Mathematics
June 2023 Practice Paper 3 (Calculator)
Higher Tier

Time: 1 hour 30 minutes

You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser, calculator. Tracing paper may be used.

Total Marks

Instructions

• Use **black** ink or ball-point pen.
• **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
• Answer **all** questions.
• Answer the questions in the spaces provided
– there may be more space than you need.
• **Calculators may be used.**
• Diagrams are **NOT** accurately drawn, unless otherwise indicated.
• You must **show all your working.**

Information

• The total mark for this paper is 80
• The marks for **each** question are shown in brackets
– use this as a guide as to how much time to spend on each question.

Advice

• Read each question carefully before you start to answer it.
• Keep an eye on the time.
• Try to answer every question.
• Check your answers if you have time at the end.

Higher Tier Formulae Sheet

Perimeter, area and volume

Where a and b are the lengths of the parallel sides and h is their perpendicular separation:

$$\text{Area of a trapezium} = \frac{1}{2}(a+b)h$$

Volume of a prism = area of cross section × length

Where r is the radius and d is the diameter:

$$\text{Circumference of a circle} = 2\pi r = \pi d$$

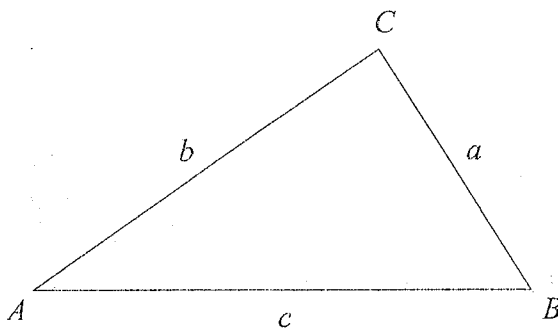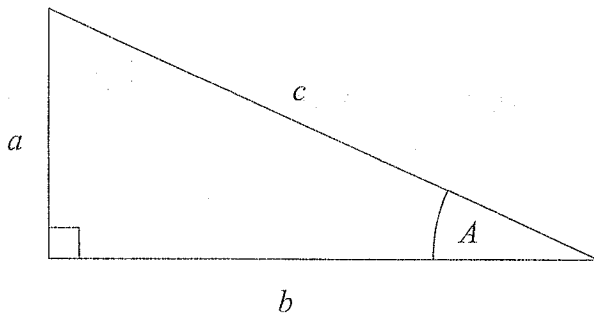$$\text{Area of a circle} = \pi r^2$$

Quadratic formula

The solution of $ax^2 + bx + c = 0$

where $a \neq 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Pythagoras' Theorem and Trigonometry

In any right-angled triangle where a, b and c are the length of the sides and c is the hypotenuse:

$$a^2 + b^2 = c^2$$

In any right-angled triangle ABC where a, b and c are the length of the sides and c is the hypotenuse:

$$\sin A = \frac{a}{c} \qquad \cos A = \frac{b}{c} \qquad \tan A = \frac{a}{b}$$

In any triangle ABC where a, b and c are the length of the sides:

sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$

$$\text{Area of triangle} = \frac{1}{2}ab \sin C$$

Compound Interest

Where P is the principal amount, r is the interest rate over a given period and n is number of times that the interest is compounded:

$$\text{Total accrued} = P\left(1 + \frac{r}{100}\right)^n$$

Probability

Where P (A) is the probability of outcome A and P (B) is the probability of outcome B:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A \text{ and } B) = P(A \text{ given } B)\, P(B)$$

END OF EXAM AID

1 $A = 2^3 \times 3^2 \times 5$ $B = 2^2 \times 3^3 \times 5$

Write down the highest common factor (HCF) of A and B

They both have: $2^2 \times 3^2 \times 5$ $=$ 180

............180............

(Total for Question 1 is 1 mark)

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2   Verity buys 12 bottles of apple juice for a total cost of £15
Verity sells all 12 bottles at £1.75 each bottle.

Work out Verity's percentage profit.

% Profit $= \dfrac{Profit}{original} \times 100$

$12 \times 1.75 = 21$

$Profit = 21 - 15 = £6$

% Profit $= \dfrac{6}{15} \times 100$

............40............ %

(Total for Question 2 is 3 marks)

3   The table shows the populations of five countries.

Country	Population
India	1.4×10^9
Turkey	8.4×10^7
Denmark	5.8×10^6
Estonia	1.3×10^6
Iceland	3.4×10^5

(a) Work out the difference between the population of India and the population of Turkey.
Give your answer in standard form.

$$1.4 \times 10^9 - 8.4 \times 10^7 = 1316000000$$

$$1.316 \times 10^9$$

(2)

Given that

$$\text{population of Iceland} = \frac{1}{k} \times \text{population of Denmark}$$

(b) Work out the value of k.
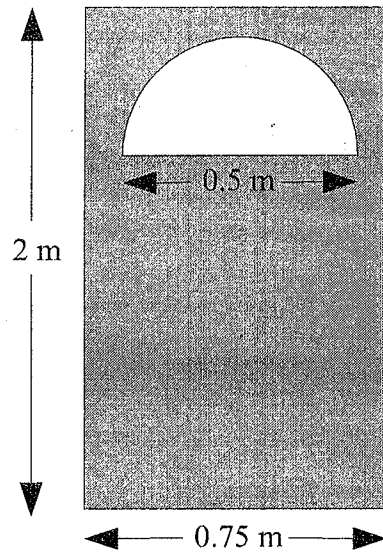Give your answer correct to the nearest whole number.

$$\frac{5.8 \times 10^6}{3.4 \times 10^5} = 17 \ (nearest \ whole \ no.)$$

$k = \underline{\quad 17 \quad}$

(2)

(Total for Question 3 is 4 marks)

4 The diagram shows the front of a wooden door with a semicircular glass window.

2 m

0.5 m

$r = 0.25$

0.75 m

Julie wants to apply 2 coats of wood varnish to the front of the door, shown shaded in the diagram.

250 millilitres of wood varnish covers 4 m² of the wood.

Work out how many millilitres of wood varnish Julie will need.
Give your answer correct to the nearest millilitre.

$$\text{Shaded Area} = 2(0.75) - \frac{\pi(0.25)^2}{2}$$

$$= 1.40 \ m^2$$

2 coats $\quad 1.40 \times 2 = 2.80 \ m^2$

250 mL covers 4 m²
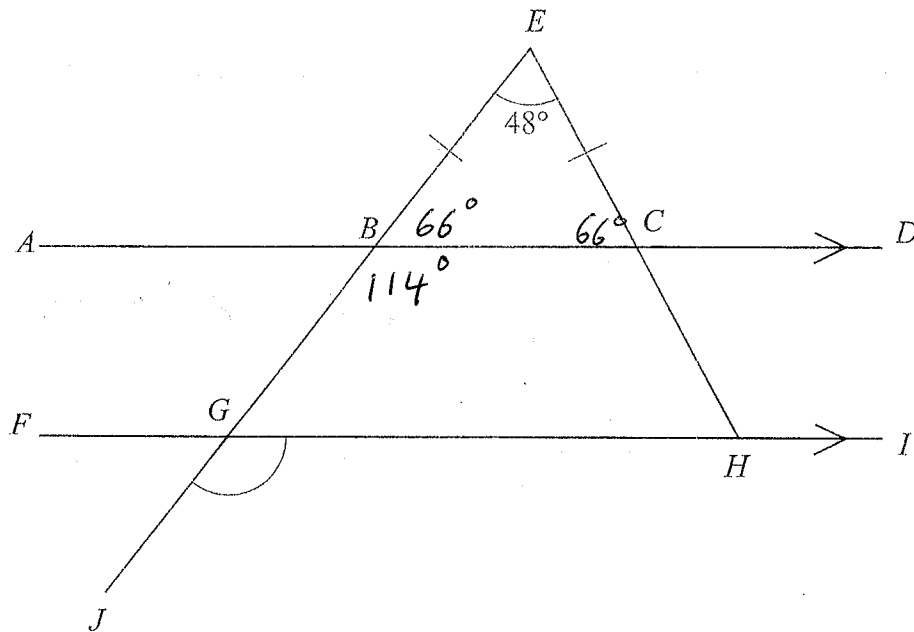$\div 4 \qquad\qquad \div 4$
62.5 mL covers 1 m²

$2.80 \times 62.5 = 175.2 \ mL$ ....**175**............ millilitres

(Total for Question 4 is 5 marks)

5

E

48°

A ——————— B / 66°   66° C ————————————→ D

114°

F ——————— G ——————————————————→ I

H

J

ABCD and *FGHI* are parallel straight lines.
EBGJ and *ECH* are straight lines.

$BE = CE$
Angle $BEC = 48°$

Work out the size of angle *JGH*.
Give a reason for each stage of your working.

$$\frac{180 - 48}{2} = 66°$$
(EBC)

Angles at the base of an
isosceles triangle are equal.

$180 - 66 = 114°$
(GBC)

Angles on a straight line

$JGH = 114°$ Corresponding angles.

_____114_____ °

(Total for Question 5 is 5 marks)

6 Adam drove 56 km from Liverpool to Manchester.
He then drove 61 km from Manchester to Sheffield.

$$Speed = \frac{distance}{time}$$

Adam's average speed from Liverpool to Manchester was 80 km/h.
Adam took 85 minutes to drive from Manchester to Sheffield.

(a) Work out Adam's average speed for his total drive from Liverpool to Sheffield.

$L \rightarrow m$

$D = 56$
$S = 80$

$Time = \frac{56}{80} = 0.7$ hours

$M \rightarrow S$

85 mins

$\frac{85}{60} = 1.41\dot{6}$ hours

$Av\ Speed = \frac{56 + 61}{0.7 + 1.41\dot{6}}$

$= 55.3$

........55.3........ km/h

(4)

Bev drove from Reading to Bristol.

Bev's average speed from Reading to Swindon was 80 km/h.
Her average speed from Swindon to Bristol was 60 km/h.

Bev says that the average speed from Reading to Bristol can be found by working out the mean of 80 km/h and 60 km/h.

(b) If Bev is correct, what does this tell you about the two parts of Bev's journey?

The time must be equal

(1)

(Total for Question 6 is 5 marks)

7   A company has to make a large number of boxes.

The company has 6 machines.
All the machines work at the same rate.
When all the machines are working, they can make all the boxes in 9 days.

2 of the machines are not working and cannot be used to make the boxes.

Work out the total number of days taken to make all the boxes.

$$6 \times 9 = 54 \text{ days of work}$$

$$\frac{54}{4} = 13.5 \text{ days}$$

13.5

8 Janet bought a new car three years ago.

At the end of the first year the value of the car had decreased by 17.5%
The value of the car decreased by 10% each year for the next two years.

At the end of the three years, the car was worth £16038

Work out the value of the car when Janet bought it three years ago.

$$x \times 0.825 \times 0.9^2 = 16038$$

$$x = \frac{16038}{0.825 \times 0.9^2}$$

$$= 24000$$

£ 24000

(Total for Question 8 is 3 marks)

9 Simplify fully $\left(\dfrac{16 x^4}{9 y^{10}}\right)^{-\frac{1}{2}}$

square root and flip

$$\left(\frac{4x^2}{3y^5}\right)^{-1}$$

$$\frac{3y^5}{4x^2}$$

$$\frac{3y^5}{4x^2}$$

(Total for Question 9 is 3 marks)

10 Expand and simplify $(2-x)(2x+3)(x+4)$

$$\left(4x + 6 - 2x^2 - 3x\right)\left(x + 4\right)$$
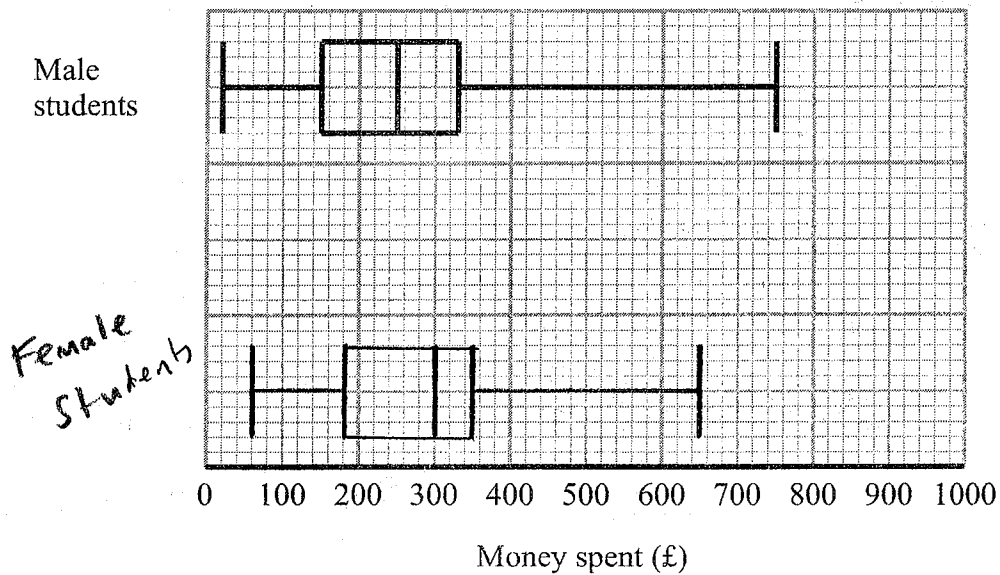
$$\left(x + 6 - 2x^2\right)\left(x + 4\right)$$

$$x^2 + 4x + 6x + 24 - 2x^3 - 8x^2$$

$$10x - 2x^3 - 7x^2 + 24$$

$$-2x^3 - 7x^2 + 10x + 24$$

(Total for Question 10 is 3 marks)

11 The box plot shows information about the distribution of the amounts of money spent by some male students on their holidays.

Male students

Female students

0   100   200   300   400   500   600   700   800   900   1000

Money spent (£)

(a) Work out the interquartile range for the amounts of money spent by these male students.

$$330 - 150$$

£ _____ 180 _____

(2)

The table below shows information about the distribution of the amounts of money spent by some female students on their holidays.

	Smallest	Lower quartile	Median	Upper quartile	Highest
Money spent (£)	60	180	300	350	650

(b) On the grid above, draw a box plot for the information in the table

(2)

Chris says,

"The box plots show that the female students spent more money than the male students."

(c) Is Chris correct?
Give a reason for your answer.

Yes — the median amount female students spent
is greater
(other comparisons are acceptable)

(1)

(Total for Question 11 is 5 marks)

12   Using algebra, show that the recurring decimal $0.18\overset{\bullet}{0}\overset{\bullet}{3} = \dfrac{119}{660}$

$$0.1803 = x$$

$$18.03 = 100x$$

$$1803.03 = 10000x$$

$$1785 = 9900x$$

$$x = \frac{1785}{9900} = \frac{119}{660}$$

13   Saffron wants to work out an estimate for the total number of fish in a lake.
On Friday, Saffron catches 210 fish from the lake.

She puts a tag on each of these fish and puts them back into the lake.
On Saturday, Saffron catches 240 fish from the same lake.

She finds that 35 of the 240 fish are tagged.
Work out an estimate for the total number of fish in the lake.
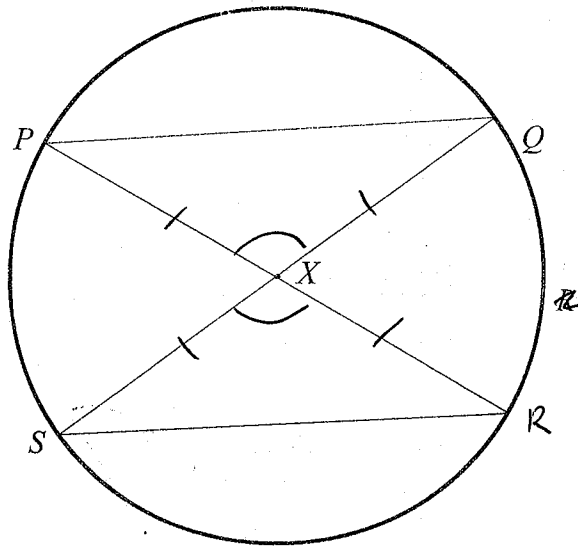
$$\frac{210}{x} = \frac{35}{240} \quad \times 6$$

$$x = 1440$$

1440

14   *P, Q, R* and *S* are four points on a circle, centre *X*.

PXR and *SXQ* are straight lines.

Prove that triangle *PQX* and triangle *SRX* are congruent.
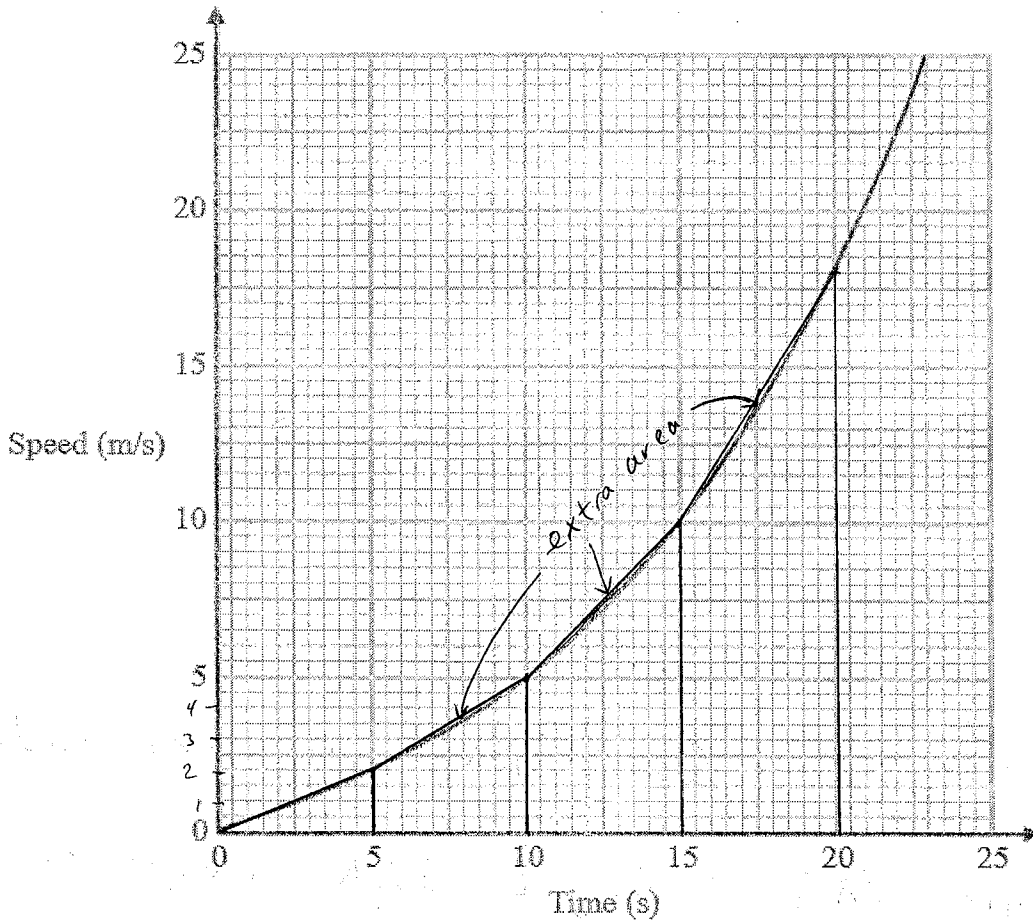
$$SX = PX = QX = RX \quad \text{all the radius}$$

Angle PXQ = Angle SXR (opposite angles)

$\therefore$ SAS

OR Angles in the same segment are equal
and ASA

15   Here is a speed-time graph for a train.

(a) Work out an estimate for the distance the train travelled in the first 20 seconds.
Use 4 strips of equal width.

$\frac{1}{2}(5)(2) = 5$

$\frac{1}{2}(2+5) \times 5 = 17.5$

$\frac{1}{2}(5+10) \times 5 = 37.5$

$\frac{1}{2}(10+18) \times 5 = 70$

$5 + 17.5 + 37.5 + 70$

_____130_____ m

(3)

(b) Is your answer to (a) an underestimate or an overestimate of the actual distance the train travelled?
Give a reason for your answer.

overestimate — the trapeziums go over the curve (extra area) (The curve is curving up)

(1)

(Total for Question 15 is 4 marks)

16 Using algebra, prove that, given any 3 consecutive even numbers, the difference between the square of the largest number and the square of the smallest number is always 8 times the middle number.

$$2n, \quad 2n+2, \quad 2n+4$$

$$(2n+4)^2 - (2n)^2$$

$$(2n+4)(2n+4) - 4n^2$$

$$4n^2 + 8n + 8n + 16 - 4n^2$$

$$16n + 16$$

$$8(2n+2) = 16n + 16$$

Both equal $16n + 16$

(Total for Question 16 is 3 marks)

17 The straight line L passes through the points $(4, -2)$ and $(6, 5)$

$$x_1 \quad y_1 \quad x_2 \quad y_2$$

The straight line M is perpendicular to L and intersects the y-axis at the point $(0, 3)$

Find the coordinates of the point where M intersects the x-axis.

$$M = \frac{\text{change in } y}{\text{change in } x}$$

$$= \frac{5 - -2}{6 - 4} = \frac{7}{2}$$

perpendicular gradient $= -\frac{2}{7}$ $\left(\begin{array}{c}\text{negative}\\\text{reciprocal}\end{array}\right)$

$$y = -\frac{2}{7}x + 3$$

Crosses x when $y = 0$

$$0 = -\frac{2}{7}x + 3$$

$$-3 = -\frac{2}{7}x$$

$$-21 = -2x$$

$$x = 10.5$$

$(\underline{10.5}, \underline{0})$

(Total for Question 17 is 4 marks)

18 Show that $\dfrac{x}{x+3} - \dfrac{2x-1}{x-3} + 1$ can be written in the form $\dfrac{ax+b}{x^2-9}$

where a and b are integers.

$$\frac{x(x-3)}{(x+3)(x-3)} - \frac{(2x-1)(x+3)}{(x+3)(x-3)} + \frac{(x+3)(x-3)}{(x+3)(x-3)}$$

$$\frac{x^2 - 3x - (2x^2 + 6x - x - 3) + x^2 - 3x + 3x - 9}{x^2 - 9}$$

$$\frac{x^2 - 3x - 2x^2 - 6x + x + 3 + x^2 - 9}{x^2 - 9}$$

$$\frac{-8x - 6}{x^2 - 9}$$

(Total for Question 18 is 4 marks)

19 (a) Express $9 + 8x - 2x^2$ in the form $a + b(x + c)^2$ where a, b and c are integers.

$$9 - 2(x^2 - 4x)$$

$$9 - 2((x-2)^2 - 4)$$

$$9 - 2(x-2)^2 + 8$$

$$17 - 2(x-2)^2$$

$$\underline{17 - 2(x - 2)^2}$$
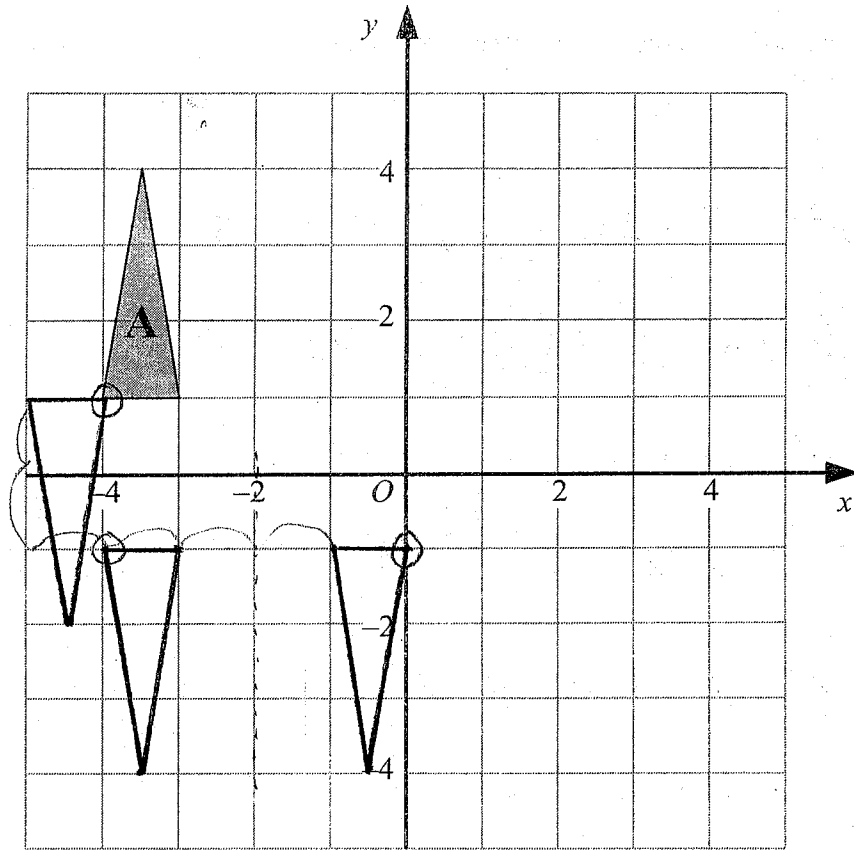
(3)

C is the curve with equation $y = 9 + 8x - 2x^2$

The point A is the maximum point on **C**

(b)   Use your answer to part (a) to write down the coordinates of A

($\underline{\quad 2 \quad}$, $\underline{\quad 17 \quad}$)

(1)

(Total for Question 19 is 4 marks)

Triangle **A** is transformed by the combined transformation of a reflection in the x-axis, followed by a

reflection in the line $x = -2$, followed by a translation with vector $\begin{pmatrix} -4 \\ 2 \end{pmatrix}$

One point on triangle **A** is invariant under the combined transformation.
Find the coordinates of this point.

You must show your working.

(-4 , 1)

(Total for Question 20 is 2 marks)

21 Prove algebraically that the straight line with equation $2x + y + 5 = 0$ is a tangent to the circle with equation $x^2 + y^2 = 5$

$$y = -2x - 5$$

$$x^2 + (-2x - 5)^2 = 5$$

$$x^2 + (-2x - 5)(-2x - 5) = 5$$

$$x^2 + 4x^2 + 10x + 10x + 25 = 5$$

$$5x^2 + 20x + 20 = 0$$

$$x^2 + 4x + 4 = 0$$

$$(x + 2)(x + 2) = 0$$
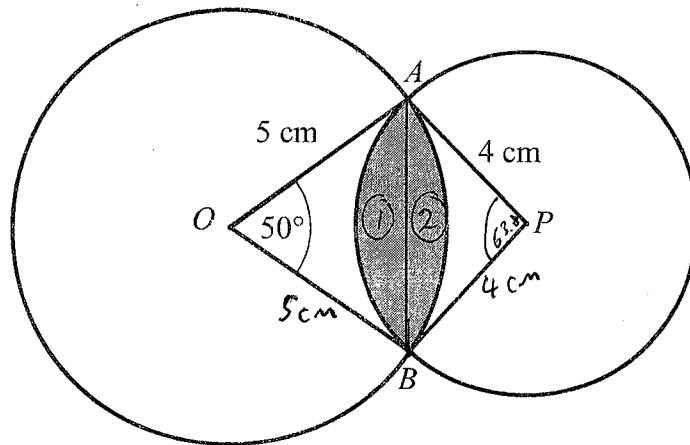
$$(x + 2)^2 = 0$$

$$x = -2$$

$$y = -2(-2) - 5$$

$$= -1$$

The line only touchs the circle at one point $(-2, -1)$ ∴ it is a tangent

22 The diagram shows two circles such that the region **R**, shown shaded in the diagram, is the region common to both circles.

One of the circles has centre O and radius 5 cm.
The other circle has centre P and radius 4 cm.
Angle $AOB = 50°$

Calculate the area of region **R**.
Give your answer correct to 3 significant figures.

$a^2 = b^2 + c^2 - 2bc \cos A$

$a^2 = 5^2 + 5^2 - 2(5)(5) \cos(50)$

$a^2 = 17.86$

$a = 4.226 \text{ cm}$

$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

$= \dfrac{4^2 + 4^2 - 4.226^2}{2(4)(4)}$

$= 0.442$

$A = \cos^{-1}(0.442)$

$= 63.8°$

Segment Area =
Area of Sector -
Area of triangle

①
$\dfrac{63.8}{360} \times \pi(4)^2 - \dfrac{1}{2}(4)(4)\sin(63.8)$

$= 1.728 \text{ cm}^2$

②
$\dfrac{50}{360} \times \pi(5)^2 - \dfrac{1}{2}(5)(5)\sin 50$

$= 1.333 \text{ cm}^2$

$1.728 + 1.333$

$= 3.06 \text{ cm}^2$

3.06 cm²

(Total for Question 22 is 6 marks)

TOTAL FOR PAPER IS 80 MARKS