Mathematics
May/June 2017 Paper 2
Paper 2 (Calculator)
Higher Tier
Time: 1 hour 30 minutes

You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser

Instructions
- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided
- there may be more space than you need.
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.
- You must **show all your working out.**

Information
- The total mark for this paper is 80
- The marks for **each** question are shown in brackets
- use this as a guide as to how much time to spend on each question.

Advice
- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

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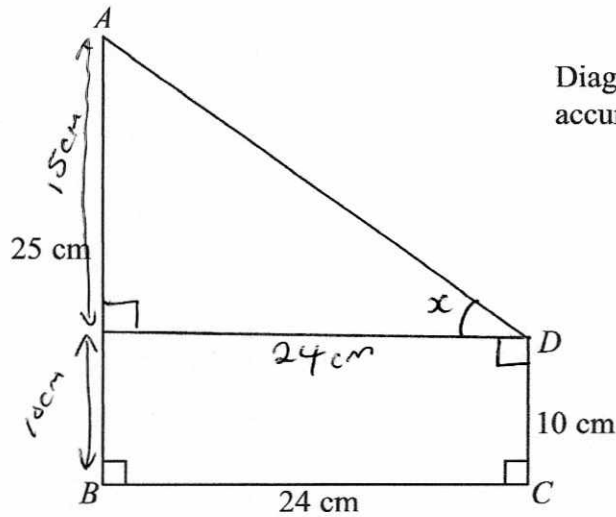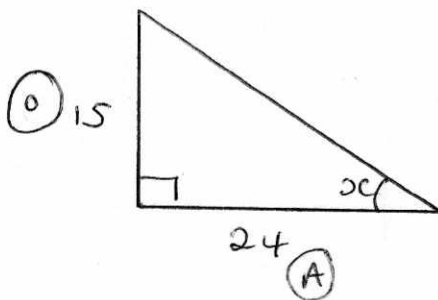**1** *ABCD* is a trapezium.

Diagram **NOT** accurately drawn

AB = 25 cm.
BC = 24 cm.
CD = 10 cm.

Angle *ABC* = angle *BCD* = 90°

Calculate the size of angle *CDA*.
Give your answer correct to 3 significant figures.

$$\tan x = \frac{O}{A}$$

$$\tan x = \frac{15}{24}$$

$$x = \tan^{-1}\left(\frac{15}{24}\right)$$

$$= 32.00538321°$$

$$CDA = 90 + "32.00538321"$$

$$= 122° \ (3sf)$$

_____122_____ °

(Total for Question 1 is 4 marks)

2 In the 2012 Paralympic Games, the total number of gold and silver medals won by Brazil was 35
The ratio of the number of gold medals that Brazil won to the number of silver medals that Brazil won was 3 : 2

How many silver medals were won by Brazil?

G   S
3 : 2

$\boxed{7}$ $\boxed{7}$ $\boxed{7}$: $\boxed{7}$ $\boxed{7}$

$\dfrac{35}{5} = 7$

$2 \times 7 = 14$

14

(Total for Question 2 is 2 marks)

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3 Jalin lives in England.
He does a search on the internet and sees the same type of camera on sale in France and in America.

In France, the camera costs 126 euros.
In America, the camera costs $165.24

Jalin finds out these exchange rates.

Exchange rates
1 euro = £0.89
£1 = $1.62

How much cheaper is the camera in America than in France?
Give your answer in pounds (£).

FRANCE: $126 \times 0.89 = £112.14$

AMERICA $165.24 \div 1.62 = £102$

$112.14 - 102 = £10.14$

£ 10.14

(Total for Question 3 is 4 marks)

4 A group of 200 adults were asked which types of magazines they read.

Their replies showed that

82 read Sports magazines
80 read Garden magazines
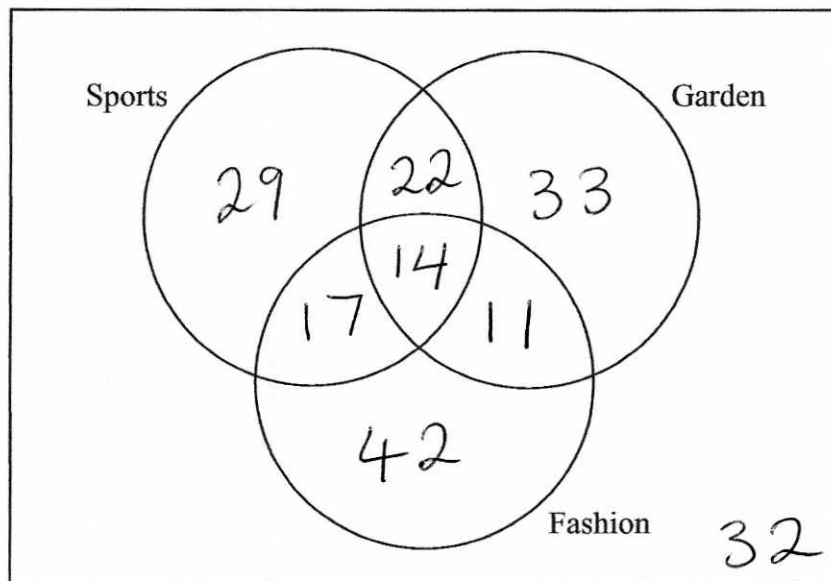84 read Fashion magazines
36 read Sports magazines and read Garden magazines
31 read Sports magazines and read Fashion magazines
25 read Garden magazines and read Fashion magazines
14 read Sports magazines and read Garden magazines and read Fashion magazines

(a) Complete the Venn diagram for this information.

(4)

One of the adults asked is to be chosen at random.

(b) Find the probability that this adult

(i) reads none of these magazine types,

$$\frac{32}{200}$$

(ii) reads exactly two of these magazine types.

$$\frac{17 + 11 + 22}{200}$$

$$\frac{50}{200}$$

(3)

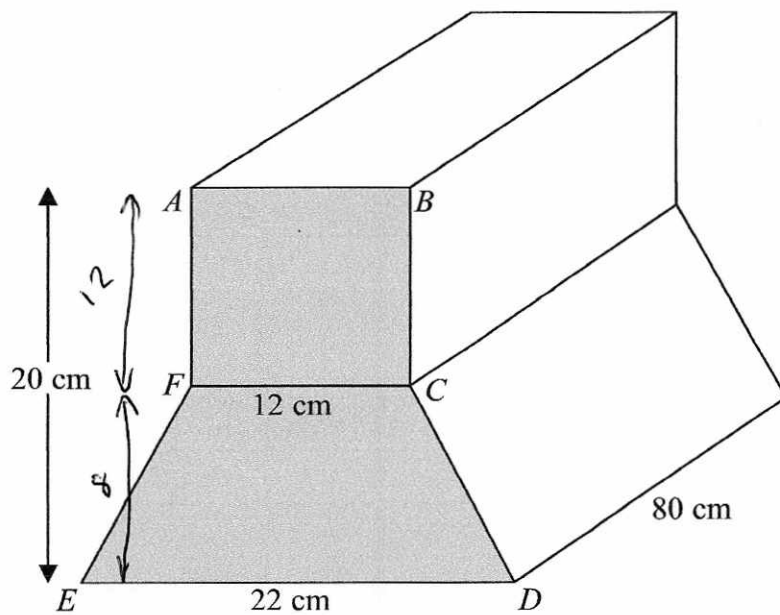**(Total for Question 4 is 7 marks)**

5 Here is a prism.

Diagram **NOT** accurately drawn

ABCDEF is a cross section of the prism.

ABCF is a square of side 12 cm.

FCDE is a trapezium.
ED = 22 cm.

The height of the prism is 20 cm.
The length of the prism is 80 cm.

Work out the total volume of the prism.

Area of Square = 12 × 12 = 144 cm²

Area of trapezium = $\frac{12 + 22}{2}$ × 8 = 136 cm²

Area of cross section = 144 + 136 = 280 cm²

280 × 80 = 22400 cm³

22400 .... cm³

(Total for Question 5 is 5 marks)

6   Liquid A has a density of 0.7 g/cm³. d
    Liquid B has a density of 1.6 g/cm³. d

    140 g of liquid A and 128 g of liquid B are mixed to make liquid C.

    Work out the density of liquid C.
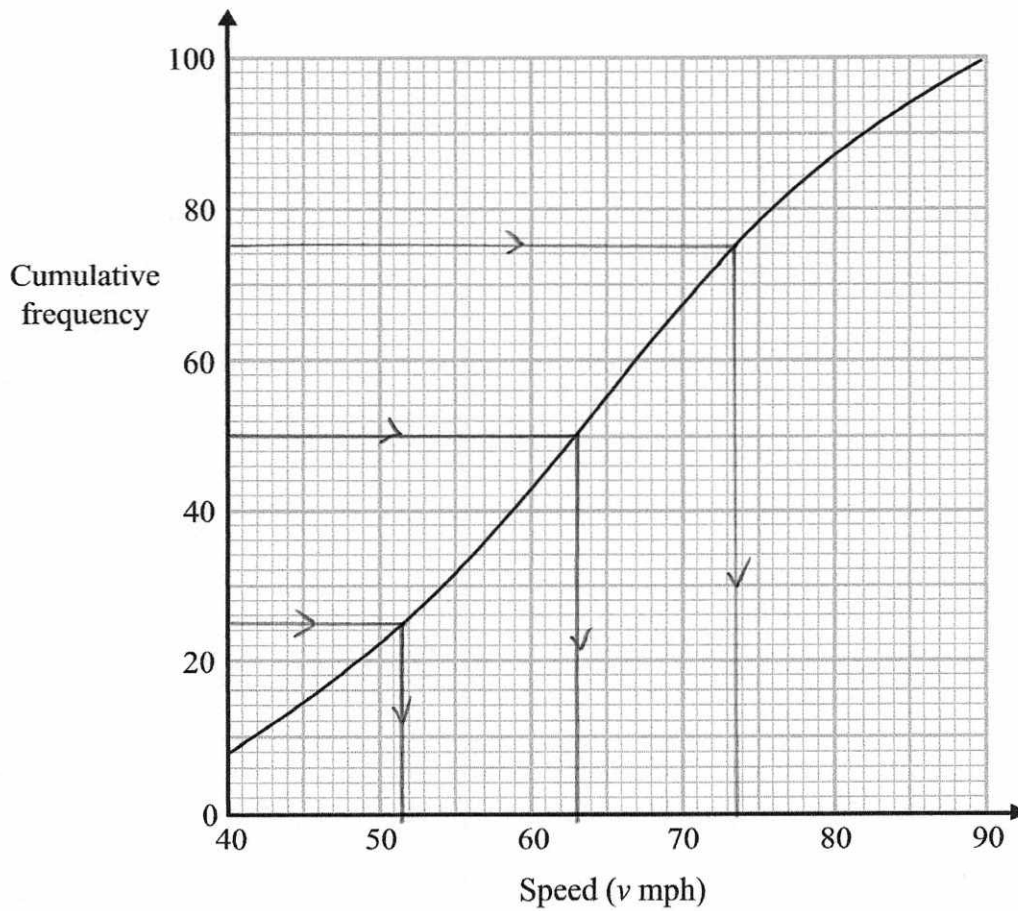
$$\text{Volume}_A = \frac{140}{0.7} = 200 \, cm^3$$

$$\text{Volume}_B = \frac{128}{1.6} = 80 \, cm^3$$

$$\text{Density}_c = \frac{140 + 128}{200 + 80} = 0.9571428571 \, g/cm^3$$
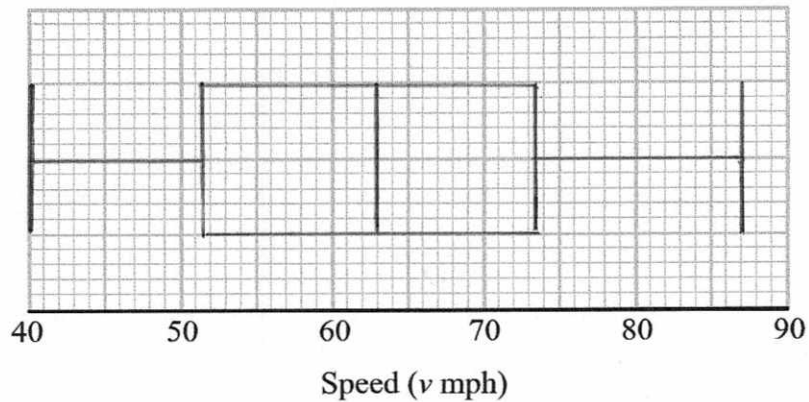
0.957   g/cm³

(Total for Question 6 is 4 marks)

7 The cumulative frequency graph gives information about the recorded speeds of 100 cars travelling along a road.

Speed (v mph)

The least recorded speed was 40 mph.
The greatest recorded speed was 87 mph.

On the grid, draw a box plot for the speeds of these cars.

Speed (v mph)

(Total for Question 7 is 3 marks)

8   Peter has £20 000 to invest in a savings account for 2 years.

He finds information about two savings accounts.

Bonus Saver	**Fixed Rate**
Compound interest	Compound interest
4% for the first year then 1.5% each year	2.5% each year

Peter wants to have as much money as possible in his savings account at the end of 2 years.

Which of these savings accounts should he choose?

BONUS SAVER

$20000 \times 1.04 \times 1.015 = £21112$

FIXED RATE

$20000 \times 1.025^2 = £21012.50$

The Bonus Saver

(Total for Question 8 is 4 marks)
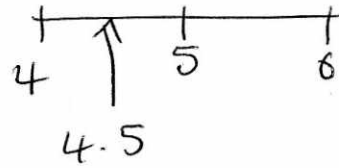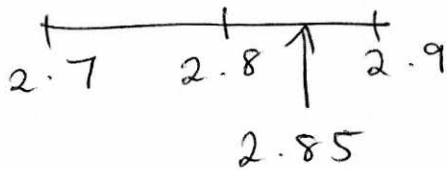
9 Rachael walks to school.

The distance to school is 2.8 km, correct to the nearest 0.1 km.

She walks at a speed of 5 km/h, correct to the nearest km/h.

Calculate the upper bound, in minutes, for the time Rachael takes to walk to school.

$$\text{Upper Time} = \frac{\text{Upper Distance}}{\text{Lower Speed}}$$

2.7   2.8   2.9
         2.85

4   5   6
   4.5

$$\frac{2.85}{4.5} = 0.6\dot{3} \text{ hours}$$

$$0.6\dot{3} \times 60 = 38 \qquad \underline{\quad 38 \quad} \text{ minutes}$$

(Total for Question 9 is 3 marks)

10 There are 30 tennis players in a tennis club.
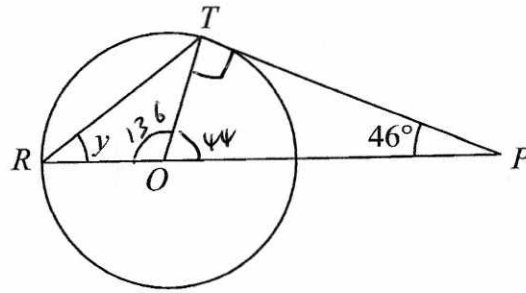
Two players are selected at random to play a tennis match.

How many different combinations of players could be selected?

$$\frac{30 \times 29}{2} = 435$$

<u>435</u>

(Total for Question 10 is 2 marks)

11

R and T are points on a circle, centre O.
ROP is a straight line.
PT is a tangent to the circle.
Angle TPO = 46°

(a) Explain why angle OTP = 90°

   tangent meets radius at 90°

(1)

(b) Work out the size of angle y.

$T\hat{O}P = 44°$ (Angles in triangle sum to 180°)

$180 - 44 = 136$ ($T\hat{O}R$)
Angles on straight line

$\dfrac{180 - 136}{2} = 22°$ Angles at the base of an isosceles triangle are equal.

22 °

(3)

(Total for Question 11 is 4 marks)

12 Use algebra to show that the recurring decimal $0.3\dot{8} = \dfrac{7}{18}$

$$0.3\dot{8} = x$$
$$3.\dot{8} = 10x$$
$$38.\dot{8} = 100x$$
$$35 = 90x$$
$$x = \frac{35}{90} = \frac{7}{18}$$

(Total for Question 12 is 2 marks)

13 Work out the formula for the nth term of the quadratic sequence: $an^2 + bn + c$

$$a + b + c \quad 11 \qquad 19 \qquad 29 \qquad 41 \ldots$$
$$\nearrow \qquad 8 \qquad 10 \qquad 12$$
$$3a + b \qquad 2 \qquad 2$$
$$2a \nearrow$$

$2a = 2$ $3a + b = 8$ $a + b + c = 11$
$\underline{a = 1}$ $3(1) + b = 8$ $1 + 5 + c = 11$
              $3 + b = 8$ $6 + c = 11$
              $\underline{b = 5}$ $\underline{c = 5}$

$$\underline{n^2 + 5n + 5}$$

(Total for Question 13 is 3 marks)

14 **L** and **M** are two mathematically similar prisms.
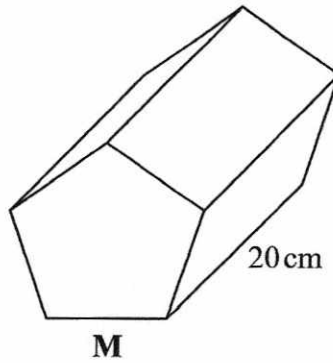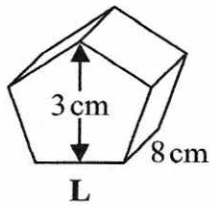
Diagram **NOT** accurately drawn

3 cm

8 cm

L

20 cm

M

Prism **L** has length 8 cm.
Prism **M** has length 20 cm.

Prism **L** has height 3 cm.

(a) Work out the height of prism **M**.

$$\text{Scale Factor} = \frac{20}{8} = 2.5$$

$$3 \times 2.5$$

7.5 cm

(2)

Prism **M** has a volume of 1875 cm^3

(b) Work out the volume of prism **L**.

Scale factor for volume = 2.5^3

$1875 \div 2.5^3 = 120$

120 cm^3

(2)

(Total for Question 14 is 4 marks)

15 The table gives information about the times, in minutes, 346 children spent playing online games in one week.

Time (*t* minutes) ⤳	Frequency	F·d
0 < *t* ⩽ 20 ²⁰	42	2.1
20 < *t* ⩽ 60 ⁴⁰	156	3.9
60 < *t* ⩽ 120 ⁶⁰	84	1.4
120 < *t* ⩽ 200 ⁸⁰	64	0.8

Draw a histogram for this information.

Time (*t* minutes)
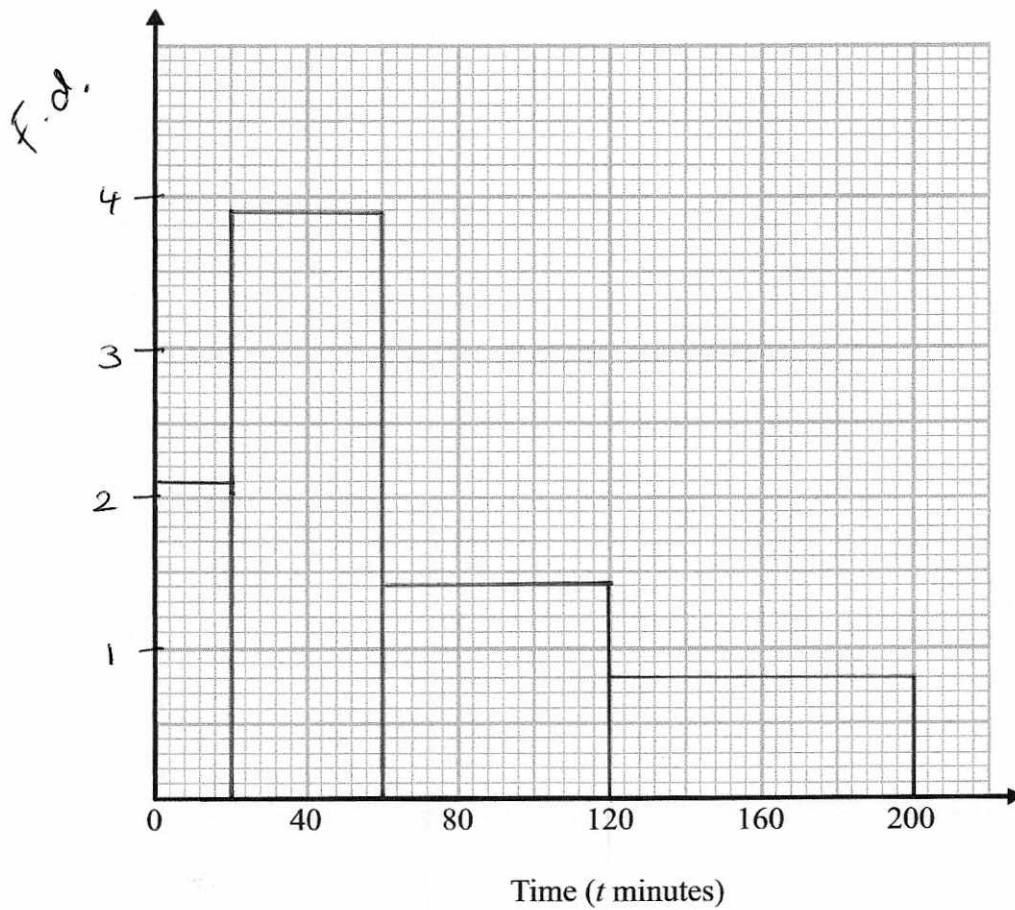
(Total for Question 15 is 4 marks)

16 Here is part of a speed-time graph for a cart.
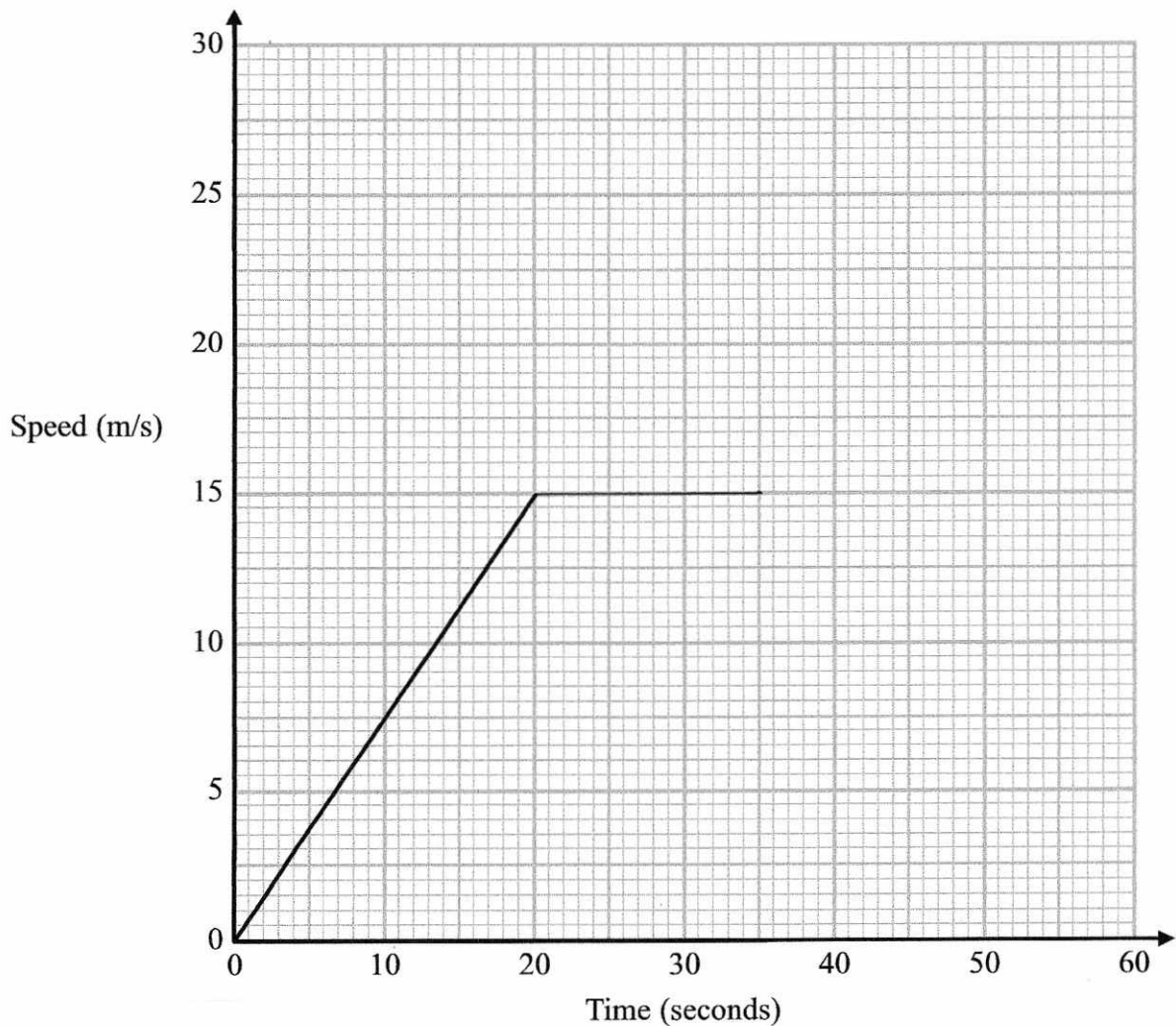
(a) Work out the acceleration of the cart during the first 20 seconds.

(gradient)

$$\frac{15}{20} = 0.75$$

.....0.75..... m/s²

(2)

When the cart has reached a speed of 15 m/s, it moves at this constant speed for 15 seconds.

(b) Show this information on the graph.

(1)

(c) Work out the total distance travelled by the cart in the first 10 seconds.

AREA

$$\frac{\overset{10}{\cancel{15} \times 7.5}}{2} = \overset{37.5}{\cancel{56.25}}$$

$\overset{37.5}{\cancel{56.25}}$ m

(2)

(Total for Question 16 is 5 marks)

17 (a) Show that $\left(5 - \sqrt{8}\right)\left(7 + \sqrt{2}\right) = 31 - 9\sqrt{2}$

Show each stage of your working.

$$35 + 5\sqrt{2} - 7\sqrt{8} - \sqrt{16}$$

$$35 + 5\sqrt{2} - 7\left(\sqrt{4}\sqrt{2}\right) - 4$$

$$31 + 5\sqrt{2} - 7\left(2\sqrt{2}\right)$$

$$31 + 5\sqrt{2} - 14\sqrt{2}$$

$$31 - 9\sqrt{2}$$

(3)

Given that c is a prime number,

(b) rationalise the denominator of $\dfrac{3c - \sqrt{c}}{\sqrt{c}}$

Simplify your answer.

$$\frac{\left(3c - \sqrt{c}\right)\left(\sqrt{c}\right)}{c}$$

$$\frac{3c\sqrt{c} - c}{c}$$

$$\frac{c\left(3\sqrt{c} - 1\right)}{c}$$

$$3\sqrt{c} - 1$$

$$\underline{3\sqrt{c} - 1}$$

(2)

(Total for Question 17 is 5 marks)

$f(x) =$ $g(x) =$

18 $f: x \mapsto 2x^2 + 1$ $g: x \mapsto \dfrac{2x}{x-1}$ where $x \neq 1$

(a) Express the composite function gf in the form gf: $x \mapsto$ …
Give your answer as simply as possible.

f in to g

$$gf(x) = \frac{2(2x^2 + 1)}{2x^2 + 1 - 1}$$

$$= \frac{4x^2 + 2}{2x^2}$$

$$= \frac{2x^2 + 1}{x^2}$$

OR $2 + \dfrac{1}{x^2}$

gf: $x \mapsto \dfrac{2x^2 + 1}{x^2}$

(2)

(b) Express the inverse function g^{-1} in the form $g^{-1}: x \mapsto$ …

$$y = \frac{2x}{x - 1}$$

Switch x and y…

$$x = \frac{2y}{y - 1}$$

Make y the subject

$$x(y - 1) = 2y$$

$$xy - x = 2y$$

$$xy = 2y + x$$

$$xy - 2y = x$$

$$y(x - 2) = x$$

$$y = \frac{x}{x - 2}$$

$g^{-1}: x \mapsto \dfrac{x}{x - 2}$

(3)

(Total for Question 18 is 5 marks)

19 (a) Show that $x^2 + 3x = 5$ can be rearranged to give: $x = \dfrac{5}{x + 3}$

$$x(x + 3) = 5$$

$$x = \frac{5}{x + 3}$$

(2)

(b) Use the iteration formula $x_{n+1} = \dfrac{5}{x_n + 3}$ with $x_0 = 1$ to find a solution for the equation $x^2 + 3x = 5$ to 1dp.

$$x_1 = \frac{5}{(1) + 3} = 1.25$$

$$x_2 = \frac{5}{(Ans) + 3} = 1.17647\ldots$$

$$x_3 = \frac{5}{(Ans) + 3} = 1.19718\ldots$$

$$x_4 = 1.191275\ldots$$

$$x = 1.2 \ (1dp)$$

1.2

(3)

(Total for Question 19 is 5 marks)

20 Solve the equation $\dfrac{3}{(x+2)} + \dfrac{4}{(x-3)} = 2$

Show clear algebraic working.

$$\frac{3(x-3)}{(x+2)(x-3)} + \frac{4(x+2)}{(x+2)(x-3)} = 2$$

$$\frac{3(x-3) + 4(x+2)}{(x+2)(x-3)} = 2$$

$$3x - 9 + 4x + 8 = 2(x+2)(x-3)$$

$$7x - 1 = 2(x^2 - 3x + 2x - 6)$$

$$7x - 1 = 2(x^2 - x - 6)$$

$$7x - 1 = 2x^2 - 2x - 12$$

$$0 = 2x^2 - 9x - 11$$

$$0 = (2x - 11)(x + 1)$$

$$x = \frac{11}{2} \qquad x = -1$$