Name: _____

Maths Genie Stage 12

Test C

Instructions

- Use **black** ink or ball-point pen.
- Answer all questions.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.
- You must **show all your working out.**
- **Calculators may be used.**

Information

- The marks for each question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end

1 x is inversely proportional to the square root of y

When $x = 14$, $y = 16$

Find the value of x when $y = 64$

$$x \propto \frac{1}{\sqrt{y}}$$

$$x = \frac{k}{\sqrt{y}}$$

$$14 = \frac{k}{\sqrt{16}}$$

$$k = 56$$

$$x = \frac{56}{\sqrt{y}}$$

$$x = \frac{56}{\sqrt{64}}$$

$$= 7$$

$x =$ 7

(Total for Question 1 is 3 marks)

---

2 There are 12 boys and x girls in a choir.
One boy and one girl will be selected to sing a duet.
Taylor says there are 174 different ways of choosing a boy and a girl.

Could Taylor be correct?
You must show your working.

$$\frac{174}{12} = 14.5$$

No. The number of girls must be a whole number, there cannot be 14.5 girls.

(Total for Question 3 is 2 marks)

3 The function f is defined such that

$$f(x) = 2x^2 - 1$$

(a) Find an expression for $f(x - 2)$

$$f(x-2) = 2(x-2)^2 - 1$$

$$= 2(x-2)(x-2) - 1$$

$$= 2(x^2 - 2x - 2x + 4) - 1$$

$$= 2x^2 - 4x - 4x + 8 - 1$$

$$= 2x^2 - 8x + 7$$

$2x^2 - 8x + 7$

(2)

(b) Hence solve: $f(x - 2) = 0$
Give your answers correct to 3 significant figures.

$$2x^2 - 8x + 7 = 0$$

$$a = 2 \quad b = -8 \quad c = 7$$

$$x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4(2)(7)}}{2(2)}$$

$$= 2.71 \text{ or } 1.29$$

$x = 2.71$ or 1.29

(3)

(Total for Question 3 is 5 marks)

4 Factorise $6x^2 - 7x - 5$

$6 \times 5 = 30$

1	30
2	15
3	10
5	6

$(6x + 3)(x - \frac{10}{6})$

$(6x + 3)(x - \frac{5}{3})$

or $(2x + 1)(3x - 5)$

$(2x + 1)(3x - 5)$

(Total for Question 4 is 2 marks)

5 Cylinder A and cylinder B are mathematically similar.
The total surface area of cylinder A is 100 cm^2 and the total surface area of cylinder B is 144 cm^2.

Cylinder A has a height of 7 cm

Calculate the height of cylinder B.

Area s.f $= \frac{144}{100} = \frac{36}{25}$

Length s.f $= \sqrt{\frac{36}{25}} = \frac{6}{5}$

$7 \times \frac{6}{5} = 8.4$ cm

8.4 cm

(Total for Question 5 is 3 marks)

6 On Monday, a company's share price increased by 15%
On Tuesday, the company's share price decreased by 10%

Katie says: "The share price has now increased by 5%".

Is Katie correct?
You must show your working.

$100 \times 1.15 \times 0.9 = 103.5$

No. The price has increased by 3.5%.

(Total for Question 6 is 2 marks)

---

7 Here are the first 5 terms of a quadratic sequence.

1 8 21 40 65

Find an expression, in terms of n, for the nth term of this sequence.

$a+b+c \rightarrow 1 \quad 8 \quad 21 \quad 40$

$3a+b \rightarrow 7 \quad 13 \quad 19$

$\rightarrow 6 \quad 6$

$2a$

$2a = 6$

$a = 3$

$3a+b = 7$

$3(3)+b = 7$

$9+b = 7$

$b = -2$

$a+b+c = 1$

$3-2+c = 1$

$1+c = 1$

$c = 0$

$3n^2 - 2n$

(Total for Question 7 is 4 marks)

8 (a) Show that the equation $5x^3 - x^2 - 8 = 0$ has a solution between $x = 1$ and $x = 2$.

$5(1)^3 - (1)^2 - 8 = -4$

$5(2)^3 - (2)^2 - 8 = 28$

change of sign $\therefore$ solution between 1 and 2

(2)

(b) Show that the equation $5x^3 - x^2 - 8 = 0$ can be rearranged to give: $x = \sqrt{\dfrac{8}{5x - 1}}$

$5x^3 - x^2 = 8$

$x^2(5x - 1) = 8$

$x^2 = \dfrac{8}{5x - 1}$

$x = \sqrt{\dfrac{8}{5x - 1}}$

(1)

(c) Starting with $x_0 = 1$, use the iteration formula $x_{n+1} = \sqrt{\dfrac{8}{5x_n - 1}}$ twice to find an estimate for the solution to $5x^3 - x^2 - 8 = 0$

$x_1 = \sqrt{\dfrac{8}{5(1) - 1}} = \sqrt{2}$

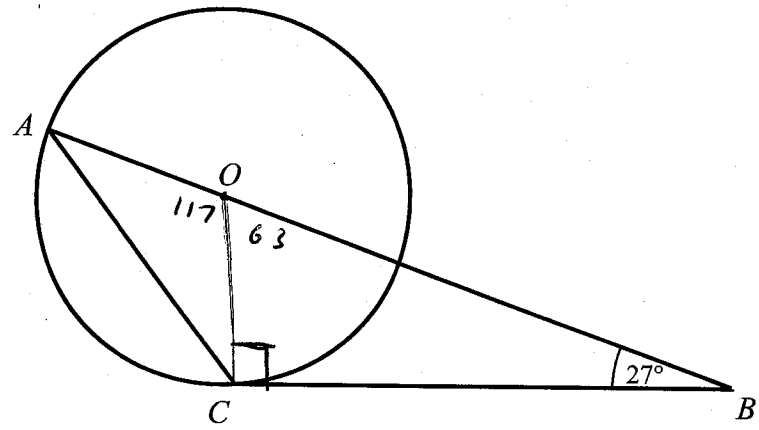
$x_2 = \sqrt{\dfrac{8}{5(\text{Ans}) - 1}} = 1.147922191$

1.147922191

(2)

(Total for Question 8 is 5 marks)

9



A and *C* are points on the circumference of a circle, centre *O*.
BC is a tangent to the circle.

Angle *ABC* = 27°

Find the size of angle *CAB*.
You must show all your working.

$OCB = 90°$ Tangent meets radius at 90°

$COB = 180 - 90 - 27 = 63°$ Angles in a triangle add to 180°

$AOC = 180 - 63 = 117°$ Angles on a straight line add to 180°

$CAB = \frac{180 - 117}{2} = 31.5°$ Angles at the base of an isosceles triangle are equal

31.5 °

(Total for Question 9 is 4 marks)