Overview

SPECIFICATION REFERENCES

G14 use standard units of measure and related concepts (length, area, volume/capacity, mass, time, money, etc.)

S1 infer properties of populations or distributions from a sample, while knowing the limitations of sampling

S2 interpret and construct tables, charts and diagrams, including frequency tables, bar charts, pie charts and pictograms for categorical data, vertical line charts for ungrouped discrete numerical data, tables and line graphs for time series data and know their appropriate use

S3 **construct and interpret diagrams for grouped discrete data and continuous data i.e. histograms with equal and unequal class intervals …**

S4 interpret, analyse and compare the distributions of data sets from univariate empirical distributions through:

- appropriate graphical representation involving discrete, continuous and grouped data …
- appropriate measures of central tendency (median, mode and modal class) and spread (range, including consideration of outliers) …

S5 apply statistics to describe a population

S6 use and interpret scatter graphs of bivariate data; recognise correlation and know that it does not indicate causation; draw estimated lines of best fit; make predictions; interpolate and extrapolate apparent trends whilst knowing the dangers of so doing

PRIOR KNOWLEDGE

Students should be able to read scales on graphs, draw circles, measure angles and plot coordinates in the first quadrant.
Students should have experience of tally charts.
Students will have used inequality notation.
Students must be able to find midpoint of two numbers.

KEYWORDS

Mean, median, mode, range, average, discrete, continuous, qualitative, quantitative, data, scatter graph, line of best fit, correlation, positive, negative, sample, population, stem and leaf, frequency, table, sort, pie chart, estimate

3a. Averages and range (G14, S2, S3, S4, S5)

Teaching time
3-5 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Design and use two-way tables for discrete and grouped data;
- Use information provided to complete a two-way table;
- Sort, classify and tabulate data and discrete or continuous quantitative data;
- Calculate mean and range, find median and mode from a small data set;
- Use a spreadsheet to calculate mean and range, and find median and mode;
- Recognise the advantages and disadvantages between measures of average;
- Construct and interpret stem and leaf diagrams (including back-to-back diagrams):
    - find the mode, median, range, as well as the greatest and least values from stem and leaf diagrams, and compare two distributions from stem and leaf diagrams (mode, median, range);
- Calculate the mean, mode, median and range from a frequency table (discrete data);
- Construct and interpret grouped frequency tables for continuous data:
    - for grouped data, find the interval which contains the median and the modal class;
    - estimate the mean with grouped data;
    - understand that the expression 'estimate' will be used where appropriate, when finding the mean of grouped data using mid-interval values.

POSSIBLE SUCCESS CRITERIA

Be able to state the median, mode, mean and range from a small data set.
Extract the averages from a stem and leaf diagram.
Estimate the mean from a table.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Students should be able to provide reasons for choosing to use a specific average to support a point of view.
Given the mean, median and mode of five positive whole numbers, can you find the numbers?
Students should be able to provide a correct solution as a counter-argument to statements involving the "averages", e.g. Susan states that the median is 15, she is wrong. Explain why.

COMMON MISCONCEPTIONS

Students often forget the difference between continuous and discrete data.
Often the $\sum(m \times f)$ is divided by the number of classes rather than $\sum f$ when estimating the mean.

NOTES

Encourage students to cross out the midpoints of each group once they have used these numbers to in $m \times f$. This helps students to avoid summing m instead of f.
Remind students how to find the midpoint of two numbers.
Emphasise that continuous data is measured, i.e. length, weight, and discrete data can be counted, i.e. number of shoes.
Designing and using data collection is no longer in the specification, but may remain a useful topic as part of the overall data handling process.

3b. Representing and interpreting data and scatter graphs (S1, S2, S3, S4, S6)

Teaching time
4-6 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Know which charts to use for different types of data sets;
- Produce and interpret composite bar charts;
- Produce and interpret comparative and dual bar charts;
- Produce and interpret pie charts:
  - find the mode and the frequency represented by each sector;
  - compare data from pie charts that represent different-sized samples;
- Produce and interpret frequency polygons for grouped data:
  - from frequency polygons, read off frequency values, compare distributions, calculate total population, mean, estimate greatest and least possible values (and range);
- Produce frequency diagrams for grouped discrete data:
  - read off frequency values, calculate total population, find greatest and least values;
- Produce histograms with equal class intervals:
  - estimate the median from a histogram with equal class width or any other information, such as the number of people in a given interval;
- Produce line graphs:
  - read off frequency values, calculate total population, find greatest and least values;
- Construct and interpret time–series graphs, comment on trends;
- Compare the mean and range of two distributions, or median or mode as appropriate;
- Recognise simple patterns, characteristics relationships in bar charts, line graphs and frequency polygons;
- Draw and interpret scatter graphs in terms of the relationship between two variables;
- Draw lines of best fit by eye, understanding what these represent;
- Identify outliers and ignore them on scatter graphs;
- Use a line of best fit, or otherwise, to predict values of a variable given values of the other variable;
- Distinguish between positive, negative and zero correlation using lines of best fit, and interpret correlation in terms of the problem;
- Understand that correlation does not imply causality, and appreciate that correlation is a measure of the strength of the association between two variables and that zero correlation does not necessarily imply 'no relationship' but merely 'no linear correlation';
- Explain an isolated point on a scatter graph;
- Use the line of best fit make predictions; interpolate and extrapolate apparent trends whilst knowing the dangers of so doing.

POSSIBLE SUCCESS CRITERIA

Use a time–series data graph to make a prediction about a future value.
Explain why same-size sectors on pie charts with different data sets do not represent the same number of items, but do represent the same proportion.
Make comparisons between two data sets.
Be able to justify an estimate they have made using a line of best fit.
Identify outliers and explain why they may occur.
Given two sets of data in a table, model the relationship and make predictions.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Many real-life situations that give rise to two variables provide opportunities for students to extrapolate and interpret the resulting relationship (if any) between the variables.
Choose which type of graph or chart to use for a specific data set and justify its use.
Evaluate statements in relation to data displayed in a graph/chart.

COMMON MISCONCEPTIONS

Students often forget the difference between continuous and discrete data.
Lines of best fit are often forgotten, but correct answers still obtained by sight.

NOTES

Interquartile range is covered in unit 14.
Misleading graphs are a useful activity for covering AO2 strand 5: Critically evaluate a given way of presenting information.
When doing time–series graphs, use examples from science, geography.
NB Moving averages are not explicitly mentioned in the programme of study but may be worth covering too.
Students need to be constantly reminded of the importance of drawing a line of best fit.
A possible extension includes drawing the line of best fit through the mean point (mean of x, mean of y).

UNIT 4: Fractions, percentages, ratio and proportion

Return to Overview

SPECIFICATION REFERENCES

N2 apply the four operations, including formal written methods, to integers, decimals and simple fractions (proper and improper), and mixed numbers – all both positive and negative; …
N3 recognise and use relationships between operations, including inverse operations (e.g. cancellation to simplify calculations and expressions); use conventional notation for priority of operations, including brackets, powers, roots and reciprocals
N8 calculate exactly with fractions …
N10 work interchangeably with terminating decimals and their corresponding fractions (such as 3.5 and $\frac{7}{2}$ or 0.375 and $\frac{3}{8}$); **change recurring decimals into their corresponding fractions and vice versa**
N11 identify and work with fractions in ratio problems
N12 interpret fractions and percentages as operators
N13 use standard units of mass, length, time, money and other measures (including standard compound measures) using decimal quantities where appropriate
R3 express one quantity as a fraction of another, where the fraction is less than 1 or greater than 1
R4 use ratio notation, including reduction to simplest form
R5 divide a given quantity into two parts in a given part:part or whole:part ratio; express the division of a quantity into two parts as a ratio; apply ratio to real contexts and problems (such as those involving conversion, comparison, scaling, mixing, concentrations)
R6 express a multiplicative relationship between two quantities as a ratio or a fraction
R7 understand and use proportion as equality of ratios
R8 relate ratios to fractions and to linear functions
R9 define percentage as 'number of parts per hundred'; interpret percentages and percentage changes as a fraction or a decimal, and interpret these multiplicatively; express one quantity as a percentage of another; compare two quantities using percentages; work with percentages greater than 100%; solve problems involving percentage change, including percentage increase/decrease, and original value problems and simple interest including in financial mathematics
R10 solve problems involving direct proportion; …

PRIOR KNOWLEDGE

Students should know the four operations of number.
Students should be able to find common factors.
Students should have a basic understanding of fractions as being 'parts of a whole'.
Students can define percentage as 'number of parts per hundred'.
Students are aware that percentages are used in everyday life.

KEYWORDS

Addition, subtraction, multiplication, division, fractions, mixed, improper, recurring, reciprocal, integer, decimal, termination, percentage, VAT, increase, decrease, multiplier, profit, loss, ratio, proportion, share, parts

4a. Fractions and percentages	**Teaching time**
(N2, N3, N8, N10 ,N12, N13, R3, R9)	11-13 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Express a given number as a fraction of another;
- Find equivalent fractions and compare the size of fractions;
- Write a fraction in its simplest form, including using it to simplify a calculation, e.g. $50 \div 20 = \frac{50}{20} = \frac{5}{2} = 2.5$;
- Find a fraction of a quantity or measurement, including within a context;
- Convert a fraction to a decimal to make a calculation easier;
- Convert between mixed numbers and improper fractions;
- Add and subtract fractions, including mixed numbers;
- Multiply and divide fractions, including mixed numbers and whole numbers and vice versa;
- Understand and use unit fractions as multiplicative inverses;
- By writing the denominator in terms of its prime factors, decide whether fractions can be converted to recurring or terminating decimals;
- Convert a fraction to a recurring decimal and vice versa;
- Find the reciprocal of an integer, decimal or fraction;
- Convert between fractions, decimals and percentages;
- Express a given number as a percentage of another number;
- Express one quantity as a percentage of another where the percentage is greater than 100%
- Find a percentage of a quantity;
- Find the new amount after a percentage increase or decrease;
- Work out a percentage increase or decrease, including: simple interest, income tax calculations, value of profit or loss, percentage profit or loss;
- Compare two quantities using percentages, including a range of calculations and contexts such as those involving time or money;
- Find a percentage of a quantity using a multiplier and use a multiplier to increase or decrease by a percentage in any scenario where percentages are used;
- Find the original amount given the final amount after a percentage increase or decrease (reverse percentages), including VAT;
- Use calculators for reverse percentage calculations by doing an appropriate division;
- Use percentages in real-life situations, including percentages greater than 100%;
- Describe percentage increase/decrease with fractions, e.g. 150% increase means $2\frac{1}{2}$ times as big;
- Understand that fractions are more accurate in calculations than rounded percentage or decimal equivalents, and choose fractions, decimals or percentages appropriately for calculations.

POSSIBLE SUCCESS CRITERIA

Express a given number as a fraction of another, including where the fraction is, for example, greater than 1, e.g. $\frac{120}{100} = 1\frac{2}{10} = 1\frac{1}{5}$.

Answer the following: James delivers 56 newspapers. $\frac{3}{8}$ of the newspapers have a magazine. How many of the newspapers have a magazine?
Prove whether a fraction is terminating or recurring.
Convert a fraction to a decimal including where the fraction is greater than 1.
Be able to work out the price of a deposit, given the price of a sofa is £480 and the deposit is 15% of the price, without a calculator.
Find fractional percentages of amounts, with and without using a calculator.

Convince me that 0.125 is $\frac{1}{8}$.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Many of these topics provide opportunities for reasoning in real-life contexts, particularly percentages:

- Calculate original values and evaluate statements in relation to this value justifying which statement is correct.

COMMON MISCONCEPTIONS

The larger the denominator, the larger the fraction.

Incorrect links between fractions and decimals, such as thinking that $\frac{1}{5}$ = 0.15, 5% = 0.5, 4% = 0.4, etc.
It is not possible to have a percentage greater than 100%.

NOTES

Ensure that you include fractions where only one of the denominators needs to be changed, in addition to where both need to be changed for addition and subtraction.
Include multiplying and dividing integers by fractions.
Use a calculator for changing fractions into decimals and look for patterns.
Recognise that every terminating decimal has its fraction with a 2 and/or 5 as a common factor in the denominator.
Use long division to illustrate recurring decimals.
Amounts of money should always be rounded to the nearest penny.
Encourage use of the fraction button.
Students should be reminded of basic percentages.
Amounts of money should always be rounded to the nearest penny, except where successive calculations are done (i.e. compound interest, which is covered in a later unit).
Emphasise the use of percentages in real-life situations.

4b. Ratio and proportion — Teaching time
(N11, N12, N13, R3, R4, R5, R6, R7, R8, R10) — 5-7 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Express the division of a quantity into a number parts as a ratio;
- Write ratios in form 1 : m or m : 1 and to describe a situation;
- Write ratios in their simplest form, including three-part ratios;
- Divide a given quantity into two or more parts in a given part : part or part : whole ratio;
- Use a ratio to find one quantity when the other is known;
- Write a ratio as a fraction and as a linear function;
- Identify direct proportion from a table of values, by comparing ratios of values;
- Use a ratio to compare a scale model to real-life object;
- Use a ratio to convert between measures and currencies, e.g. £1.00 = €1.36;
- Scale up recipes;
- Convert between currencies.

POSSIBLE SUCCESS CRITERIA

Write/interpret a ratio to describe a situation such as 1 blue for every 2 red …, 3 adults for every 10 children …
Recognise that two paints mixed red to yellow 5 : 4 and 20 : 16 are the same colour.
When a quantity is split in the ratio 3:5, what fraction does each person get?
Find amounts for three people when amount for one given.
Express the statement 'There are twice as many girls as boys' as the ratio 2 : 1 or the linear function $y = 2x$, where x is the number of boys and y is the number of girls.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Problems involving sharing in a ratio that include percentages rather than specific numbers such can provide links with other areas of Mathematics:

> In a youth club the ratio of the number of boys to the number of girls is 3 : 2 . 30% of the boys are under the age of 14 and 60% of the girls are under the age of 14. What percentage of the youth club is under the age of 14?

COMMON MISCONCEPTIONS

Students often identify a ratio-style problem and then divide by the number given in the question, without fully understanding the question.

NOTES

Three-part ratios are usually difficult for students to understand.
Also include using decimals to find quantities.
Use a variety of measures in ratio and proportion problems.
Include metric to imperial and vice versa, but give them the conversion factor, e.g. 5 miles = 8 km, 1 inch = 2.4 cm – these aren't specifically in the programme of study but are still useful.

UNIT 5: Angles, polygons, parallel lines; Right-angled triangles: Pythagoras and trigonometry

Return to Overview

SPECIFICATION REFERENCES

N7 Calculate with roots and with integer **and fractional** indices
N8 calculate exactly with fractions and **surds** …
N15 round numbers and measures to an appropriate degree of accuracy (e.g. to a specified number of decimal places or significant figures); …
A4 simplify and manipulate algebraic expressions (including those involving surds) by collecting like terms …
G1 use conventional terms and notations: points, lines, vertices, edges, planes, parallel lines, perpendicular lines, right angles, polygons, regular polygons and polygons with reflection and/or rotation symmetries; …
G3 … understand and use alternate and corresponding angles on parallel lines; derive and use the sum of angles in a triangle (e.g. to deduce and use the angle sum in any polygon, and to derive properties of regular polygons)
G4 derive and apply the properties and definitions of: special types of quadrilaterals, including square, rectangle, parallelogram, trapezium, kite and rhombus; …
G6 apply angle facts, triangle congruence, similarity and properties of quadrilaterals to conjecture and derive results about angles and sides, including Pythagoras' theorem and the fact that the base angles of an isosceles triangle are equal, and use known results to obtain simple proofs
G11 solve geometrical problems on coordinate axes
G20 know the formulae for: Pythagoras' theorem $a^2 + b^2 = c^2$, and the trigonometric ratios sine, cosine and tan; apply them to find angles and lengths in right-angled triangles … and in two dimensional figures
G21 know the exact values of sin θ and cos θ for θ = 0°, 30°, 45°, 60° and 90°; know the exact value of tan θ for θ = 0°, 30°, 45° and 60°

PRIOR KNOWLEDGE

Students should be able to rearrange simple formulae and equations, as preparation for rearranging trig formulae.
Students should recall basic angle facts.
Students should understand that fractions are more accurate in calculations than rounded percentage or decimal equivalents.

KEYWORDS

Quadrilateral, angle, polygon, interior, exterior, proof, tessellation, symmetry, parallel, corresponding, alternate, co-interior, vertices, edge, face, sides, Pythagoras' Theorem, sine, cosine, tan, trigonometry, opposite, hypotenuse, adjacent, ratio, elevation, depression, segment, length

5a. Polygons, angles and parallel lines (G1, G3, G4, G6, G11)

Teaching time
5-7 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Classify quadrilaterals by their geometric properties and distinguish between scalene, isosceles and equilateral triangles;
- Understand 'regular' and 'irregular' as applied to polygons;
- Understand the proof that the angle sum of a triangle is 180°, and derive and use the sum of angles in a triangle;
- Use symmetry property of an isosceles triangle to show that base angles are equal;
- Find missing angles in a triangle using the angle sum in a triangle AND the properties of an isosceles triangle;
- Understand a proof of, and use the fact that, the exterior angle of a triangle is equal to the sum of the interior angles at the other two vertices;
- Explain why the angle sum of a quadrilateral is 360°; use the angle properties of quadrilaterals and the fact that the angle sum of a quadrilateral is 360°;
- Understand and use the angle properties of parallel lines and find missing angles using the properties of corresponding and alternate angles, giving reasons;
- Use the angle sums of irregular polygons;
- Calculate and use the sums of the interior angles of polygons, use the sum of angles in a triangle to deduce and use the angle sum in any polygon and to derive the properties of regular polygons;
- Use the sum of the exterior angles of any polygon is 360°;
- Use the sum of the interior angles of an n-sided polygon;
- Use the sum of the interior angle and the exterior angle is 180°;
- Find the size of each interior angle, or the size of each exterior angle, or the number of sides of a regular polygon, and use the sum of angles of irregular polygons;
- Calculate the angles of regular polygons and use these to solve problems;
- Use the side/angle properties of compound shapes made up of triangles, lines and quadrilaterals, including solving angle and symmetry problems for shapes in the first quadrant, more complex problems and using algebra;
- Use angle facts to demonstrate how shapes would 'fit together', and work out interior angles of shapes in a pattern.

POSSIBLE SUCCESS CRITERIA

Name all quadrilaterals that have a specific property.
Given the size of its exterior angle, how many sides does the polygon have?
What is the same, and what is different between families of polygons?

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Multi-step "angle chasing"-style problems that involve justifying how students have found a specific angle will provide opportunities to develop a chain of reasoning.
Geometrical problems involving algebra whereby equations can be formed and solved allow students the opportunity to make and use connections with different parts of mathematics.

COMMON MISCONCEPTIONS

Some students will think that all trapezia are isosceles, or a square is only square if 'horizontal', or a 'non-horizontal' square is called a diamond.

Pupils may believe, incorrectly, that:

- perpendicular lines have to be horizontal/vertical;
- all triangles have rotational symmetry of order 3;
- all polygons are regular.

Incorrectly identifying the 'base angles' (i.e. the equal angles) of an isosceles triangle when not drawn horizontally.

NOTES

Demonstrate that two line segments that do not meet could be perpendicular – if they are extended and they would meet at right angles.

Students must be encouraged to use geometrical language appropriately, 'quote' the appropriate reasons for angle calculations and show step-by-step deduction when solving multi-step problems.

Emphasise that diagrams in examinations are seldom drawn accurately.

Use tracing paper to show which angles in parallel lines are equal.

Students must use co-interior, not supplementary, to describe paired angles inside parallel lines. (NB Supplementary angles are any angles that add to 180, not specifically those in parallel lines.)

Use triangles to find angle sums of polygons; this could be explored algebraically as an investigation.

5b. Pythagoras' Theorem and trigonometry (A4, N7, N8, N15, G6, G20, G21)

Teaching time
5-7 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Understand, recall and use Pythagoras' Theorem in 2D;
- Given three sides of a triangle, justify if it is right-angled or not;
- Calculate the length of the hypotenuse in a right-angled triangle (including decimal lengths and a range of units);
- Find the length of a shorter side in a right-angled triangle;
- Calculate the length of a line segment AB given pairs of points;
- Give an answer to the use of Pythagoras' Theorem in surd form;
- Understand, use and recall the trigonometric ratios sine, cosine and tan, and apply them to find angles and lengths in general triangles in 2D figures;
- Use the trigonometric ratios to solve 2D problems;
- Find angles of elevation and depression;
- Know the exact values of sin θ and cos θ for θ = 0°, 30°, 45°, 60° and 90°; know the exact value of tan θ for θ = 0°, 30°, 45° and 60°.

POSSIBLE SUCCESS CRITERIA

Does 2, 3, 6 give a right-angled triangle?
Justify when to use Pythagoras' Theorem and when to use trigonometry.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Combined triangle problems that involve consecutive application of Pythagoras' Theorem or a combination of Pythagoras' Theorem and the trigonometric ratios.
In addition to abstract problems, students should be encouraged to apply Pythagoras' Theorem and/or the trigonometric ratios to real-life scenarios that require them to evaluate whether their answer fulfils certain criteria, e.g. the angle of elevation of 6.5 m ladder cannot exceed 65°. What is the greatest height it can reach?

COMMON MISCONCEPTIONS

Answers may be displayed on a calculator in surd form.
Students forget to square root their final answer, or round their answer prematurely.

NOTES

Students may need reminding about surds.
Drawing the squares on the three sides will help when deriving the rule.
Scale drawings are not acceptable.
Calculators need to be in degree mode.
To find in right-angled triangles the exact values of sin θ and cos θ for θ = 0°, 30°, 45°, 60° and 90°, use triangles with angles of 30°, 45° and 60°.
Use a suitable mnemonic to remember SOHCAHTOA.
Use Pythagoras' Theorem and trigonometry together.

UNIT 6: Real-life and algebraic linear graphs, quadratic and cubic graphs, the equation of a circle, plus rates of change and area under graphs made from straight lines

Return to Overview

SPECIFICATION REFERENCES

N13 use standard units of mass, length, time, money and other measures (including standard compound measures) using decimal quantities where appropriate

N15 round numbers and measures to an appropriate degree of accuracy (e.g. to a specified number of decimal places or significant figures); …

A8 work with coordinates in all four quadrants

A9 plot graphs of equations that correspond to straight-line graphs in the coordinate plane; use the form $y = mx + c$ to identify parallel **and perpendicular lines**; find the equation of the line through two given points, or through one point with a given gradient

A10 identify and interpret gradients and intercepts of linear functions graphically and algebraically

A11 identify and interpret roots, intercepts, turning points of quadratic functions graphically; …

A12 recognise, sketch and interpret graphs of linear functions, quadratic functions, simple cubic functions, the reciprocal function $y = \frac{1}{x}$ with $x \neq 0$, …

A14 plot and interpret … graphs of non-standard functions in real contexts to find approximate solutions to problems such as simple kinematic problems involving distance, speed and acceleration

A15 **calculate or estimate gradients of graphs and areas under graphs (including quadratic and non-linear graphs) and interpret results in cases such as distance–time graphs, velocity–time graphs … (this does not include calculus)**

A16 **recognise and use the equation of a circle with centre at the origin; find the equation of a tangent to a circle at a given point**

A17 solve linear equations in one unknown … (including those with the unknown on both sides of the equation); find approximate solutions using a graph

R1 change freely between related standard units (e.g. time, length, area, volume/capacity, mass) and compound units (e.g. speed, rates of pay, prices, density, pressure) in numerical and algebraic contexts

R8 relate ratios to fractions and to linear functions

R10 solve problems involving direct … proportion, including graphical … representations

R11 use compound units such as speed, … unit pricing, …

PRIOR KNOWLEDGE

Students can identify coordinates of given points in the first quadrant or all four quadrants.
Students can use Pythagoras' Theorem and calculate the area of compound shapes.
Students can use and draw conversion graphs for these units.
Students can use function machines and inverse operations.

KEYWORDS

Coordinate, axes, 3D, Pythagoras, graph, speed, distance, time, velocity, quadratic, solution, root, function, linear, circle, cubic, approximate, gradient, perpendicular, parallel, equation

6a. Graphs: the basics and real-life graphs (N13, N15, A8, A10, A14, A15, R1, R11)

Teaching time
5-7 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Identify and plot points in all four quadrants;
- Draw and interpret straight-line graphs for real-life situations, including ready reckoner graphs, conversion graphs, fuel bills, fixed charge and cost per item;
- Draw distance–time and velocity–time graphs;
- Use graphs to calculate various measures (of individual sections), including: unit price (gradient), average speed, distance, time, acceleration; including using enclosed areas by counting squares or using areas of trapezia, rectangles and triangles;
- Find the coordinates of the midpoint of a line segment with a diagram given and coordinates;
- Find the coordinates of the midpoint of a line segment from coordinates;
- Calculate the length of a line segment given the coordinates of the end points;
- Find the coordinates of points identified by geometrical information.
- Find the equation of the line through two given points.

POSSIBLE SUCCESS CRITERIA

Interpret a description of a journey into a distance–time or speed–time graph.
Calculate various measures given a graph.
Calculate an end point of a line segment given one coordinate and its midpoint.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Speed/distance graphs can provide opportunities for interpreting non-mathematical problems as a sequence of mathematical processes, whilst also requiring students to justify their reasons why one vehicle is faster than another.
Calculating the length of a line segment provides links with other areas of mathematics.

COMMON MISCONCEPTIONS

Where line segments cross the y-axis, finding midpoints and lengths of segments is particularly challenging as students have to deal with negative numbers.

NOTES

Careful annotation should be encouraged: it is good practice to label the axes and check that students understand the scales.
Use various measures in the distance–time and velocity–time graphs, including miles, kilometres, seconds, and hours, and include large numbers in standard form.
Ensure that you include axes with negative values to represent, for example, time before present time, temperature or depth below sea level.
Metric-to-imperial measures are not specifically included in the programme of study, but it is a useful skill and ideal for conversion graphs.
Emphasise that velocity has a direction.
Coordinates in 3D can be used to extend students.

6b. Linear graphs and coordinate geometry (A9, A10, A12, A17, R8, R10)

Teaching time 7-9 hours

OBJECTIVES

By the end of the unit, students should be able to:

- Plot and draw graphs of $y = a$, $x = a$, $y = x$ and $y = -x$, drawing and recognising lines parallel to axes, plus $y = x$ and $y = -x$;
- Identify and interpret the gradient of a line segment;
- Recognise that equations of the form $y = mx + c$ correspond to straight-line graphs in the coordinate plane;
- Identify and interpret the gradient and y-intercept of a linear graph given by equations of the form $y = mx + c$;
- Find the equation of a straight line from a graph in the form $y = mx + c$;
- Plot and draw graphs of straight lines of the form $y = mx + c$ with and without a table of values;
- Sketch a graph of a linear function, using the gradient and y-intercept (i.e. without a table of values);
- Find the equation of the line through one point with a given gradient;
- Identify and interpret gradient from an equation $ax + by = c$;
- Find the equation of a straight line from a graph in the form $ax + by = c$;
- Plot and draw graphs of straight lines in the form $ax + by = c$;
- Interpret and analyse information presented in a range of linear graphs:
  - use gradients to interpret how one variable changes in relation to another;
  - find approximate solutions to a linear equation from a graph;
  - identify direct proportion from a graph;
  - find the equation of a line of best fit (scatter graphs) to model the relationship between quantities;
- Explore the gradients of parallel lines and lines perpendicular to each other;
- Interpret and analyse a straight-line graph and generate equations of lines parallel and perpendicular to the given line;
- Select and use the fact that when $y = mx + c$ is the equation of a straight line, then the gradient of a line parallel to it will have a gradient of m and a line perpendicular to this line will have a gradient of $-\frac{1}{m}$.

POSSIBLE SUCCESS CRITERIA

Find the equation of the line passing through two coordinates by calculating the gradient first.
Understand that the form $y = mx + c$ or $ax + by = c$ represents a straight line.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Given an equation of a line provide a counter argument as to whether or not another equation of a line is parallel or perpendicular to the first line.
Decide if lines are parallel or perpendicular without drawing them and provide reasons.

COMMON MISCONCEPTIONS

Students can find visualisation of a question difficult, especially when dealing with gradients resulting from negative coordinates.

NOTES

Encourage students to sketch what information they are given in a question – emphasise that it is a sketch.
Careful annotation should be encouraged – it is good practice to label the axes and check that students understand the scales.

6c. Quadratic, cubic and other graphs (A11, A12, A16) | **Teaching time** 5-7 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Recognise a linear, quadratic, cubic, reciprocal and circle graph from its shape;
- Generate points and plot graphs of simple quadratic functions, then more general quadratic functions;
- Find approximate solutions of a quadratic equation from the graph of the corresponding quadratic function;
- Interpret graphs of quadratic functions from real-life problems;
- Draw graphs of simple cubic functions using tables of values;
- Interpret graphs of simple cubic functions, including finding solutions to cubic equations;
- Draw graphs of the reciprocal function $y = \frac{1}{x}$ with $x \neq 0$ using tables of values;
- Draw circles, centre the origin, equation $x^2 + y^2 = r^2$.

POSSIBLE SUCCESS CRITERIA

Select and use the correct mathematical techniques to draw linear, quadratic, cubic and reciprocal graphs.
Identify a variety of functions by the shape of the graph.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Match equations of quadratics and cubics with their graphs by recognising the shape or by sketching.

COMMON MISCONCEPTIONS

Students struggle with the concept of solutions and what they represent in concrete terms.

NOTES

Use lots of practical examples to help model the quadratic function, e.g. draw a graph to model the trajectory of a projectile and predict when/where it will land.
Ensure axes are labelled and pencils used for drawing.
Graphical calculations or appropriate ICT will allow students to see the impact of changing variables within a function.

UNIT 7: Perimeter, area and volume, plane shapes and prisms, circles, cylinders, spheres, cones; Accuracy and bounds

Return to Overview

SPECIFICATION REFERENCES

N8 calculate exactly with … multiples of π; …

N14 estimate answers; check calculations using approximation and estimation, including answers obtained using technology

N15 round numbers and measures to an appropriate degree of accuracy (e.g. to a specified number of decimal places or significant figures); use inequality notation to specify simple error intervals due to truncation or rounding

N16 apply and interpret limits of accuracy, **including upper and lower bounds**

R1 change freely between related standard units (e.g. time, length, area, volume/capacity, mass) … in numerical and algebraic contexts

G1 use conventional terms and notations: points, lines, vertices, edges, planes, parallel lines, perpendicular lines, right angles, polygons, regular polygons and polygons with reflection and/or rotation symmetries; …

G9 identify and apply circle definitions and properties, including: centre, radius, chord, diameter, circumference, tangent, arc, sector and segment

G12 identify properties of the faces, surfaces, edges and vertices of: cubes, cuboids, prisms, cylinders, pyramids, cones and spheres

G13 construct and interpret plans and elevations of 3D shapes.

G14 use standard units of measure and related concepts (length, area, volume/capacity, mass, time, money, etc)

G16 know and apply formulae to calculate: area of triangles, parallelograms, trapezia; volume of cuboids and other right prisms (including cylinders)

G17 know the formulae: circumference of a circle = $2\pi r = \pi d$, area of a circle = πr^2; calculate: perimeters of 2D shapes, including circles; areas of circles and composite shapes; surface area and volume of spheres, pyramids, cones and composite solids

G18 calculate arc lengths, angles and areas of sectors of circles

PRIOR KNOWLEDGE

Students should know the names and properties of 3D forms.
The concept of perimeter and area by measuring lengths of sides will be familiar to students.
Students should be able to substitute numbers into an equation and give answers to an appropriate degree of accuracy.
Students should know the various metric units.

KEYWORDS

Triangle, rectangle, parallelogram, trapezium, area, perimeter, formula, length, width, prism, compound, measurement, polygon, cuboid, volume, nets, isometric, symmetry, vertices, edge, face, circle, segment, arc, sector, cylinder, circumference, radius, diameter, pi, composite, sphere, cone, capacity, hemisphere, segment, frustum, bounds, accuracy, surface area

7a. Perimeter, area and circles (N8, N14, N15, R1, G1, G9, G14, G16, G17, G18)

Teaching time
4-6 hours

OBJECTIVES

By the end of the unit, students should be able to:

- Recall and use the formulae for the area of a triangle, rectangle, trapezium and parallelogram using a variety of metric measures;
- Calculate the area of compound shapes made from triangles, rectangles, trapezia and parallelograms using a variety of metric measures;
- Find the perimeter of a rectangle, trapezium and parallelogram using a variety of metric measures;
- Calculate the perimeter of compound shapes made from triangles and rectangles;
- Estimate area and perimeter by rounding measurements to 1 significant figure to check reasonableness of answers;
- Recall the definition of a circle and name and draw parts of a circle;
- Recall and use formulae for the circumference of a circle and the area enclosed by a circle (using circumference = $2\pi r = \pi d$ and area of a circle = πr^2) using a variety of metric measures;
- Use $\pi \approx 3.142$ or use the π button on a calculator;
- Calculate perimeters and areas of composite shapes made from circles and parts of circles (including semicircles, quarter-circles, combinations of these and also incorporating other polygons);
- Calculate arc lengths, angles and areas of sectors of circles;
- Find radius or diameter, given area or circumference of circles in a variety of metric measures;
- Give answers in terms of π;
- Form equations involving more complex shapes and solve these equations.

POSSIBLE SUCCESS CRITERIA

Calculate the area and/or perimeter of shapes with different units of measurement.

Understand that answers in terms of π are more accurate.

Calculate the perimeters and/or areas of circles, semicircles and quarter-circles given the radius or diameter and vice versa.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Using compound shapes or combinations of polygons that require students to subsequently interpret their result in a real-life context.

Know the impact of estimating their answers and whether it is an overestimate or underestimate in relation to a given context.

Multi-step problems, including the requirement to form and solve equations, provide links with other areas of mathematics.

COMMON MISCONCEPTIONS

Students often get the concepts of area and perimeter confused.

Shapes involving missing lengths of sides often result in incorrect answers.

Diameter and radius are often confused, and recollection of area and circumference of circles involves incorrect radius or diameter.

NOTES

Encourage students to draw a sketch where one isn't provided.
Emphasise the functional elements with carpets, tiles for walls, boxes in a larger box, etc. Best value and minimum cost can be incorporated too.
Ensure that examples use different metric units of length, including decimals.
Emphasise the need to learn the circle formulae; "Cherry Pie's Delicious" and "Apple Pies are too" are good ways to remember them.
Ensure that students know it is more accurate to leave answers in terms of π, but only when asked to do so.

7b. 3D forms and volume, cylinders, cones and spheres
(N8, N15, G12, G13, G14, G16, G17)

Teaching time
6-8 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Find the surface area of prisms using the formulae for triangles and rectangles, and other (simple) shapes with and without a diagram;
- Draw sketches of 3D solid and identify planes of symmetry of 3D solids, and sketch planes of symmetry;
- Recall and use the formula for the volume of a cuboid or prism made from composite 3D solids using a variety of metric measures;
- Convert between metric measures of volume and capacity, e.g. 1 ml = 1 cm^3;
- Use volume to solve problems;
- Estimating surface area, perimeter and volume by rounding measurements to 1 significant figure to check reasonableness of answers;
- Use $\pi \approx 3.142$ or use the π button on a calculator;
- Find the volume and surface area of a cylinder;
- Recall and use the formula for volume of pyramid;
- Find the surface area of a pyramid;
- Use the formulae for volume and surface area of spheres and cones;
- Solve problems involving more complex shapes and solids, including segments of circles and frustums of cones;
- Find the surface area and volumes of compound solids constructed from cubes, cuboids, cones, pyramids, spheres, hemispheres, cylinders;
- Give answers in terms of π;
- Form equations involving more complex shapes and solve these equations.

POSSIBLE SUCCESS CRITERIA

Given dimensions of a rectangle and a pictorial representation of it when folded, work out the dimensions of the new shape.
Work out the length given the area of the cross-section and volume of a cuboid.
Understand that answers in terms of π are more accurate.
Given two solids with the same volume and the dimensions of one, write and solve an equation in terms of π to find the dimensions of the other, e.g. a sphere is melted down to make ball bearings of a given radius, how many will it make?

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Combinations of 3D forms such as a cone and a sphere where the radius has to be calculated given the total height.

COMMON MISCONCEPTIONS

Students often get the concepts of surface area and volume confused.

NOTES

Encourage students to draw a sketch where one isn't provided.
Use lots of practical examples to ensure that students can distinguish between surface area and volume. Making solids using multi-link cubes can be useful.
Solve problems including examples of solids in everyday use.
Scaffold drawing 3D shapes by initially using isometric paper.
Whilst not an explicit objective, it is useful for students to draw and construct nets and show how they fold to make 3D solids, allowing students to make the link between 3D shapes and their nets. This will enable students to understand that there is often more than one net that can form a 3D shape.
Formulae for curved surface area and volume of a sphere, and surface area and volume of a cone will be given on the formulae page of the examinations.
Ensure that students know it is more accurate to leave answers in terms of π but only when asked to do so.

7c. Accuracy and bounds (N15, N16)

Teaching time 4-6 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Calculate the upper and lowers bounds of numbers given to varying degrees of accuracy;
- Calculate the upper and lower bounds of an expression involving the four operations;
- Find the upper and lower bounds in real-life situations using measurements given to appropriate degrees of accuracy;
- Find the upper and lower bounds of calculations involving perimeters, areas and volumes of 2D and 3D shapes;
- Calculate the upper and lower bounds of calculations, particularly when working with measurements;
- Use inequality notation to specify an error bound.

POSSIBLE SUCCESS CRITERIA

Round 16,000 people to the nearest 1000.
Round 1100 g to 1 significant figure.
Work out the upper and lower bounds of a formula where all terms are given to 1 decimal place.
Be able to justify that measurements to the nearest whole unit may be inaccurate by up to one half in either direction.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

This sub-unit provides many opportunities for students to evaluate their answers and provide counter-arguments in mathematical and real-life contexts, in addition to requiring them to understand the implications of rounding their answers.

COMMON MISCONCEPTIONS

Students readily accept the rounding for lower bounds, but take some convincing in relation to upper bounds.

NOTES

Students should use 'half a unit above' and 'half a unit below' to find upper and lower bounds.
Encourage use a number line when introducing the concept.

UNIT 8: Transformations; Constructions: triangles, nets, plan and elevation, loci, scale drawings and bearings

Return to Overview

SPECIFICATION REFERENCES

R2 use scale factors, scale diagrams and maps
R6 express a multiplicative relationship between two quantities as a ratio or a fraction
G1 … draw diagrams from written description
G2 use the standard ruler and compass constructions (perpendicular bisector of a line segment, constructing a perpendicular to a given line from/at a given point, bisecting a given angle); use these to construct given figures and solve loci problems; know that the perpendicular distance from a point to a line is the shortest distance to the line
G3 apply the properties of angles at a point, angles at a point on a straight line, vertically opposite angles; understand and use alternate and corresponding angles on parallel lines; derive and use the sum of angles in a triangle (e.g. to deduce and use the angle sum in any polygon, and to derive properties of regular polygons)
G5 use the basic congruence criteria for triangles (SSS, SAS, ASA, RHS)
G6 apply angle facts, triangle congruence, similarity and properties of quadrilaterals to conjecture and derive results about angles and sides, …
G7 identify, describe and construct congruent and similar shapes, including on a coordinate axis, by considering rotation, reflection, translation and enlargement (including fractional **and negative** scale factors)
G8 **describe the changes and invariance achieved by combinations of rotations, reflections and translations**
G12 identify properties of the faces, surfaces, edges and vertices of: cubes, cuboids, prisms, cylinders, pyramids, cones and spheres
G13 construct and interpret plans and elevations of 3D shapes
G15 measure line segments and angles in geometric figures, including interpreting maps and scale drawings and use of bearings
G19 apply the concepts of congruence and similarity, including the relationships between lengths … in similar figures
G24 describe translations as 2D vectors
G25 apply addition and subtraction of vectors, multiplication of vectors by a scalar, and diagrammatic and column representations of vectors; …

PRIOR KNOWLEDGE

Students should be able to recognise 2D shapes.
Students should be able to plot coordinates in four quadrants and linear equations parallel to the coordinate axes.

KEYWORDS

Rotation, reflection, translation, transformation, enlargement, scale factor, vector, centre, angle, direction, mirror line, centre of enlargement, describe, distance, congruence, similar, combinations, single, corresponding, constructions, compasses, protractor, bisector, bisect, line segment, perpendicular, loci, bearing

8a. Transformations | **Teaching time**
(R6, G5, G6, G7, G8, G24, G25) | 5-7 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Distinguish properties that are preserved under particular transformations;
- Recognise and describe rotations – know that that they are specified by a centre and an angle;
- Rotate 2D shapes using the origin or any other point (not necessarily on a coordinate grid);
- Identify the equation of a line of symmetry;
- Recognise and describe reflections on a coordinate grid – know to include the mirror line as a simple algebraic equation, $x = a$, $y = a$, $y = x$, $y = -x$ and lines not parallel to the axes;
- Reflect 2D shapes using specified mirror lines including lines parallel to the axes and also $y = x$ and $y = -x$;
- Recognise and describe single translations using column vectors on a coordinate grid;
- Translate a given shape by a vector;
- Understand the effect of one translation followed by another, in terms of column vectors (to introduce vectors in a concrete way);
- Enlarge a shape on a grid without a centre specified;
- Describe and transform 2D shapes using enlargements by a positive integer, positive fractional, and negative scale factor;
- Know that an enlargement on a grid is specified by a centre and a scale factor;
- Identify the scale factor of an enlargement of a shape;
- Enlarge a given shape using a given centre as the centre of enlargement by counting distances from centre, and find the centre of enlargement by drawing;
- Find areas after enlargement and compare with before enlargement, to deduce multiplicative relationship (area scale factor); given the areas of two shapes, one an enlargement of the other, find the scale factor of the enlargement (whole number values only);
- Use congruence to show that translations, rotations and reflections preserve length and angle, so that any figure is congruent to its image under any of these transformations;
- Describe and transform 2D shapes using combined rotations, reflections, translations, or enlargements;
- Describe the changes and invariance achieved by combinations of rotations, reflections and translations.

POSSIBLE SUCCESS CRITERIA

Recognise similar shapes because they have equal corresponding angles and/or sides scaled up in same ratio.
Understand that translations are specified by a distance and direction (using a vector).
Recognise that enlargements preserve angle but not length.
Understand that distances and angles are preserved under rotations, reflections and translations so that any shape is congruent to its image.
Understand that similar shapes are enlargements of each other and angles are preserved.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Students should be given the opportunity to explore the effect of reflecting in two parallel mirror lines and combining transformations.

COMMON MISCONCEPTIONS

Students often use the term 'transformation' when describing transformations instead of the required information.
Lines parallel to the coordinate axes often get confused.

NOTES

Emphasise the need to describe the transformations fully, and if asked to describe a 'single' transformation students should not include two types.
Find the centre of rotation, by trial and error and by using tracing paper. Include centres on or inside shapes.
Area of similar shapes is covered in unit 12.

8b. Constructions, loci and bearings (R2, G1, G2, G3, G12, G13, G15, G19)

Teaching time 6-8 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Draw 3D shapes using isometric grids;
- Understand and draw front and side elevations and plans of shapes made from simple solids;
- Given the front and side elevations and the plan of a solid, draw a sketch of the 3D solid;
- Use and interpret maps and scale drawings, using a variety of scales and units;
- Read and construct scale drawings, drawing lines and shapes to scale;
- Estimate lengths using a scale diagram;
- Understand, draw and measure bearings;
- Calculate bearings and solve bearings problems, including on scaled maps, and find/mark and measure bearings
- Use the standard ruler and compass constructions:
  - bisect a given angle;
  - construct a perpendicular to a given line from/at a given point;
  - construct angles of 90°, 45°;
  - perpendicular bisector of a line segment;
- Construct:
  - a region bounded by a circle and an intersecting line;
  - a given distance from a point and a given distance from a line;
  - equal distances from two points or two line segments;
  - regions which may be defined by 'nearer to' or 'greater than';
- Find and describe regions satisfying a combination of loci, including in 3D;
- Use constructions to solve loci problems including with bearings;
- Know that the perpendicular distance from a point to a line is the shortest distance to the line.

POSSIBLE SUCCESS CRITERIA

Able to read and construct scale drawings.
When given the bearing of a point A from point B, can work out the bearing of B from A.
Know that scale diagrams, including bearings and maps, are 'similar' to the real-life examples.
Able to sketch the locus of point on a vertex of a rotating shape as it moves along a line, of a point on the circumference and at the centre of a wheel.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Interpret a given plan and side view of a 3D form to be able to produce a sketch of the form.
Problems involving combinations of bearings and loci can provide a rich opportunity to link with other areas of mathematics and allow students to justify their findings.

COMMON MISCONCEPTIONS

Correct use of a protractor may be an issue.

NOTES

Drawings should be done in pencil.
Relate loci problems to real-life scenarios, including mobile phone masts and coverage.
Construction lines should not be erased.

UNIT 9: Algebra: Solving quadratic equations and inequalities, solving simultaneous equations algebraically

Return to Overview

SPECIFICATION REFERENCES

N1 order positive and negative integers, decimals and fractions; use the symbols =, ≠, <, >, ≤, ≥

N8 calculate exactly with … **surds**; … **simplify surd expressions involving squares (e.g. √12 = √(4 × 3) = √4 × √3 = 2√3)**

A4 simplify and manipulate algebraic expressions (including those involving surds …) by: … **factorising quadratic expressions of the form $ax^2 + bx + c$**

A9 … find the equation of the line through two given points, or through one point with a given gradient

A11 identify and interpret roots … of quadratic functions algebraically …

A18 solve quadratic equations **(including those that require rearrangement)** algebraically by factorising, **by completing the square and by using the quadratic formula**; …

A19 solve two simultaneous equations in two variables (linear/linear **or linear/quadratic**) algebraically; find approximate solutions using a graph

A21 … derive an equation (or two simultaneous equations), solve the equation(s) and interpret the solution.

A22 solve linear inequalities in one **or two** variable**(s)**, **and quadratic inequalities in one variable**; represent the solution set on a number line, **using set notation and on a graph**

PRIOR KNOWLEDGE

Students should understand the ≥ and ≤ symbols.
Students can substitute into, solve and rearrange linear equations.
Students should be able to factorise simple quadratic expressions.
Students should be able to recognise the equation of a circle.

KEYWORDS

Quadratic, solution, root, linear, solve, simultaneous, inequality, completing the square, factorise, rearrange, surd, function, solve, circle, sets, union, intersection

9a. Solving quadratics and simultaneous equations (N8, A4, A9, A11, A18, A19, A21)

Teaching time
6-8 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Factorise quadratic expressions in the form $ax^2 + bx + c$;
- Set up and solve quadratic equations;
- Solve quadratic equations by factorisation and completing the square;
- Solve quadratic equations that need rearranging;
- Solve quadratic equations by using the quadratic formula;
- Find the exact solutions of two simultaneous equations in two unknowns;
- Use elimination or substitution to solve simultaneous equations;
- Solve exactly, by elimination of an unknown, two simultaneous equations in two unknowns:
    - linear / linear, including where both need multiplying;
    - linear / quadratic;
    - linear / $x^2 + y^2 = r^2$;
- Set up and solve a pair of simultaneous equations in two variables for each of the above scenarios, including to represent a situation;
- Interpret the solution in the context of the problem;

POSSIBLE SUCCESS CRITERIA

Solve $3x^2 + 4 = 100$.
Know that the quadratic formula can be used to solve all quadratic equations, and often provides a more efficient method than factorising or completing the square.
Have an understanding of solutions that can be written in surd form.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Problems that require students to set up and solve a pair of simultaneous equations in a real-life context, such as 2 adult tickets and 1 child ticket cost £28, and 1 adult ticket and 3 child tickets cost £34. How much does 1 adult ticket cost?

COMMON MISCONCEPTIONS

Using the formula involving negatives can result in incorrect answers.
If students are using calculators for the quadratic formula, they can come to rely on them and miss the fact that some solutions can be left in surd form.

NOTES

Remind students to use brackets for negative numbers when using a calculator, and remind them of the importance of knowing when to leave answers in surd form.
Link to unit 2, where quadratics were solved algebraically (when $a = 1$).
The quadratic formula must now be known; it will not be given in the exam paper.
Reinforce the fact that some problems may produce one inappropriate solution which can be ignored.
Clear presentation of working out is essential.
Link with graphical representations.

9b. Inequalities (N1, A22)	**Teaching time** 5–7 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Show inequalities on number lines;
- Write down whole number values that satisfy an inequality;
- Solve simple linear inequalities in one variable, and represent the solution set on a number line;
- Solve two linear inequalities in x, find the solution sets and compare them to see which value of x satisfies both solve linear inequalities in two variables algebraically;
- Use the correct notation to show inclusive and exclusive inequalities.

POSSIBLE SUCCESS CRITERIA

Use inequality symbols to compare numbers.
Given a list of numbers, represent them on a number line using the correct notation.
Solve equations involving inequalities.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Problems that require student to justify why certain values in a solution can be ignored.

COMMON MISCONCEPTIONS

When solving inequalities students often state their final answer as a number quantity, and exclude the inequality or change it to =.
Some students believe that –6 is greater than –3.

NOTES

Emphasise the importance of leaving their answer as an inequality (and not changing it to =).
Link to units 2 and 9a, where quadratics and simultaneous equations were solved.
Students can leave their answers in fractional form where appropriate.
Ensure that correct language is used to avoid reinforcing misconceptions: for example, 0.15 should never be read as 'zero point fifteen', and 5 > 3 should be read as 'five is greater than 3', not '5 is bigger than 3'.

UNIT 10: Probability

Teaching Time
7-9 hours

Return to Overview

SPECIFICATION REFERENCES

N5 apply systematic listing strategies, **including use of the product rule for counting** …
P1 record, describe and analyse the frequency of outcomes of probability experiments using tables and frequency trees
P2 apply ideas of randomness, fairness and equally likely events to calculate expected outcomes of multiple future experiments
P3 relate relative expected frequencies to theoretical probability, using appropriate language and the 0–1 probability scale
P4 apply the property that the probabilities of an exhaustive set of outcomes sum to one; apply the property that the probabilities of an exhaustive set of mutually exclusive events sum to one
P5 understand that empirical unbiased samples tend towards theoretical probability distributions, with increasing sample size
P6 enumerate sets and combinations of sets systematically, using tables, grids, Venn diagrams and tree diagrams
P7 construct theoretical possibility spaces for single and combined experiments with equally likely outcomes and use these to calculate theoretical probabilities
P8 calculate the probability of independent and dependent combined events, including using tree diagrams and other representations, and know the underlying assumptions
P9 **calculate and interpret conditional probabilities through representation using expected frequencies with two-way tables, tree diagrams and Venn diagrams**

PRIOR KNOWLEDGE

Students should understand that a probability is a number between 0 and 1, and distinguish between events which are impossible, unlikely, even chance, likely, and certain to occur.
Students should be able to mark events and/or probabilities on a probability scale of 0 to 1.
Students should know how to add and multiply fractions and decimals.
Students should have experience of expressing one number as a fraction of another number.

KEYWORDS

Probability, mutually exclusive, conditional, tree diagrams, sample space, outcomes, theoretical, relative frequency, Venn diagram, fairness, experimental

OBJECTIVES

By the end of the unit, students should be able to:

- Write probabilities using fractions, percentages or decimals;
- Understand and use experimental and theoretical measures of probability, including relative frequency to include outcomes using dice, spinners, coins, etc;
- Estimate the number of times an event will occur, given the probability and the number of trials;
- Find the probability of successive events, such as several throws of a single dice;
- List all outcomes for single events, and combined events, systematically;
- Draw sample space diagrams and use them for adding simple probabilities;
- Know that the sum of the probabilities of all outcomes is 1;

- Use $1 - p$ as the probability of an event not occurring where p is the probability of the event occurring;
- Work out probabilities from Venn diagrams to represent real-life situations and also ‘abstract’ sets of numbers/values;
- Use union and intersection notation;
- Find a missing probability from a list or two-way table, including algebraic terms;
- Understand conditional probabilities and decide if two events are independent;
- Draw a probability tree diagram based on given information, and use this to find probability and expected number of outcome;
- Understand selection with or without replacement;
- Calculate the probability of independent and dependent combined events;
- Use a two-way table to calculate conditional probability;
- Use a tree diagram to calculate conditional probability;
- Use a Venn diagram to calculate conditional probability;
- Compare experimental data and theoretical probabilities;
- Compare relative frequencies from samples of different sizes.

POSSIBLE SUCCESS CRITERIA

If the probability of outcomes are x, $2x$, $4x$, $3x$, calculate x.
Draw a Venn diagram of students studying French, German or both, and then calculate the probability that a student studies French given that they also study German.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Students should be given the opportunity to justify the probability of events happening or not happening in real-life and abstract contexts.

COMMON MISCONCEPTIONS

Probability without replacement is best illustrated visually and by initially working out probability ‘with’ replacement.
Not using fractions or decimals when working with probability trees.

NOTES

Encourage students to work ‘across’ the branches, working out the probability of each successive event. The probability of the combinations of outcomes should = 1.
Use problems involving ratio and percentage, similar to:

- A bag contains balls in the ratio 2 : 3 : 4. A ball is taken at random. Work out the probability that the ball will be … ;
- In a group of students 55% are boys, 65% prefer to watch film A, 10% are girls who prefer to watch film B. One student picked at random. Find the probability that this is a boy who prefers to watch film A (P6).

Emphasise that, were an experiment repeated, it will usually lead to different outcomes, and that increasing sample size generally leads to better estimates of probability and population characteristics.

UNIT 11: Multiplicative reasoning: direct and inverse proportion, relating to graph form for direct, compound measures, repeated proportional change

Teaching Time
6-8 hours

Return to Overview

SPECIFICATION REFERENCES

N3 recognise and use relationships between operations, including inverse operations (e.g. cancellation to simplify calculations and expressions); …
N12 interpret fractions and percentages as operators
N13 use standard units of mass, length, time, money and other measures (including standard compound measures) using decimal quantities where appropriate
R1 change freely between related standard units (e.g. time, length, area, volume/capacity, mass) and compound units (e.g. speed, rates of pay, prices, density, pressure) in numerical and algebraic contexts
R6 express a multiplicative relationship between two quantities as a ratio or a fraction
R10 solve problems involving direct and inverse proportion, including graphical and algebraic representations
R11 use compound units such as speed, rates of pay, unit pricing, density and pressure
R14 … recognise and interpret graphs that illustrate direct and inverse proportion
R16 set up, solve and interpret the answers in growth and decay problems, including compound interest **and work with general iterative processes**

PRIOR KNOWLEDGE

Students should be able to find a percentage of an amount and relate percentages to decimals.
Students should be able to rearrange equations and use these to solve problems.
Knowledge of speed = distance/time, density = mass/volume.

KEYWORDS

Ration, proportion, best value, unitary, proportional change, compound measure, density, mass, volume, speed, distance, time, density, mass, volume, pressure, acceleration, velocity, inverse, direct, constant of proportionality

OBJECTIVES

By the end of the unit, students should be able to:

- Express a multiplicative relationship between two quantities as a ratio or a fraction, e.g. when A:B are in the ratio 3:5, A is $\frac{3}{5}B$. When $4a = 7b$, then $a = \frac{7b}{4}$ or a:b is 7:4;
- Solve proportion problems using the unitary method;
- Work out which product offers best value and consider rates of pay;
- Work out the multiplier for repeated proportional change as a single decimal number;
- Represent repeated proportional change using a multiplier raised to a power, use this to solve problems involving compound interest and depreciation;
- Understand and use compound measures and:
  - convert between metric speed measures;
  - convert between density measures;
  - convert between pressure measures;
- Use kinematics formulae from the formulae sheet to calculate speed, acceleration, etc (with variables defined in the question);

- Calculate an unknown quantity from quantities that vary in direct or inverse proportion;
- Recognise when values are in direct proportion by reference to the graph form, and use a graph to find the value of k in $y = kx$;
- Set up and use equations to solve word and other problems involving direct proportion (this is covered in more detail in unit 19);
- Relate algebraic solutions to graphical representation of the equations;
- Recognise when values are in inverse proportion by reference to the graph form;
- Set up and use equations to solve word and other problems involving inverse proportion, and relate algebraic solutions to graphical representation of the equations.

POSSIBLE SUCCESS CRITERIA

Change g/cm^3 to kg/m^3, kg/m^2 to g/cm^2, m/s to km/h.
Solve word problems involving direct and inverse proportion.
Understand direct proportion as: as x increases, y increases.
Understand inverse proportion as: as x increases, y decreases.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Speed/distance type problems that involve students justifying their reasons why one vehicle is faster than another.
Calculations involving value for money are a good reasoning opportunity that utilise different skills.
Working out best value of items using different currencies given an exchange rate.

NOTES

Include fractional percentages of amounts with compound interest and encourage use of single multipliers.
Amounts of money should be rounded to the nearest penny, but emphasise the importance of not rounding until the end of the calculation if doing in stages.
Use a formula triangle to help students see the relationship for compound measures – this will help them evaluate which inverse operations to use.
Help students to recognise the problem they are trying to solve by the unit measurement given, e.g. km/h is a unit of speed as it is speed divided by a time.
Kinematics formulae involve a constant acceleration (which could be zero).
Encourage students to write down the initial equation of proportionality and, if asked to find a formal relating two quantities, the constant of proportionality must be found.

UNIT 12: Similarity and congruence in 2D and 3D

Teaching Time
5-7 hours

Back to Overview

SPECIFICATION REFERENCES

R6 express a multiplicative relationship between two quantities as a ratio or a fraction
R12 compare lengths, areas and volumes using ratio notation; make links to similarity (including trigonometric ratios) and scale factors
G5 use the basic congruence criteria for triangles (SSS, SAS, ASA, RHS)
G6 apply angle facts, triangle congruence, similarity and properties of quadrilaterals to conjecture and derive results about angles and sides, including … the fact that the base angles of an isosceles triangle are equal, and use known results to obtain simple proofs
G17 … calculate: surface area and volume of spheres, pyramids, cones and composite solids
G19 apply the concepts of congruence and similarity, including the relationships between lengths, **areas and volumes** in similar figures

PRIOR KNOWLEDGE

Students should be able to recognise and enlarge shapes and calculate scale factors.
Students should have knowledge of how to calculate area and volume in various metric measures.
Students should be able to measure lines and angles, and use compasses, ruler and protractor to construct standard constructions.

KEYWORDS

Congruence, side, angle, compass, construction, shape, volume, length, area, volume, scale factor, enlargement, similar, perimeter, frustum

OBJECTIVES

By the end of the unit, students should be able to:

- Understand and use SSS, SAS, ASA and RHS conditions to prove the congruence of triangles using formal arguments, and to verify standard ruler and pair of compasses constructions;
- Solve angle problems by first proving congruence;
- Understand similarity of triangles and of other plane shapes, and use this to make geometric inferences;
- Prove that two shapes are similar by showing that all corresponding angles are equal in size and/or lengths of sides are in the same ratio/one is an enlargement of the other, giving the scale factor;
- Use formal geometric proof for the similarity of two given triangles;
- Understand the effect of enlargement on angles, perimeter, area and volume of shapes and solids;
- Identify the scale factor of an enlargement of a similar shape as the ratio of the lengths of two corresponding sides, using integer or fraction scale factors;
- Write the lengths, areas and volumes of two shapes as ratios in their simplest form;
- Find missing lengths, areas and volumes in similar 3D solids;
- Know the relationships between linear, area and volume scale factors of mathematically similar shapes and solids;

- Use the relationship between enlargement and areas and volumes of simple shapes and solids;
- Solve problems involving frustums of cones where you have to find missing lengths first using similar triangles.

POSSIBLE SUCCESS CRITERIA

Recognise that all corresponding angles in similar shapes are equal in size when the corresponding lengths of sides are not.
Understand that enlargement does not have the same effect on area and volume.
Understand, from the experience of constructing them, that triangles satisfying SSS, SAS, ASA and RHS are unique, but SSA triangles are not.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Multi-step questions which require calculating missing lengths of similar shapes prior to calculating area of the shape, or using this information in trigonometry or Pythagoras problems.

COMMON MISCONCEPTIONS

Students commonly use the same scale factor for length, area and volume.

NOTES

Encourage students to model consider what happens to the area when a 1 cm square is enlarged by a scale factor of 3.
Ensure that examples involving given volumes are used, requiring the cube root being calculated to find the length scale factor.
Make links between similarity and trigonometric ratios.

UNIT 13: Sine and cosine rules, $\frac{1}{2}ab \sin C$, trigonometry and Pythagoras' Theorem in 3D, trigonometric graphs, and accuracy and bounds

Return to Overview

SPECIFICATION REFERENCES

N16 apply and interpret limits of accuracy, **including upper and lower bounds**

A8 work with coordinates in all four quadrants

A12 recognise, sketch and interpret graphs of linear functions, quadratic functions, simple cubic functions, the reciprocal function $y = \frac{1}{x}$ with $x \neq 0$, **exponential, functions $y = k^x$ for positive values of k, and the trigonometric functions (with arguments in degrees) $y = \sin x$, $y = \cos x$ and $y = \tan x$ for angles of any size**

A13 **sketch translations and reflections of a given function**

G11 solve geometrical problems on coordinate axes

G20 know the formulae for: Pythagoras' Theorem $a^2 + b^2 = c^2$ and the trigonometric ratios, sine, cosine and tan; apply them to find angles and lengths in right-angled triangles **and, where possible, general triangles** in two **and three** dimensional figures

G21 know the exact values of sin θ and cos θ for θ = 0°, 30°, 45° , 60° and 90°; know the exact value of tan θ for θ = 0°, 30°, 45° and 60°

G22 **know and apply the sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$, and cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$, to find unknown lengths and angles**

G23 **know and apply Area = $\frac{1}{2}ab \sin C$ to calculate the area, sides or angles of any triangle**

PRIOR KNOWLEDGE

Students should be able to use axes and coordinates to specify points in all four quadrants.
Students should be able to recall and apply Pythagoras' Theorem and trigonometric ratios.
Students should be able to substitute into formulae.

KEYWORDS

Axes, coordinates, sine, cosine, tan, angle, graph, transformations, side, angle, inverse, square root, 2D, 3D, diagonal, plane, cuboid

13a. Graphs of trigonometric functions (A8, A12, A13, G21)

Teaching time
5–7 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Recognise, sketch and interpret graphs of the trigonometric functions (in degrees) $y = \sin x$, $y = \cos x$ and $y = \tan x$ for angles of any size.
- Know the exact values of sin θ and cos θ for θ = 0°, 30°, 45° , 60° and 90° and exact value of tan θ for θ = 0°, 30°, 45° and 60° and find them from graphs.
- Apply to the graph of $y = \mathrm{f}(x)$ the transformations $y = -\mathrm{f}(x)$, $y = \mathrm{f}(-x)$ for sine, cosine and tan functions $\mathrm{f}(x)$.
- Apply to the graph of $y = \mathrm{f}(x)$ the transformations $y = \mathrm{f}(x) + a$, $y = \mathrm{f}(x + a)$ for sine, cosine and tan functions $\mathrm{f}(x)$.

POSSIBLE SUCCESS CRITERIA

Match the characteristic shape of the graphs to their functions and transformations.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Match a given list of events/processes with their graph.
Calculate and justify specific coordinates on a transformation of a trigonometric function.

NOTES

Translations and reflections of functions are included in this specification, but not rotations or stretches.
This work could be supported by the used of graphical calculators or suitable ICT.
Students need to recall the above exact values for sin, cos and tan.

13b. Further trigonometry (N16, G11, G20, G22, G23)

Teaching time 8-10 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Know and apply Area = $\frac{1}{2}ab \sin C$ to calculate the area, sides or angles of any triangle.
- Know the sine and cosine rules, and use to solve 2D problems (including involving bearings).
- Use the sine and cosine rules to solve 3D problems.
- Understand the language of planes, and recognise the diagonals of a cuboid.
- Solve geometrical problems on coordinate axes.
- Understand, recall and use trigonometric relationships and Pythagoras' Theorem in right-angled triangles, and use these to solve problems in 3D configurations.
- Calculate the length of a diagonal of a cuboid.
- Find the angle between a line and a plane.

POSSIBLE SUCCESS CRITERIA

Find the area of a segment of a circle given the radius and length of the chord.
Justify when to use the cosine rule, sine rule, Pythagoras' Theorem or normal trigonometric ratios to solve problems.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Triangles formed in a semi-circle can provide links with other areas of mathematics.

COMMON MISCONCEPTIONS

Not using the correct rule, or attempting to use 'normal trig' in non-right-angled triangles.
When finding angles students will be unable to rearrange the cosine rule or fail to find the inverse of cos θ.

NOTES

The cosine rule is used when we have SAS and used to find the side opposite the 'included' angle or when we have SSS to find an angle.
Ensure that finding angles with 'normal trig' is refreshed prior to this topic.
Students may find it useful to be reminded of simple geometrical facts, i.e. the shortest side is always opposite the shortest angle in a triangle.
The sine and cosine rules and general formula for the area of a triangle are not given on the formulae sheet.
In multi-step questions emphasise the importance of not rounding prematurely and using exact values where appropriate.
Whilst 3D coordinates are not included in the programme of study, they provide a visual introduction to trigonometry in 3D.

UNIT 14: Statistics and sampling, cumulative frequency and histograms

Return to Overview

SPECIFICATION REFERENCES

S1 infer properties of populations or distributions from a sample, whilst knowing the limitations of sampling apply statistics to describe a population

S3 **interpret and construct diagrams for grouped discrete data and continuous data, i.e. histograms with equal and unequal class intervals and cumulative frequency graphs, and know their appropriate use**

S4 interpret, analyse and compare the distributions of data sets from univariate empirical distributions through:

- Appropriate graphical representation involving discrete, continuous and grouped data, **including box plots**
- appropriate measures of central tendency (median, mean, mode and modal class) and spread (range, including consideration of outliers, **quartiles and inter-quartile range)**

PRIOR KNOWLEDGE

Students should understand the different types of data: discrete/continuous.
Students should have experience of inequality notation.
Students should be able to multiply a fraction by a number.
Students should understand the data handling cycle.

KEYWORDS

Sample, population, fraction, decimal, percentage, bias, stratified sample, random, cumulative frequency, box plot, histogram, frequency density, frequency, mean, median, mode, range, lower quartile, upper quartile, interquartile range, spread, comparison, outlier

14a. Collecting data (S1)

Teaching time
5–7 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Specify the problem and plan:
  - decide what data to collect and what analysis is needed;
  - understand primary and secondary data sources;
  - consider fairness;
- Understand what is meant by a sample and a population;
- Understand how different sample sizes may affect the reliability of conclusions drawn;
- Identify possible sources of bias and plan to minimise it;
- Write questions to eliminate bias, and understand how the timing and location of a survey can ensure a sample is representative (see note);

POSSIBLE SUCCESS CRITERIA

Explain why a sample may not be representative of a whole population.
Carry out their own statistical investigation and justify how sources of bias have been eliminated.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

When using a sample of a population to solve contextual problem, students should be able to justify why the sample may not be representative the whole population.

NOTES

Emphasise the difference between primary and secondary sources and remind students about the difference between discrete and continuous data.
Discuss sample size and mention that a census is the whole population (the UK census takes place every 10 years in a year ending with a 1 – the next one is due in 2021).
Specifying the problem and planning for data collection is not included in the programme of study, but is a prerequisite to understanding the context of the topic.
Writing a questionnaire is also not included in the programme of study, but remains a good topic for demonstrating bias and ways to reduce bias in terms of timing, location and question types.

14b. Cumulative frequency, box plots and histograms (S1, S3, S4)

Teaching time
5-7 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Use statistics found in all graphs/charts in this unit to describe a population;
- Know the appropriate uses of cumulative frequency diagrams;
- Construct and interpret cumulative frequency tables, cumulative frequency graphs/diagrams and from the graph:
  - estimate frequency greater/less than a given value;
  - find the median and quartile values and interquartile range;
- Compare the mean and range of two distributions, or median and interquartile range, as appropriate;
- Interpret box plots to find median, quartiles, range and interquartile range and draw conclusions;
- Produce box plots from raw data and when given quartiles, median and identify any outliers;
- Know the appropriate uses of histograms;
- Construct and interpret histograms from class intervals with unequal width;
- Use and understand frequency density;
- From histograms:
  - complete a grouped frequency table;
  - understand and define frequency density;
- Estimate the mean and median from a histogram with unequal class widths or any other information from a histogram, such as the number of people in a given interval.

POSSIBLE SUCCESS CRITERIA

Construct cumulative frequency graphs, box plots and histograms from frequency tables.
Compare two data sets and justify their comparisons based on measures extracted from their diagrams where appropriate in terms of the context of the data.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Interpret two or more data sets from box plots and relate the key measures in the context of the data.
Given the size of a sample and its box plot calculate the proportion above/below a specified value.

COMMON MISCONCEPTIONS

Labelling axes incorrectly in terms of the scales, and also using 'Frequency' instead of 'Frequency Density' or 'Cumulative Frequency'.
Students often confuse the methods involved with cumulative frequency, estimating the mean and histograms when dealing with data tables.

NOTES

Ensure that axes are clearly labelled.
As a way to introduce measures of spread, it may be useful to find mode, median, range and interquartile range from stem and leaf diagrams (including back-to-back) to compare two data sets.
As an extension, use the formula for identifying an outlier, (i.e. if data point is below LQ – 1.5 × IQR or above UQ + 1.5 × IQR, it is an outlier). Get them to identify outliers in the data, and give bounds for data.

UNIT 15: Quadratics, expanding more than two brackets, sketching graphs, graphs of circles, cubes and quadratics

Teaching time
6-8 hours

Return to Overview

SPECIFICATION REFERENCES

N8 Calculate exactly with … **surds** …
A4 simplify and manipulate algebraic expressions … by: expanding products of two **or more** binomials
A11 identify and interpret roots, intercepts, turning points of quadratic functions graphically; … **identify turning points by completing the square**
A12 recognise, sketch and interpret graphs of … quadratic functions, simple cubic functions …
A18 solve quadratic equations (**including those that require rearrangement)** …; find approximate solutions using a graph
A19 solve two simultaneous equations in two variables (linear/linear **or linear/quadratic**) algebraically; find approximate solutions using a graph
A20 **find approximate solutions to equations numerically using iteration**
A21 … derive an equation (or two simultaneous equations), solve the equation(s) and interpret the solution.
A22 solve linear inequalities in one or **two** variable**(s), and quadratic inequalities in one variable**; represent the solution set on a number line, **using set notation and on a graph**

PRIOR KNOWLEDGE

Students should be able to solve quadratics and linear equations.
Students should be able to solve simultaneous equations algebraically.

KEYWORDS

Sketch, estimate, quadratic, cubic, function, factorising, simultaneous equation, graphical, algebraic

OBJECTIVES

By the end of the unit, students should be able to:

- Sketch a graph of a quadratic function, by factorising or by using the formula, identifying roots and y-intercept, turning point;
- Be able to identify from a graph if a quadratic equation has any real roots;
- Find approximate solutions to quadratic equations using a graph;
- Expand the product of more than two linear expressions;
- Sketch a graph of a quadratic function and a linear function, identifying intersection points;
- Sketch graphs of simple cubic functions, given as three linear expressions;
- Solve simultaneous equations graphically:
  - find approximate solutions to simultaneous equations formed from one linear function and one quadratic function using a graphical approach;
  - find graphically the intersection points of a given straight line with a circle;
  - solve simultaneous equations representing a real-life situation graphically, and interpret the solution in the context of the problem;
- Solve quadratic inequalities in one variable, by factorising and sketching the graph to find critical values;

- Represent the solution set for inequalities using set notation, i.e. curly brackets and 'is an element of' notation;
  - for problems identifying the solutions to two different inequalities, show this as the intersection of the two solution sets, i.e. solution of $x^2 - 3x - 10 < 0$ as $\{x: -3 < x < 5\}$;
- Solve linear inequalities in two variables graphically;
- Show the solution set of several inequalities in two variables on a graph;
- Use iteration with simple converging sequences.

POSSIBLE SUCCESS CRITERIA

Expand $x(x - 1)(x + 2)$.
Expand $(x - 1)^3$.
Expand $(x + 1)(x + 2)(x - 1)$.
Sketch $y = (x + 1)^2(x - 2)$.
Interpret a pair of simultaneous equations as a pair of straight lines and their solution as the point of intersection.

Be able to state the solution set of $x^2 - 3x - 10 < 0$ as $\{x: x < -3\} \cup \{x: x > 5\}$.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Match equations to their graphs and to real-life scenarios.
"Show that"-type questions will allow students to show a logical and clear chain of reasoning.

COMMON MISCONCEPTIONS

When estimating values from a graph, it is important that students understand it is an 'estimate'.
It is important to stress that when expanding quadratics, the x terms are also collected together.
Quadratics involving negatives sometimes cause numerical errors.

NOTES

The extent of algebraic iteration required needs to be confirmed.
You may want to extend the students to include expansions of more than three linear expressions.
Practise expanding 'double brackets' with all combinations of positives and negatives.
Set notation is a new topic.

UNIT 16: Circle theorems and circle geometry

Return to Overview

SPECIFICATION REFERENCES

A16 **recognise and use the equation of a circle with centre at the origin; find the equation of a tangent to a circle at a given point**

G9 identify and apply circle definitions and properties, including: centre, radius, chord, diameter, circumference, tangent, arc, sector and segment

G10 **apply and prove the standard circle theorems concerning angles, radii, tangents and chords, and use them to prove related results**

PRIOR KNOWLEDGE

Students should have practical experience of drawing circles with compasses.
Students should recall the words, centre, radius, diameter and circumference.
Students should recall the relationship of the gradient between two perpendicular lines.
Students should be able to find the equation of the straight line, given a gradient and a coordinate.

KEYWORDS

Radius, centre, tangent, circumference, diameter, gradient, perpendicular, reciprocal, coordinate, equation, substitution, chord, triangle, isosceles, angles, degrees, cyclic quadrilateral, alternate, segment, semicircle, arc, theorem

16a. Circle theorems (G9, G10)

Teaching time
4-6 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Recall the definition of a circle and identify (name) and draw parts of a circle, including sector, tangent, chord, segment;
- Prove and use the facts that:
  - the angle subtended by an arc at the centre of a circle is twice the angle subtended at any point on the circumference;
  - the angle in a semicircle is a right angle;
  - the perpendicular from the centre of a circle to a chord bisects the chord;
  - angles in the same segment are equal;
  - alternate segment theorem;
  - opposite angles of a cyclic quadrilateral sum to 180°;
- Understand and use the fact that the tangent at any point on a circle is perpendicular to the radius at that point;
- Find and give reasons for missing angles on diagrams using:
  - circle theorems;
  - isosceles triangles (radius properties) in circles;
  - the fact that the angle between a tangent and radius is 90°;
  - the fact that tangents from an external point are equal in length.

POSSIBLE SUCCESS CRITERIA

Justify clearly missing angles on diagrams using the various circle theorems.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Problems that involve a clear chain of reasoning and provide counter-arguments to statements. Can be linked to other areas of mathematics by incorporating trigonometry and Pythagoras' Theorem.

COMMON MISCONCEPTIONS

Much of the confusion arises from mixing up the diameter and the radius.

NOTES

Reasoning needs to be carefully constructed and correct notation should be used throughout.
Students should label any diagrams clearly, as this will assist them; particular emphasis should be made on labelling any radii in the first instance.

16b. Circle geometry (A16)

Teaching time
4-6 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Select and apply construction techniques and understanding of loci to draw graphs based on circles and perpendiculars of lines;
- Find the equation of a tangent to a circle at a given point, by:
  - finding the gradient of the radius that meets the circle at that point (circles all centre the origin);
  - finding the gradient of the tangent perpendicular to it;
  - using the given point;
- Recognise and construct the graph of a circle using $x^2 + y^2 = r^2$ for radius r centred at the origin of coordinates.

POSSIBLE SUCCESS CRITERIA

Find the gradient of a radius of a circle drawn on a coordinate grid and relate this to the gradient of the tangent.
Justify the relationship between the gradient of a tangent and the radius.
Produce an equation of a line given a gradient and a coordinate.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Justify if a straight-line graph would pass through a circle drawn on a coordinate grid.

COMMON MISCONCEPTIONS

Students find it difficult working with negative reciprocals of fractions and negative fractions.

NOTES

Work with positive gradients of radii initially and review reciprocals prior to starting this topic.
It is useful to start this topic through visual proofs, working out the gradient of the radius and the tangent, before discussing the relationship.

UNIT 17: Changing the subject of formulae (more complex), algebraic fractions, solving equations arising from algebraic fractions, rationalising surds, proof

Teaching time
6-8 hours

Return to Overview

SPECIFICATION REFERENCES

N8 … **simplify surd expressions involving squares (e.g. $\sqrt{12} = \sqrt{(4 \times 3)} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$) and rationalise denominators**

A4 simplify and manipulate algebraic expressions (including those involving surds **and algebraic fractions**) by:
- collecting like terms
- multiplying a single term over a bracket
- taking out common factors
- expanding products of two **or more** binomials
- factorising quadratic expressions of the form $x^2 + bx + c$, including the difference of two squares; **factorising quadratic expressions of the form $ax^2 + bx + c$**
- simplifying expressions involving sums, products and powers, including the laws of indices

A5 … rearrange formulae to change the subject

A6 … argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments **and proofs**

A7 where appropriate, interpret simple expressions as functions with inputs and outputs; **interpret the reverse process as the 'inverse function'; interpret the succession of two functions as a 'composite function' (the use of formal function notation is expected)**

A18 solve quadratic equations **(including those that require rearrangement)** algebraically by factorising, …

PRIOR KNOWLEDGE

Students should be able to simplify surds.
Students should be able to use negative numbers with all four operations.
Students should be able to recall and use the hierarchy of operations.

KEYWORDS

Rationalise, denominator, surd, rational, irrational, fraction, equation, rearrange, subject, proof, function notation, inverse, evaluate

OBJECTIVES

By the end of the unit, students should be able to:
- Rationalise the denominator involving surds;
- Simplify algebraic fractions;
- Multiply and divide algebraic fractions;
- Solve quadratic equations arising from algebraic fraction equations;
- Change the subject of a formula, including cases where the subject occurs on both sides of the formula, or where a power of the subject appears;

- Change the subject of a formula such as $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$, where all variables are in the denominators;
- Solve 'Show that' and proof questions using consecutive integers (n, n + 1), squares a^2, b^2, even numbers 2n, odd numbers 2n +1;
- Use function notation;
- Find f(x) + g(x) and f(x) – g(x), 2f(x), f(3x) etc algebraically;
- Find the inverse of a linear function;
- Know that $f^{-1}(x)$ refers to the inverse function;
- For two functions f(x) and g(x), find gf(x).

POSSIBLE SUCCESS CRITERIA

Rationalise: $\frac{1}{\sqrt{3} - 1}$, $\frac{1}{\sqrt{3}}$, (√18 + 10) +√2.
Explain the difference between rational and irrational numbers.
Given a function, evaluate f(2).
When g(x) = 3 – 2x, find $g^{-1}(x)$.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Formal proof is an ideal opportunity for students to provide a clear logical chain of reasoning providing links with other areas of mathematics.

COMMON MISCONCEPTIONS

√3 x √3 = 9 is often seen.
When simplifying involving factors, students often use the 'first' factor that they find and not the LCM.

NOTES

It is useful to generalise $\sqrt{m} \times \sqrt{m} = m$.
Revise the difference of two squares to show why we use, for example, (√3 – 2) as the multiplier to rationalise (√3 + 2).
Link collecting like terms to simplifying surds (Core 1 textbooks are a good source for additional work in relation to simplifying surds).
Practice factorisation where the factor may involve more than one variable.
Emphasise that, by using the LCM for the denominator, the algebraic manipulation is easier.

UNIT 18: Vectors and geometric proof

Teaching time
8-10 hours

Return to Overview

SPECIFICATION REFERENCES

G25 apply addition and subtraction of vectors, multiplication of vectors by a scalar, and diagrammatic and column representations of vectors; **use vectors to construct geometric arguments and proof**

PRIOR KNOWLEDGE

Students will have used vectors to describe translations and will have knowledge of Pythagoras' Theorem and the properties of triangles and quadrilaterals.

KEYWORDS

Vector, direction, magnitude, scalar, multiple, parallel, collinear, proof, ratio, column vector

OBJECTIVES

By the end of the unit, students should be able to:

- Understand and use vector notation, including column notation, and understand and interpret vectors as displacement in the plane with an associated direction.
- Understand that 2**a** is parallel to **a** and twice its length, and that **a** is parallel to –**a** in the opposite direction.
- Represent vectors, combinations of vectors and scalar multiples in the plane pictorially.
- Calculate the sum of two vectors, the difference of two vectors and a scalar multiple of a vector using column vectors (including algebraic terms).
- Find the length of a vector using Pythagoras' Theorem.
- Calculate the resultant of two vectors.
- Solve geometric problems in 2D where vectors are divided in a given ratio.
- Produce geometrical proofs to prove points are collinear and vectors/lines are parallel.

POSSIBLE SUCCESS CRITERIA

Add and subtract vectors algebraically and use column vectors.
Solve geometric problems and produce proofs.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

"Show that"-type questions are an ideal opportunity for students to provide a clear logical chain of reasoning providing links with other areas of mathematics, in particular algebra.
Find the area of a parallelogram defined by given vectors.

COMMON MISCONCEPTIONS

Students find it difficult to understand that parallel vectors are equal as they are in different locations in the plane.

NOTES

Students find manipulation of column vectors relatively easy compared to pictorial and algebraic manipulation methods – encourage them to draw any vectors they calculate on the picture.
Geometry of a hexagon provides a good source of parallel, reverse and multiples of vectors.
Remind students to underline vectors or use an arrow above them, or they will be regarded as just lengths.
Extend geometric proofs by showing that the medians of a triangle intersect at a single point.
3D vectors or **i**, **j** and **k** notation can be introduced and further extension work can be found in GCE Mechanics 1 textbooks.

UNIT 19: Direct and indirect proportion: using statements of proportionality, reciprocal and exponential graphs, rates of change in graphs, functions, transformations of graphs

Return to Overview

SPECIFICATION REFERENCES

A7 where appropriate, interpret simple expressions as functions with inputs and outputs; …

A12 recognise, sketch and interpret graphs of the reciprocal function $y = \frac{1}{x}$ with $x \neq 0$, **exponential functions $y = k^x$ for positive values of k** …

A13 **sketch translations and reflections of a given function**

A14 plot and interpret reciprocal graphs and **exponential graphs** …

A15 **calculate or estimate gradients of graphs and areas under graphs (including quadratic and other non-linear graphs) and interpret results in cases such distance–time graphs, velocity–time graphs and graphs in financial contexts (this does not include calculus)**

R7 understand and use proportion as equality of ratios

R10 solve problems involving direct and inverse proportion, including graphical and algebraic representations

R13 understand that X is inversely proportional to Y is equivalent to X is proportional to $\frac{1}{Y}$; **construct and** interpret equations that describe direct and inverse proportion

R14 interpret the gradient of a straight line graph as a rate of change; recognise and interpret graphs that illustrate direct and inverse proportion

R15 **interpret the gradient at a point on a curve as the instantaneous rate of change; apply the concepts of average and instantaneous rate of change (gradients of chords and tangents) in numerical, algebraic and graphical contexts (this does not include calculus**

R16 set up, solve and interpret the answers in growth and decay problems …

PRIOR KNOWLEDGE

Students should be able to draw linear and quadratic graphs.
Students should be able to calculate the gradient of a linear function between two points.
Students should recall transformations of trigonometric functions.
Students should have knowledge of writing statements of direct proportion and forming an equation to find values.

KEYWORDS

Reciprocal, linear, gradient, quadratic, exponential, functions, direct, indirect, proportion, estimate, area, rate of change, distance, time, velocity, transformations, cubic, transformation, constant of proportionality

19a. Reciprocal and exponential graphs; Gradient and area under graphs (R14, R15, A7, A12, A13, A14, A15)

Teaching time
6-8 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Recognise, sketch and interpret graphs of the reciprocal function $y = \frac{1}{x}$ with $x \neq 0$
- State the value of x for which the equation is not defined;
- Recognise, sketch and interpret graphs of exponential functions $y = k^x$ for positive values of k and integer values of x;
- Use calculators to explore exponential growth and decay;
- Set up, solve and interpret the answers in growth and decay problems;
- Interpret and analyse transformations of graphs of functions and write the functions algebraically, e.g. write the equation of f(x) + a, or f(x – a):
  - apply to the graph of y = f(x) the transformations y = –f(x), y = f(–x) for linear, quadratic, cubic functions;
  - apply to the graph of y = f(x) the transformations y = f(x) + a, y = f(x + a) for linear, quadratic, cubic functions;
- Estimate area under a quadratic or other graph by dividing it into trapezia;
- Interpret the gradient of linear or non-linear graphs, and estimate the gradient of a quadratic or non-linear graph at a given point by sketching the tangent and finding its gradient;
- Interpret the gradient of non-linear graph in curved distance–time and velocity–time graphs:
  - for a non-linear distance–time graph, estimate the speed at one point in time, from the tangent, and the average speed over several seconds by finding the gradient of the chord;
  - for a non-linear velocity–time graph, estimate the acceleration at one point in time, from the tangent, and the average acceleration over several seconds by finding the gradient of the chord;
- Interpret the gradient of a linear or non-linear graph in financial contexts;
- Interpret the area under a linear or non-linear graph in real-life contexts;
- Interpret the rate of change of graphs of containers filling and emptying;
- Interpret the rate of change of unit price in price graphs.

POSSIBLE SUCCESS CRITERIA

Explain why you cannot find the area under a reciprocal or tan graph.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Interpreting many of these graphs in relation to their specific contexts.

COMMON MISCONCEPTIONS

The effects of transforming functions is often confused.

NOTES

Translations and reflections of functions are included in this specification, but not rotations or stretches.
Financial contexts could include percentage or growth rate.
When interpreting rates of change with graphs of containers filling and emptying, a steeper gradient means a faster rate of change.
When interpreting rates of change of unit price in price graphs, a steeper graph means larger unit price.

19b. Direct and inverse proportion (R7, R10, R13, R16)

Teaching time
6-8 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Recognise and interpret graphs showing direct and indirect proportion;
- Identify direct proportion from a table of values, by comparing ratios of values, for x squared and x cubed relationships;
- Write statements of proportionality for quantities proportional to the square, cube or other power of another quantity;
- Set up and use equations to solve word and other problems involving direct proportion;
- Use $y = kx$ to solve direct proportion problems, including questions where students find k, and then use k to find another value;
- Solve problems involving inverse proportion using graphs by plotting and reading values from graphs;
- Solve problems involving inverse proportionality;
- Set up and use equations to solve word and other problems involving direct proportion or inverse proportion.

POSSIBLE SUCCESS CRITERIA

Understand that when two quantities are in direct proportion, the ratio between them remains constant.
Know the symbol for 'is proportional to'.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Justify and infer relationships in real-life scenarios to direct and inverse proportion such as ice cream sales and sunshine.

COMMON MISCONCEPTIONS

Direct and inverse proportion can get mixed up.

NOTES

Consider using science contexts for problems involving inverse proportionality, e.g. volume of gas inversely proportional to the pressure or frequency is inversely proportional to wavelength.

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