Name: ______________________________

Maths Genie Stage 14

Test A

Instructions

- Use **black** ink or ball-point pen.
- Answer all questions.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.
- You must **show all your working out.**
- **Calculators may be used.**

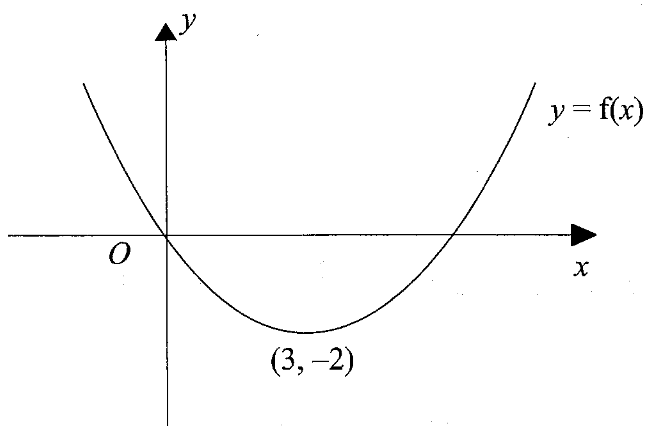
Information

- The marks for each question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end

1 The graph of $y = f(x)$ is shown below.



The coordinates of the minimum point of this curve are $(3, -2)$.

Write down the coordinates of the minimum point of the curve with equation

(a) $y = f(x + 3)$

$(0, -2)$
(1)

(b) $y = -f(x)$

$(3, 2)$
(1)

(c) $y = f(-x)$

$(-3, -2)$
(1)

(Total for Question 1 is 3 marks)

2 The point A has the coordinates (2,4)
$x_1 \ y_1$
The point B has the coordinates (6,10)
$x_2 \ y_2$

Find the equation of the perpendicular bisector to AB.

midpoint of $AB = \left(\frac{2+6}{2}, \frac{4+10}{2}\right)$

$= (4,7)$

gradient of $AB = \frac{10-4}{6-2}$

$= \frac{6}{4}$

$= \frac{3}{2}$

perp gradient $= -\frac{2}{3}$

$y = -\frac{2}{3}x + c$

$7 = -\frac{2}{3}(4) + c$

$7 = -\frac{8}{3} + c$

$21 = -8 + 3c$

$29 = 3c$

$c = \frac{29}{3}$

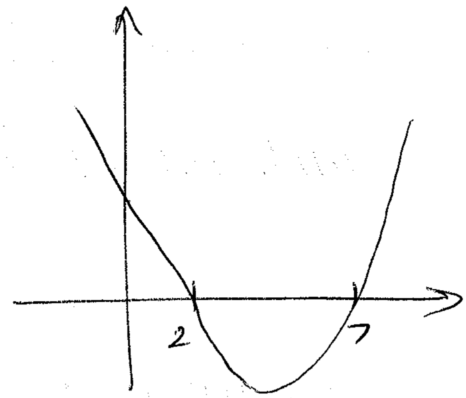
$y = -\frac{2}{3}x + \frac{29}{3}$

(Total for Question 2 is 4 marks)

3 Solve $x^2 - 9x + 14 \leq 0$

$(x-2)(x-7) \leq 0$

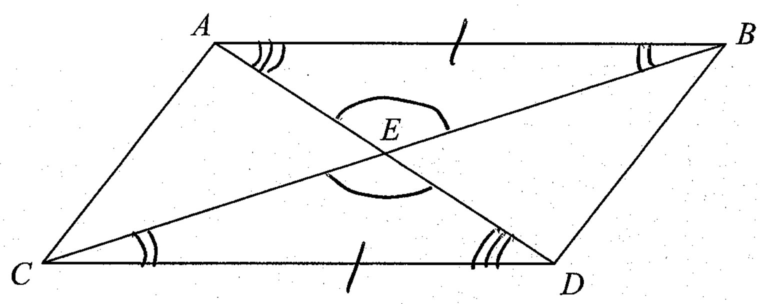
$x = 2$ and $x = 7$



$2 \leq x \leq 7$

(Total for Question 3 is 3 marks)

4 *ABCD* is a parallelogram
E is the point where the diagonals *AD* and *BC* meet.



Prove that triangle *ABE* is congruent to triangle *CDE*.

~~Angle AEB = Angle CED~~

~~opposite angles are~~ equal

AB = CD opposite sides of a parallelogram are equal

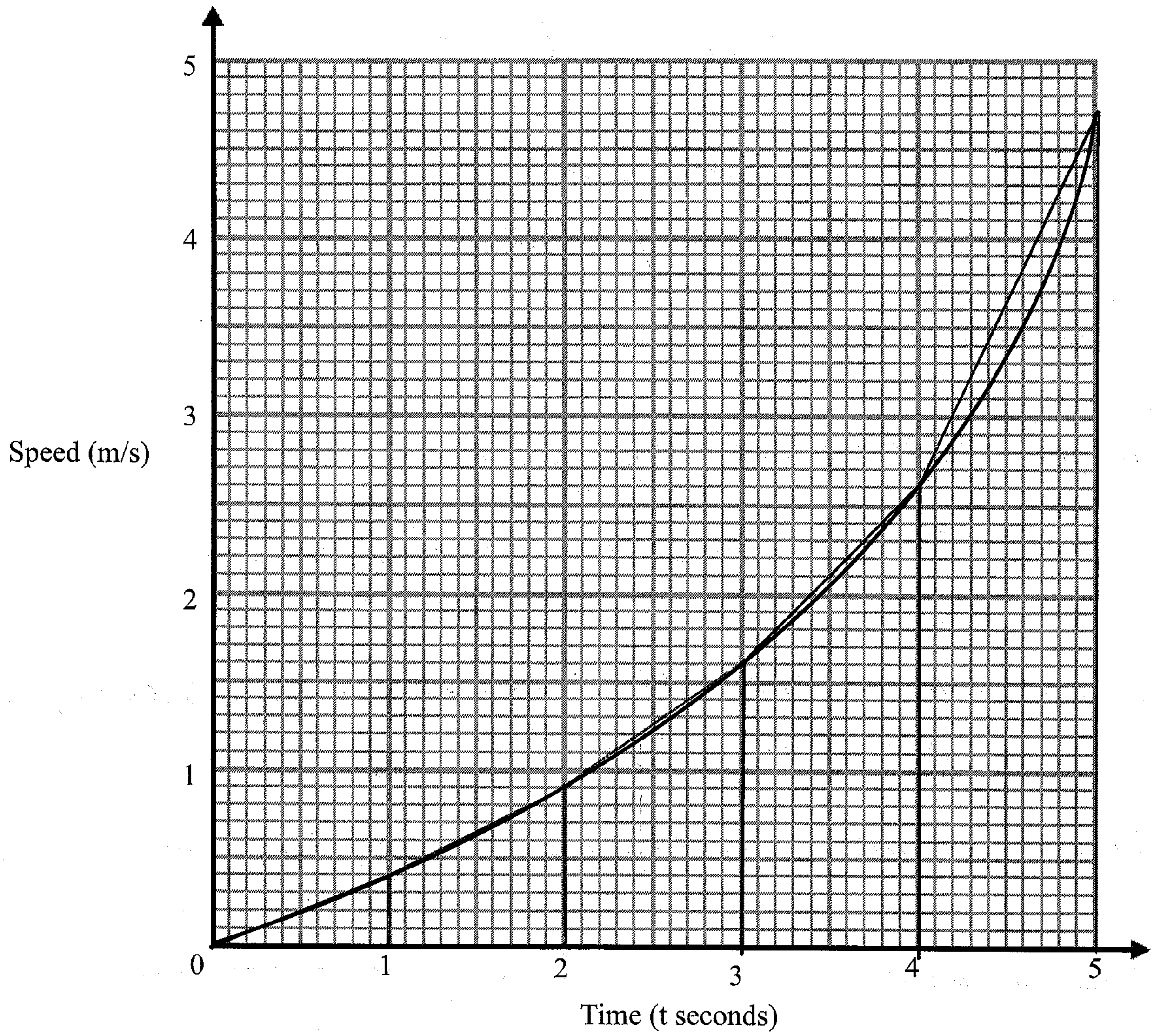
BAE = CDE alternate angles are equal

ABE = DCE ———— " ————

ASA

(Total for Question 4 is 3 marks)

5 Here is a speed-time graph.



Use 5 strips of equal width to find an estimate for the distance travelled in 5 seconds.

$\frac{1}{2}(1)(0.4) = 0.2$

$\frac{1}{2}(0.4+0.9)\times 1 = 0.65$

$\frac{1}{2}(0.9+1.6)\times 1 = 1.25$

$\frac{1}{2}(1.6+2.6)\times 1 = 2.1$

$\frac{1}{2}(2.6+4.7)\times 1 = 3.65$

$0.2+0.65+1.25$
$+2.1+3.65$
$= \underline{\underline{7.85}}$

7.85 .... m

(Total for Question 5 is 3 marks)

6 Solve the simultaneous equations

$$x^2 + y^2 = 17$$
$$y = 3x - 1$$

$$x^2 + (3x - 1)^2 = 17$$

$$x^2 + (3x - 1)(3x - 1) = 17$$

$$x^2 + 9x^2 - 3x - 3x + 1 = 17$$

$$10x^2 - 6x + 1 = 17$$

$$10x^2 - 6x - 16 = 0$$

$$5x^2 - 3x - 8 = 0$$

$$(5x - 8)(x + 1) = 0$$

$$x = \frac{8}{5} \qquad x = -1$$

$$y = 3\left(\frac{8}{5}\right) - 1 \qquad y = 3(-1) - 1$$

$$= \frac{24}{5} - 1 \qquad = -4$$

$$= \frac{19}{5}$$

$x = \frac{8}{5}$ and $y = \frac{19}{5}$ or $x = -1$ and $y = -4$

(Total for Question 6 is 5 marks)

7 There are some red counters and some blue counters in a bag.

The ratio of red counters to blue counters is 3:2

Two counters are removed at random.

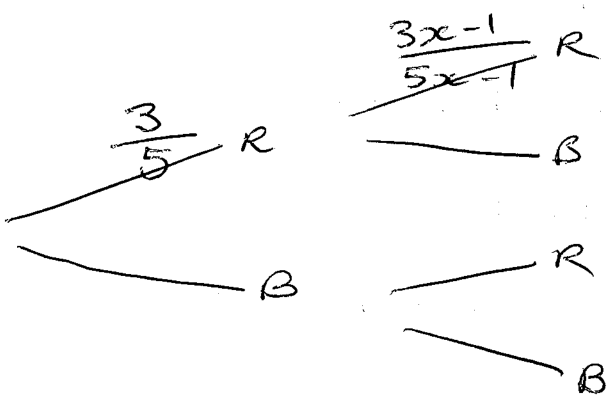
The probability that both the counters taken are red is $\frac{6}{17}$

Work how many blue counters are in the bag.

$3x$ Red

$2x$ Blue

$5x$ total.



$\frac{6}{17}$

$$\frac{3}{5} \times \frac{3x-1}{5x-1} = \frac{6}{17}$$

$$\frac{3(3x-1)}{5(5x-1)} = \frac{6}{17}$$

$$51(3x-1) = 30(5x-1)$$

$$17(3x-1) = 10(5x-1)$$

$$51x - 17 = 50x - 10$$

$$x - 17 = -10$$

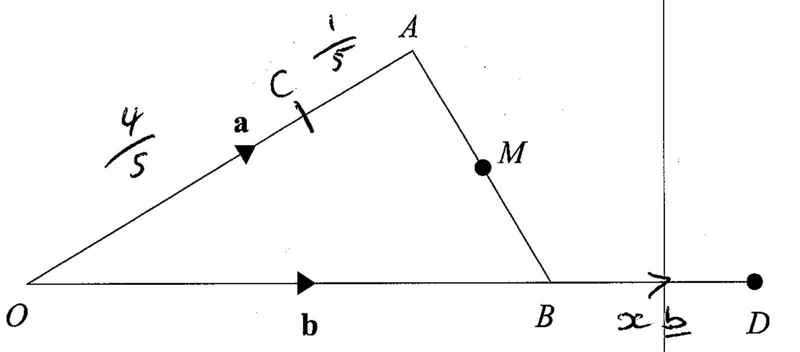
$$x = 7$$

$2 \times 7 = 14$ Blue Counters

14

(Total for Question 7 is 5 marks)

8



$\overrightarrow{OA} = a$

$\overrightarrow{OB} = b$

C is the point on OA such that OC:CA = 4:1
M is the midpoint of AB

Given that C, M and D are on the same straight line find OB:BD

Let $\overrightarrow{BD} = x\underline{b}$

$\overrightarrow{AB} = -a + b$

$\overrightarrow{AM} = -\frac{1}{2}a + \frac{1}{2}b$

$\overrightarrow{CM} = \frac{1}{5}a - \frac{1}{2}a + \frac{1}{2}b$

$= -\frac{3}{10}a + \frac{1}{2}b$

$\overrightarrow{CD} = k(-\frac{3}{10}a + \frac{1}{2}b)$

$\overrightarrow{CD} = -\frac{4}{5}a + b + xb$

$-\frac{3}{10}ka + \frac{1}{2}kb = -\frac{4}{5}a + (1+x)b$

$\underline{\underline{a}}$

$-\frac{3}{10}k = \frac{-4}{5}$

$k = \frac{40}{15} = \frac{8}{3}$

$\underline{\underline{b}}$

$\frac{1}{2}\left(\frac{8}{3}\right) = 1 + x$

$\frac{8}{6} = 1 + x$

$\frac{4}{3} = 1 + x$

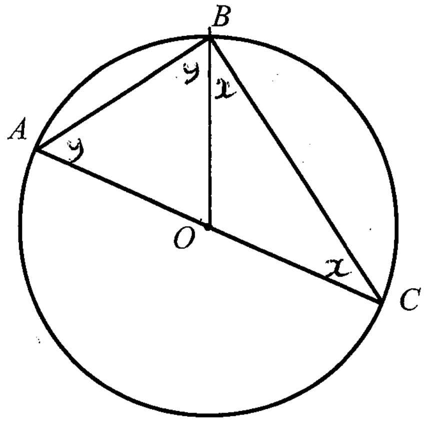
$x = \frac{1}{3}$

$\overrightarrow{BD} = \frac{1}{3}\overrightarrow{OB}$

$OB : BD = \underline{\underline{3 : 1}}$

(Total for Question 8 is 5 marks)

9



A, B and C are points on the circumference of a circle, centre O.
AOC is a diameter of the circle.

Prove that angle ABC is 90°
You must **not** use any circle theorems in your proof.

Let $OBC = x$ Let $OAB = y$

$OCB = x$ Angles at the base of an isosceles triangle are equal

$BAO = y$ —— " ——

$BOC = 180 - 2x$ Angles in a triangle add to 180°

$BOA = 180 - 2y$ —— " ——

$180 - 2x + 180 - 2y = 180$ Angles on a straight line add to 180°

$360 - 2x - 2y = 180$

$180 = 2x + 2y$

$90 = x + y$

$ABC = x + y$

$= \underline{90°}$

(Total for Question 9 is 4 marks)