Write your name here

Surname	Other Names

Mathematics

November 2022 Practice Paper 2 (Calculator)
Higher Tier

Time: 1 hour 30 minutes

You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser, calculator. Tracing paper may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided – there may be more space than you need.
- **Calculators may be used**.
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.
- You must **show all your working.**

Information

- The total mark for this paper is 80
- The marks for **each** question are shown in brackets – use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

Higher Tier Formulae Sheet

Perimeter, area and volume

Where a and b are the lengths of the parallel sides and h is their perpendicular separation:

Area of a trapezium = $\frac{1}{2}(a + b)\, h$

Volume of a prism = area of cross section × length

Where r is the radius and d is the diameter:

Circumference of a circle = $2\pi r = \pi d$

Area of a circle = πr^2

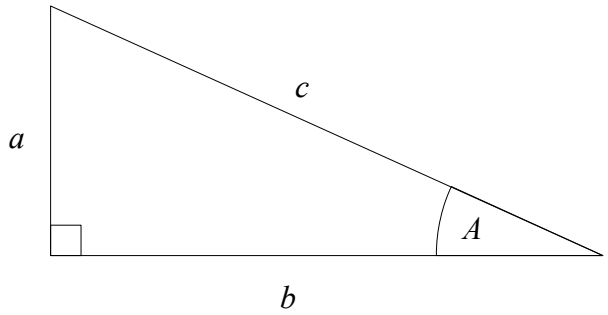
Quadratic formula

The solution of $ax^2 + bx + c = 0$

where $a \neq 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Pythagoras' Theorem and Trigonometry

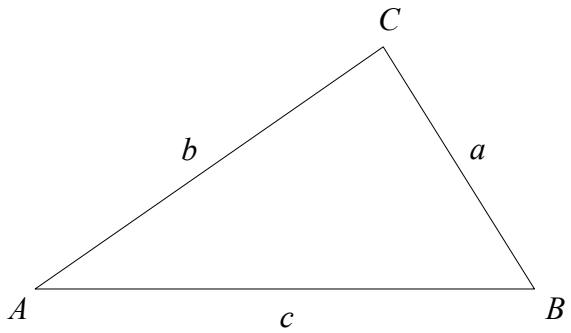


In any right-angled triangle where a, b and c are the length of the sides and c is the hypotenuse:

$$a^2 + b^2 = c^2$$

In any right-angled triangle ABC where a, b and c are the length of the sides and c is the hypotenuse:

$$\sin A = \frac{a}{c} \qquad \cos A = \frac{b}{c} \qquad \tan A = \frac{a}{b}$$



In any triangle ABC where a, b and c are the length of the sides:

sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle = $\frac{1}{2}ab \sin C$

Compound Interest

Where P is the principal amount, r is the interest rate over a given period and n is number of times that the interest is compounded:

Total accrued = $P\left(1 + \frac{r}{100}\right)^n$

Probability

Where P (A) is the probability of outcome A and P (B) is the probability of outcome B:

P (A or B) = P (A) + P (B) – P (A and B)

P (A and B) = P (A given B) P(B)

END OF EXAM AID

1 Solve $a^2 - 10a + 16 = 0$

16

1 16

2 8

4 4

$(a-2)(a-8) = 0$

$a = 2 \quad a = 8$

$a = 2$ or $a = 8$

(Total for Question 1 is 3 marks)

---

2 Here are a list of ingredients for making 12 flapjacks.

225 g of butter
75g of sugar
4 tbsp of honey
350g of oats

Connor wants to make 20 flapjacks.

How much of each ingredient will Connor need?

Butter $\frac{225}{12} \times 20 = 375$ g

Sugar $\frac{75}{12} \times 20 = 125$ g

Honey $\frac{4}{12} \times 20 = 6\frac{2}{3}$ tbsp $[6.\dot{6}]$

Oats $\frac{350}{12} \times 20 = 583.\dot{3}$ g

butter ..... 375 ..... g

sugar ..... 125 ..... g

honey ..... $6\frac{2}{3}$ ..... tbsp

oats ..... $583.\dot{3}$ ..... g

(Total for Question 2 is 3 marks)

3 Here are the first 5 terms of a sequence.

9 14 19 24 29

Find an expression, in terms of n, for the nth term of this sequence.

$5n$ 5 10 15 20 25

$5n + 4$

(Total for Question 3 is 2 marks)

4 Here is a list of seven numbers.
One of the numbers is hidden.

11	6	7	10	7	9	?

The mean of the numbers is 9.

Find the value of the hidden number.

$7 \times 9 = 63$ (sum of 7 numbers)

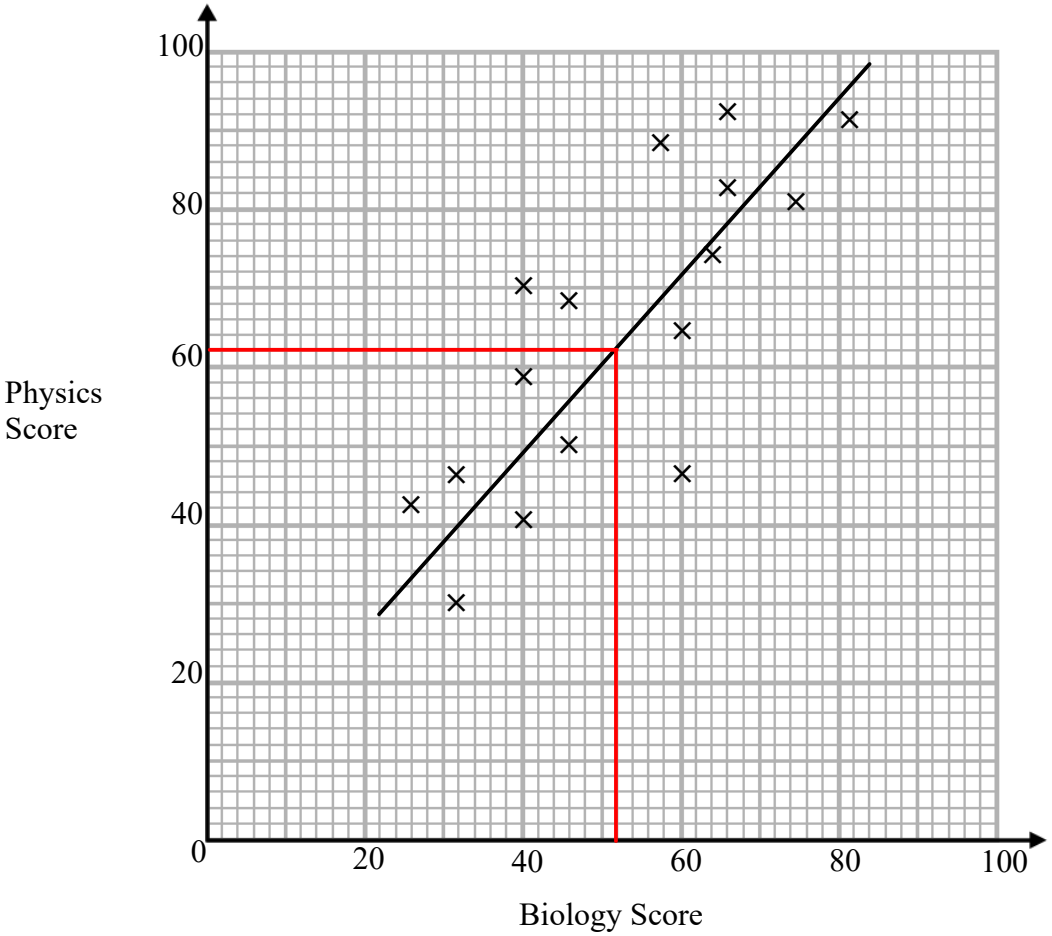
$11 + 6 + 7 + 10 + 7 + 9 = 50$

$63 - 50 = 13$

13

(Total for Question 4 is 2 marks)

5 The scatter graph shows the scores of 16 students on their Biology and Physics tests.



(a) What type of correlation does the scatter graph show?

positive

(1)

(b) Another students scored 52 marks on their Biology test.
Estimate the Physics score for this student.

62

(2)

(Total for Question 5 is 3 marks)

6 In a sale, the normal price of a TV is reduced by 20%.
The sale price of the TV is £660

Work out the normal price of the TV.

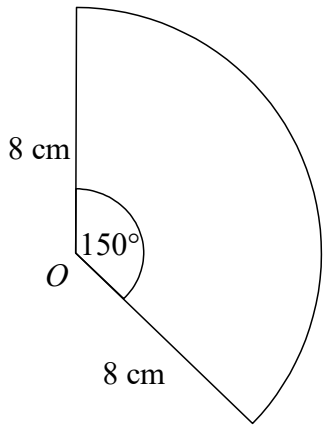
$$x \times 0.8 = 660$$
$$x = \frac{660}{0.8}$$
$$= 825$$

£ 825

(Total for Question 6 is 2 marks)

---

7 The diagram shows a sector, centre *O*.
The radius of the circle is 8 cm.
The angle of the sector is 150°.



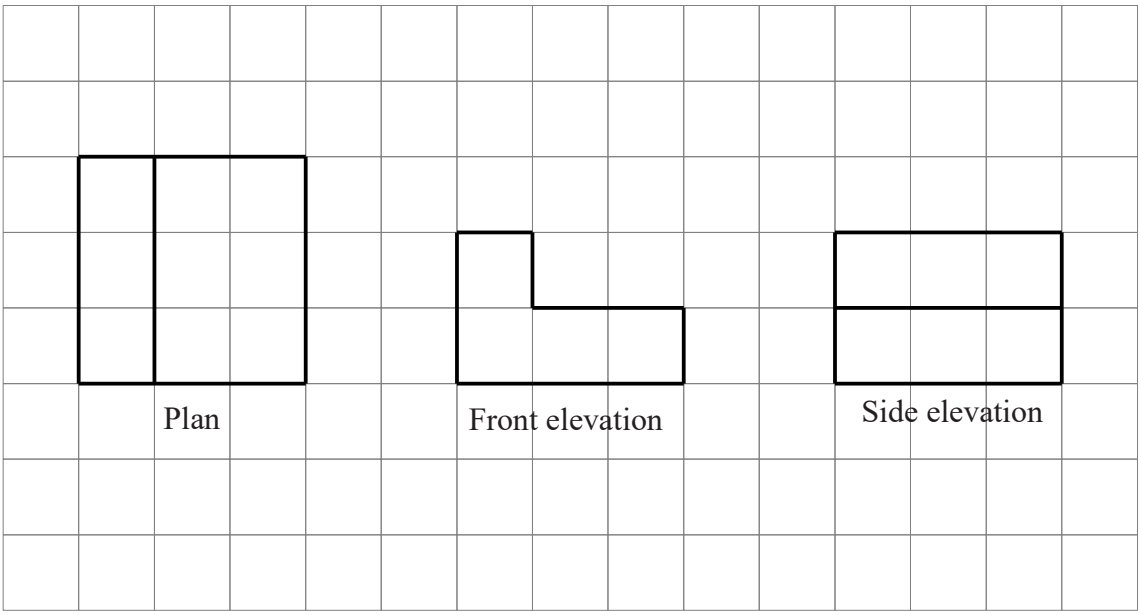
Calculate the area of the sector.
Give your answer correct to 3 significant figures.

$$\frac{150}{360} \times \pi(8)^2 = 83.8 \text{ cm}^2$$

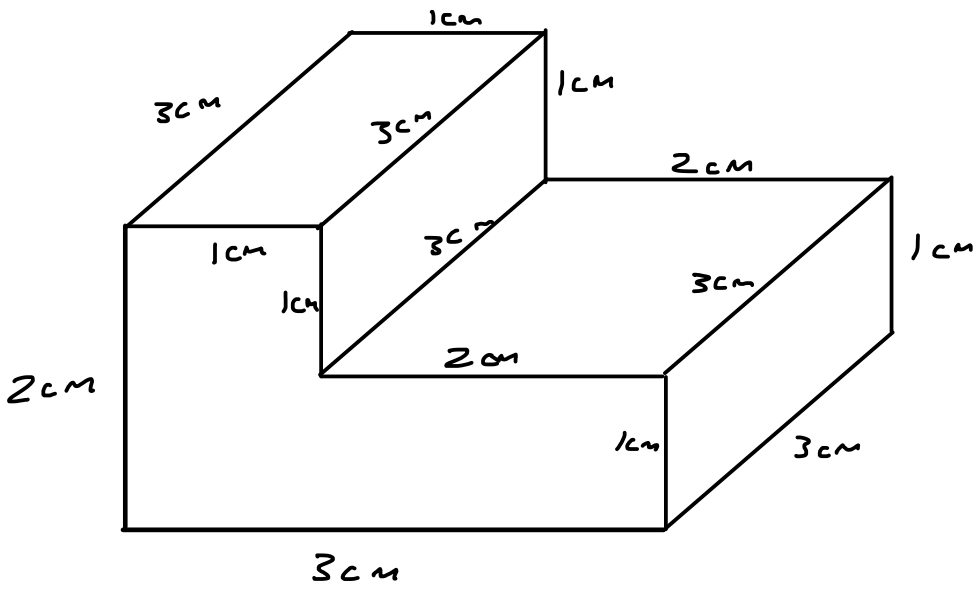
83.8 cm^2

(Total for Question 7 is 2 marks)

8 The diagram shows the plan, front elevation and side elevation of a solid shape, drawn on a centimetre grid.



In the space below, draw a sketch of the solid shape.
Give the dimensions of the solid on your sketch.



(Total for Question 8 is 2 marks)

9 Matt wants to invest £8000 for three years. He can choose between Bank A and Bank B.

Bank A	**Bank B**
1.2% compound interest per annum	2% compound interest in the first year 1% compound interest for each extra year

Which bank will give Matt the most interest after three years.
You must show your working.

A

8000×1.012^3

$= 8291.47$

B

$8000 \times 1.02 \times 1.01^2$

$= 8324.02$

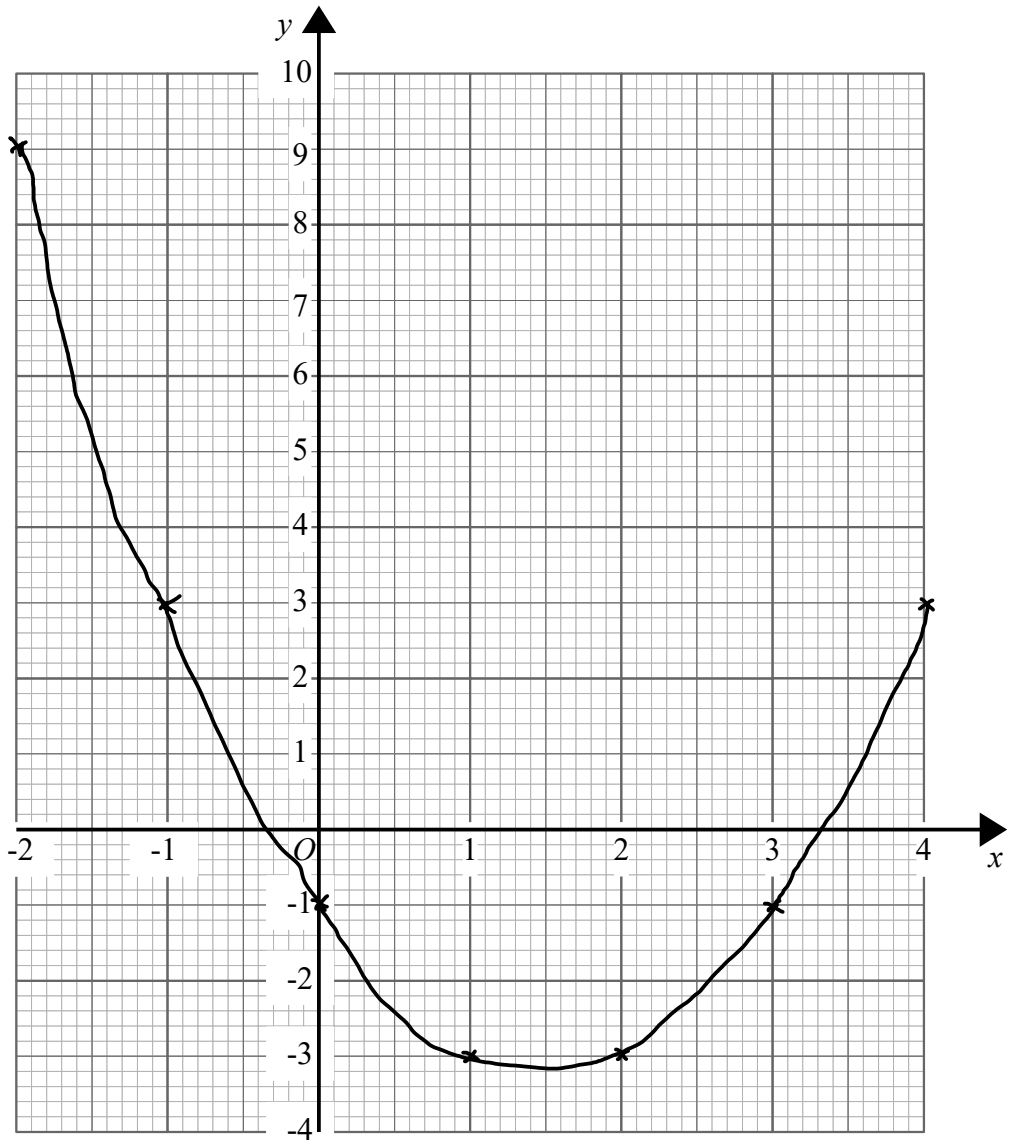
Bank B

(Total for Question 9 is 4 marks)

10 Complete the table of values for $y = x^2 - 3x - 1$

x	-2	-1	0	1	2	3	4
y	9	3	-1	-3	-3	-1	3

(2)



(a) On the grid draw the graph of $y = x^2 - 3x - 1$ for values of x from –2 to 4 (2)

(b) Use the graph to find an estimate of the turning point of the graph $y = x^2 - 3x - 1$

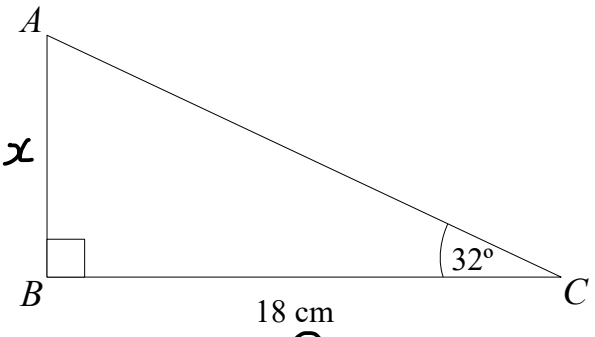
(1.5, -3.2)

(2)

(Total for Question 10 is 6 marks)

(-3.25)

11



Calculate the area of triangle *ABC*.

$\tan\theta = \frac{O}{A}$

$\tan(32) = \frac{x}{18}$

$x = 18\tan(32)$

$= 11.2476\ldots$

$\text{Area} = \frac{1}{2}bh$

$= \frac{1}{2}(18)(11.2476)$

$= 101.2288\ldots$

…….101……..cm^2

(Total for Question 11 is 4 marks)

$x_1\ y_1 \quad x_2\ y_2$

12 Line A passes through the points (-2, 1) and (4, 10)
Find the equation of the line parallel to A that passes through (2,7)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{10 - 1}{4 - -2}$$

$$= \frac{3}{2}$$

$y = \frac{3}{2}x + c \qquad (2, 7)$
$\qquad\qquad\quad x \quad y$

$7 = \frac{3}{2}(2) + c$

$7 = 3 + c$

$c = 4$

$y = \frac{3}{2}x + 4$

(Total for Question 12 is 3 marks)

---

13 Prove algebraically that the recurring decimal $0.68\dot{1}$ can be written as $\frac{15}{22}$

$0.68\dot{1} = x$

$6.8\dot{1} = 10x$

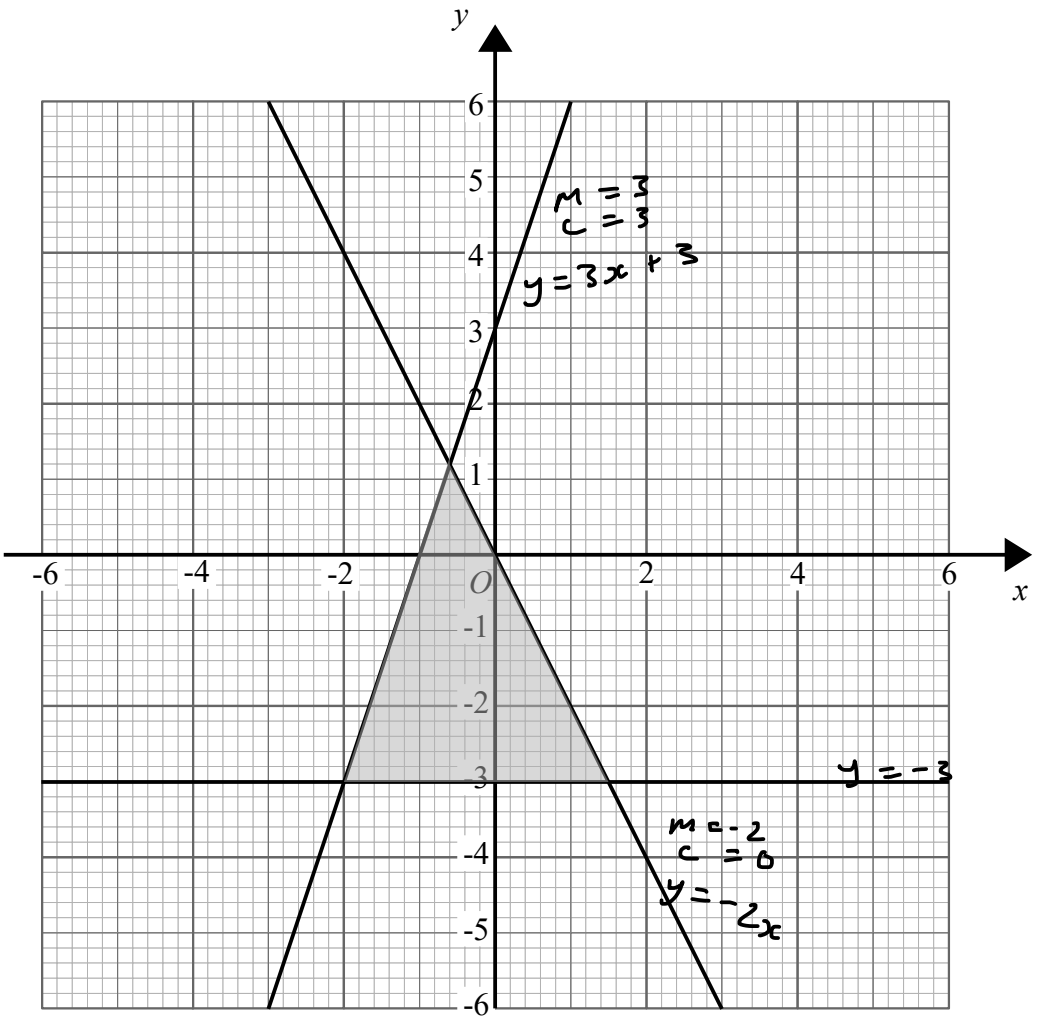
$681.8\dot{1} = 1000x$

$675 = 990x$

$x = \frac{675}{990} = \frac{15}{22}$

(Total for Question 13 is 2 marks)

14



Write down the three inequalities that define the shaded region

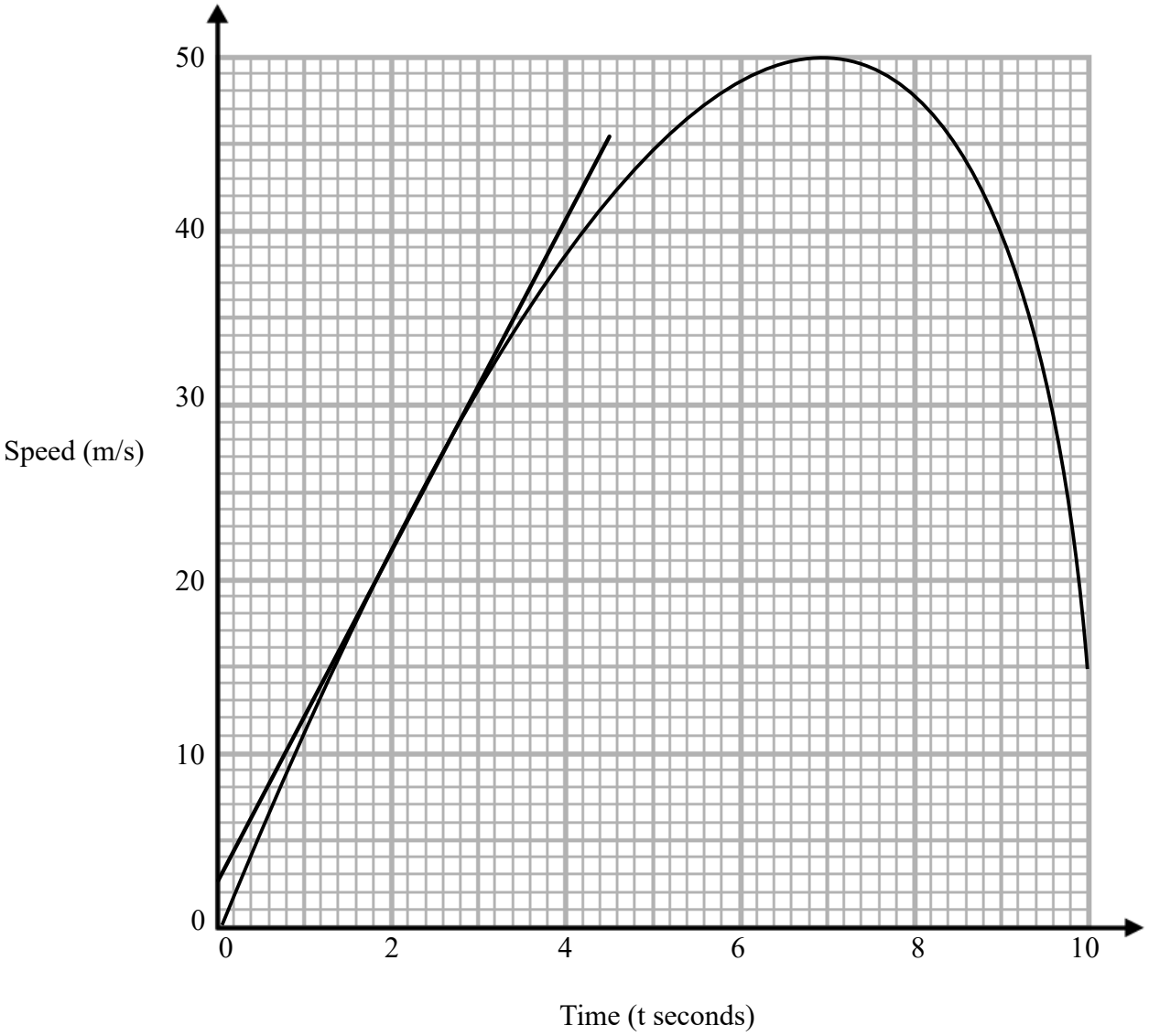
$y \geqslant -3$

$y \leqslant -2x$

$y \leqslant 3x + 3$

(Total for Question 14 is 4 marks)

15 Here is a speed-time graph.



(a) Work out an estimate for the gradient when t = 2.

(0, 3) (4, 41) $\frac{41-3}{4} = 9.5$

9.5

(2)

(b) What does the gradient of this curve represent?

The acceleration.

(1)

(Total for Question 15 is 3 marks)

16 Karen buys a pack of 8 bottles of water.
The pack costs £1.25

Karen sells all 8 bottles of water for 50p each.

Work out Karen's percentage profit.

$8 \times 50 = £4.00$

Profit $= 4 - 1.25 = 2.75$

$\frac{\text{change}}{\text{original}} \times 100$

$\frac{2.75}{1.25} \times 100 = 220\%$

.....220..... %

(Total for Question 16 is 2 marks)

17 Greg bought a new car for £18 000.
In the first year the value of the car depreciates by 30%.
In the second year and the third year the car depreciates by 14%

Work out the value of the car after three years.

$18000 \times 0.7 \times 0.86^2$

$= £9318.96$

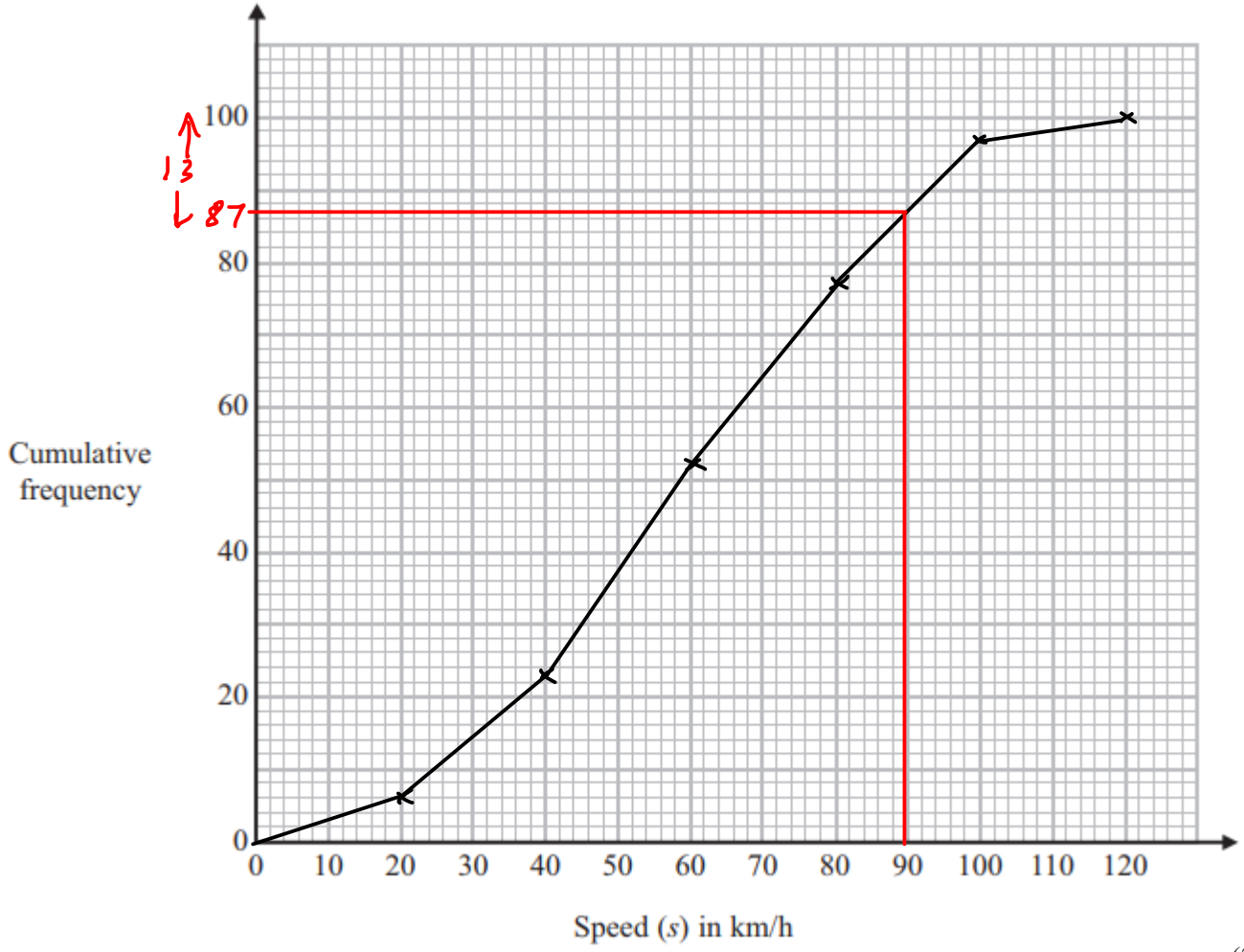
£.....9318.96.....

(Total for Question 17 is 3 marks)

18 The frequency table shows the speeds of 100 cars.

Speed (km/h)	Frequency	CF
$0 < s \leqslant 20$	6	6
$20 < s \leqslant 40$	17	23
$40 < s \leqslant 60$	29	52
$60 < s \leqslant 80$	25	77
$80 < s \leqslant 100$	20	97
$100 < s \leqslant 120$	3	100

(a) On the grid, plot a cumulative frequency graph for this information.



(2)

(b) Find an estimate for the number of cars travelling over 90 km/h.

13

(1)

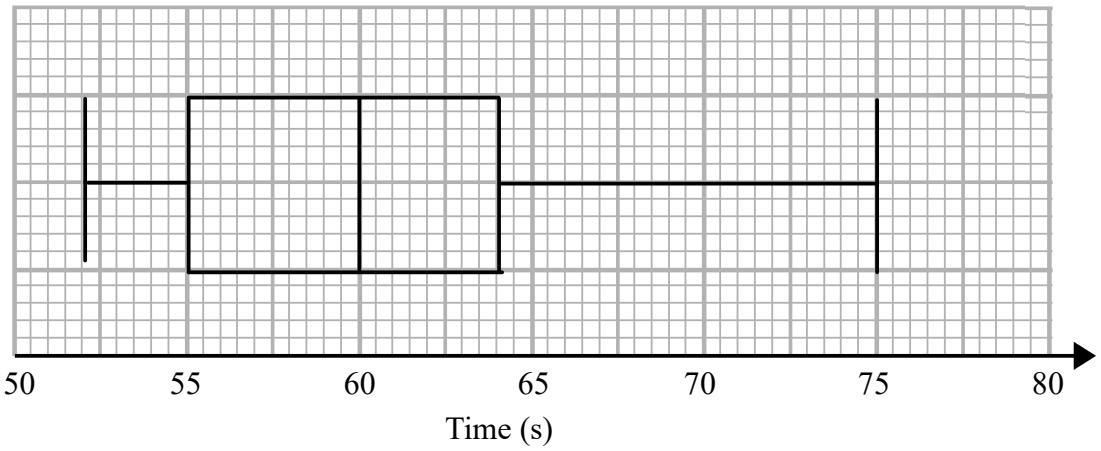
(Total for Question 18 is 3 marks)

19 The times, in seconds, of 15 students running a race are recorded below.

52 54 54 55 58 58 59 60 60 61 61 64 67 70 75

Lowest L.Q Median U.Q Highest

Draw a box plot for this information.



(Total for Question 19 is 2 marks)

---

20 Beth wants to estimate the number of frogs in a lake.

She catches a sample of 80 frogs, marks them and puts them back in the lake.
Later that day, in a second sample of 80 frogs, she finds that 10 of them are marked.

Work out an estimate for the number of frogs in the lake

$$\frac{80}{x} = \frac{10}{80}$$

$$\frac{80}{x} = \frac{1}{8}$$

$$640 = x$$

640

(Total for Question 20 is 2 marks)

21 The number of rabbits in a field t days from now is P_t where

$P_0 = 220$
$P_{t+1} = 1.15(P_t - 20)$

Work out the number of rabbits in the garden 3 days from now.

$P_1 = 1.15(220 - 20) = 230$

$P_2 = 1.15(\text{Ans} - 20) = 241.5$

$P_3 = 1.15(\text{Ans} - 20) = 254.725$

[255]

255

(Total for Question 21 is 3 marks)

22 X and Y are two geometrically similar solid shapes.

The total surface area of shape X is 450 cm^2
The total surface area of shape Y is 800 cm^2

The volume of shape X is 1350 cm^3

Calculate the volume of shape Y.

$\frac{800}{450} = \frac{16}{9}$ scale factor for area

$\sqrt{\frac{16}{9}} = \frac{4}{3}$ scale factor for length

$\left(\frac{4}{3}\right)^3 = \frac{64}{27}$ (volume scale factor)

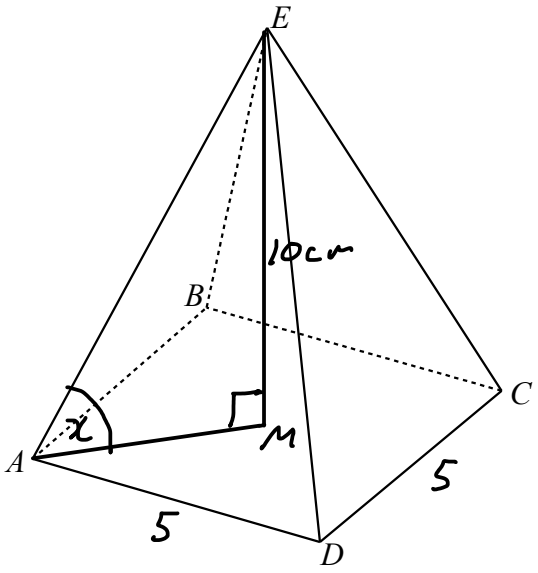
$1350 \times \frac{64}{27} = 3200$

3200 cm^3

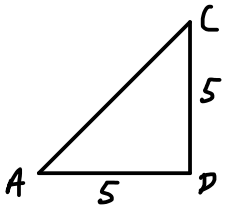
(Total for Question 22 is 3 marks)

23 The diagram shows a pyramid.
The base of the pyramid $ABCD$ is a square.

$AB = 5$ cm
The point E is 10 cm vertically above the base.

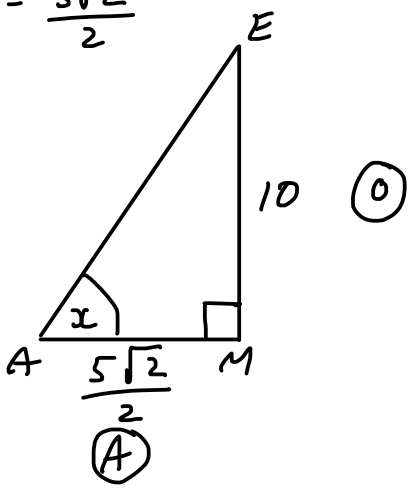


Calculate the size of angle EAC.



$AC^2 = 5^2 + 5^2$
$AC^2 = 50$
$AC = \sqrt{50} = 5\sqrt{2}$

$AM = \frac{5\sqrt{2}}{2}$



(0)

$\tan x = \frac{10}{\frac{5\sqrt{2}}{2}}$

$x = \tan^{-1}(2\sqrt{2})$

$= 70.5287...$

(A)

70.5 °

(Total for Question 23 is 4 marks)

24 Given that $f(x) = 2x - 4$ and $g(x) = 3x + 5$

(a) Find $gf(3)$

$f(3) = 2(3) - 4$

$= 2$

$g(2) = 3(2) + 5$

$= 11$

11

(2)

(b) Work out an expression for $f^{-1}(x)$

$f(x) = 2x - 4$

$f(x) + 4 = 2x$

$\frac{f(x) + 4}{2} = x$

$f^{-1}(x) = \frac{x + 4}{2}$

$f^{-1}(x) = \frac{x + 4}{2}$

(2)

(Total for Question 24 is 4 marks)

25 A circle has the equation $x^2 + y^2 = 12.25$

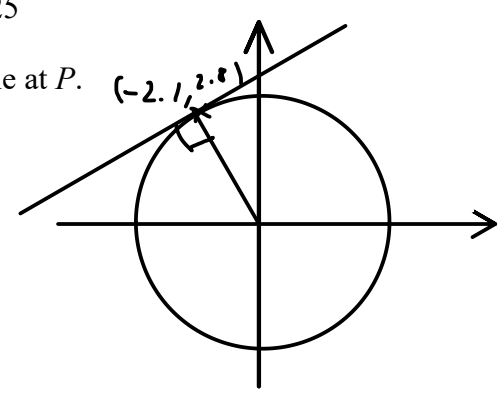
(a) Write down the length of the radius of the circle.

$\sqrt{12.25}$

3.5

(1)

P is the point (-2.1, 2.8) on the circle $x^2 + y^2 = 12.25$



(b) Work out the equation of the tangent to the circle at P.

(0, 0) (-2.1, 2.8)

$m\ (\text{radius}) = \frac{2.8 - 0}{-2.1 - 0}$

$= -\frac{4}{3}$

$m\ (\text{tangent}) = \frac{3}{4}$ (perpendicular)

$y = \frac{3}{4}x + c \quad (-2.1, 2.8)$

$2.8 = \frac{3}{4}(-2.1) + c$

$2.8 = -\frac{63}{40} + c$

$c = \frac{35}{8}$

$y = \frac{3}{4}x + \frac{35}{8}$

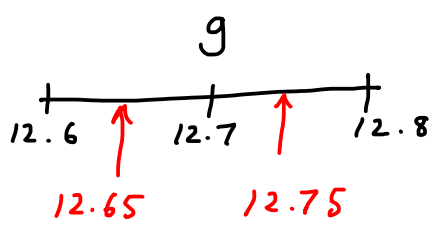
(4)

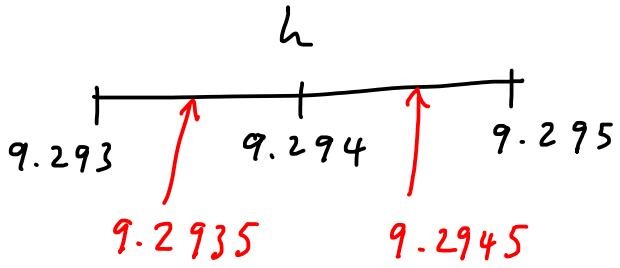
(Total for Question 25 is 5 marks)

26 $f = \dfrac{\sqrt{g}}{h}$

g = 12.7 correct to 3 significant figures
h = 9.294 correct to 3 decimal places

By considering bounds, work out the value of f to a suitable degree of accuracy.
Give a reason for your answer.





upper $f = \dfrac{\sqrt{12.75}}{9.2935}$

$= 0.384216$

Lower $f = \dfrac{\sqrt{12.65}}{9.2945}$

$= 0.382665$

Both round to 0.38 (2 dp)

0.38

(Total for Question 26 is 4 marks)