Write your name here

Surname	Other Names

Mathematics

2022 Paper 3 (Calculator)
Higher Tier

Time: 1 hour 30 minutes

You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser, calculator. Tracing paper may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided – there may be more space than you need.
- **Calculators may be used**.
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.
- You must **show all your working.**

Information

- The total mark for this paper is 80
- The marks for **each** question are shown in brackets – use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

Higher Tier Formulae Sheet

Perimeter, area and volume

Where a and b are the lengths of the parallel sides and h is their perpendicular separation:

Area of a trapezium = $\frac{1}{2}(a + b)\, h$

Volume of a prism = area of cross section × length

Where r is the radius and d is the diameter:

Circumference of a circle = $2\pi r = \pi d$

Area of a circle = πr^2

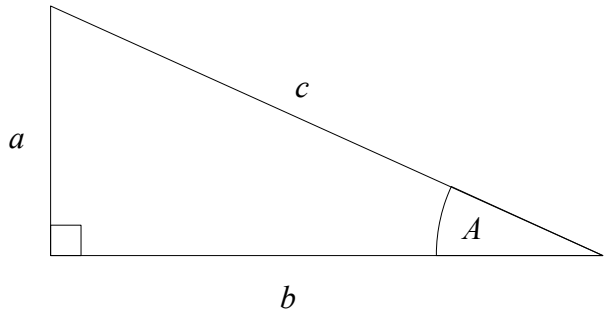
Quadratic formula

The solution of $ax^2 + bx + c = 0$

where $a \neq 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Pythagoras' Theorem and Trigonometry

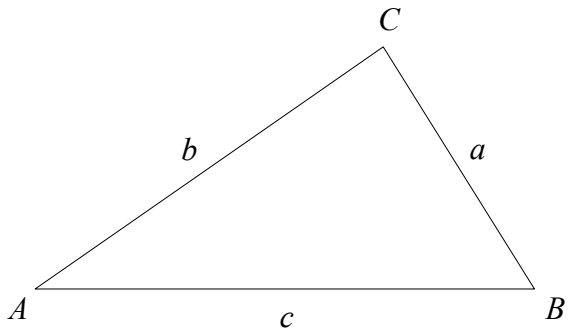


In any right-angled triangle where a, b and c are the length of the sides and c is the hypotenuse:

$$a^2 + b^2 = c^2$$

In any right-angled triangle ABC where a, b and c are the length of the sides and c is the hypotenuse:

$$\sin A = \frac{a}{c} \qquad \cos A = \frac{b}{c} \qquad \tan A = \frac{a}{b}$$



In any triangle ABC where a, b and c are the length of the sides:

sine rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

cosine rule: $a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle = $\frac{1}{2}ab \sin C$

Compound Interest

Where P is the principal amount, r is the interest rate over a given period and n is number of times that the interest is compounded:

Total accrued = $P\left(1 + \frac{r}{100}\right)^n$

Probability

Where P (A) is the probability of outcome A and P (B) is the probability of outcome B:

P (A or B) = P (A) + P (B) – P (A and B)

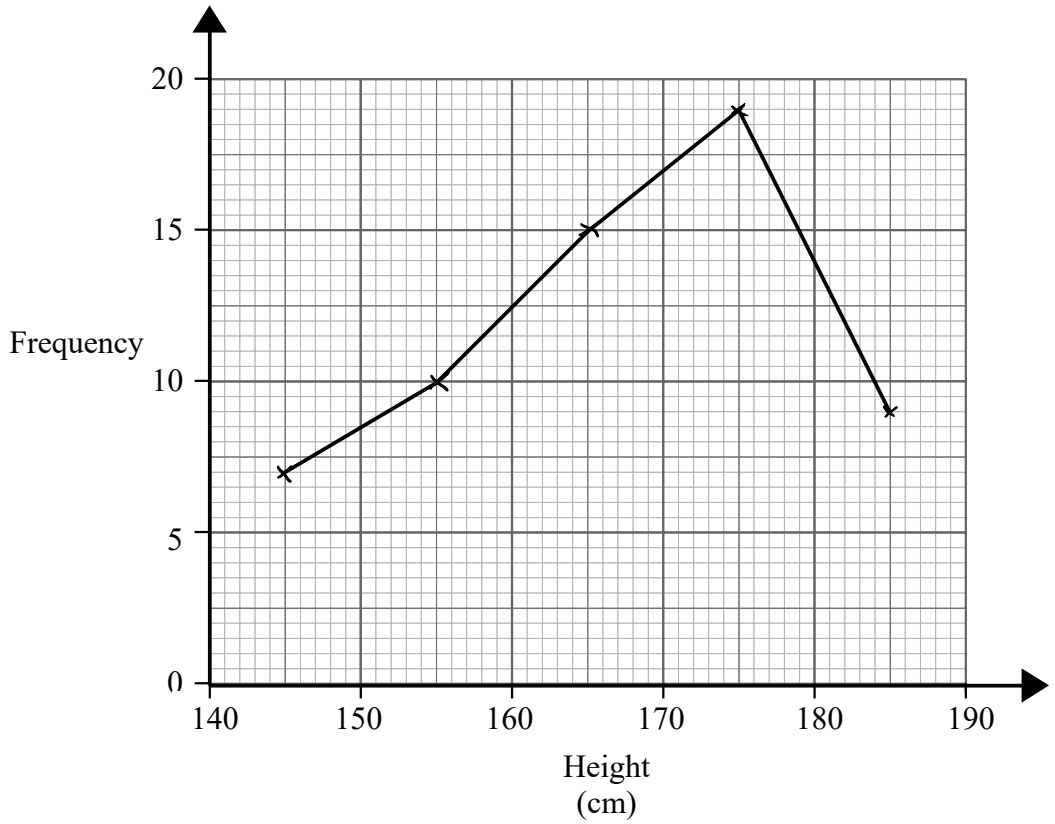
P (A and B) = P (A given B) P(B)

END OF EXAM AID

1 The frequency table shows the heights, in cm, of some tomato plants.

Height (cm)	Frequency
$140 < h \leqslant 150$	7
$150 < h \leqslant 160$	10
$160 < h \leqslant 170$	15
$170 < h \leqslant 180$	19
$180 < h \leqslant 190$	9

Draw a frequency polygon to show this information.



(Total for Question 1 is 2 marks)

2 Banana computers sold 19.3 million computers in 2017.

In 2018, they sold 18.2 million computers.

Work out the percentage decrease in the number of computers sold.

Give your answer to three significant figures.

$$\frac{\text{change}}{\text{original}} \times 100$$

$$\frac{19.3 - 18.2}{19.3} \times 100 = 5.699$$

$$= 5.70\%$$

5.70 ..... %

(Total for Question 2 is 3 marks)

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3 The value of a house increased by 6%.
The house then had a value of £265 000

Work out the value of the house before the increase.

$$x \times 1.06 = 265000$$

$$x = \frac{265000}{1.06}$$

$$= 250\,000$$

£ 250 000

(Total for Question 3 is 2 marks)

4 $s = ut + \frac{1}{2}at^2$

$u = -5$
$a = 4$
$t = 3$

(a) Work out the value of s.

$$s = (-5)(3) + \frac{1}{2}(4)(3)^2$$

$s =$ 3

(2)

(b) Make a the subject of $s = ut + \frac{1}{2}at^2$

$$2s = 2ut + at^2$$
$$2s - 2ut = at^2$$
$$\frac{2s - 2ut}{t^2} = a$$

$a = \frac{2s - 2ut}{t^2}$

(2)

(Total for Question 4 is 4 marks)

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5 There are 120 people in a school canteen.
40% of the people in the canteen are in year 11 students.

The number of year 11 students in the canteen is three times the number of year 10 students.
The rest of the people in the canteen are year 9 students.

the number of year 9 students : the number of year 10 students = n : 1

Work out the value of n.
You must show how you get your answer.

$40\% \times 120 = 48$ (Yr 11)

$\frac{48}{3} = 16$ (Yr 10)

$120 - 48 - 16 = 56$ (Yr 9)

$56 : 16$
$\div 16 \quad \div 16$
$\frac{7}{2} : 1$

$n = \frac{7}{2}$

(Total for Question 5 is 2 marks)

6 Amy drives 300 miles from London to Newcastle.
She drives the first 165 miles at an average speed of 60 mph.
From this point it takes Amy 3 hours and 5 minutes to complete her journey.

$\text{speed} = \frac{\text{distance}}{\text{time}}$

What was Amy's average speed for the whole journey?
Give your answer correct to 3 significant figures.

First 165 miles $d = 165$
$s = 60$

$\text{time} = \frac{d}{s}$

$= \frac{165}{60}$

$= 2.75$ (2 hrs 45 mins)

Whole Journey:

$s = \frac{\text{total distance}}{\text{total time}}$

$= \frac{300}{2\text{hrs}45 + 3\text{hrs}5}$

$= \frac{300}{5\text{hrs}50\text{mins}}$

$= 51.4$ mph

2hrs 45mins + 3hrs 5mins
= 5hrs 50 mins
or $5.8\dot{3}$ hrs

…51.4… mph

(Total for Question 6 is 4 marks)

7 Potatoes cost £9 for a 12.5 kg bag at a farm shop.
The same type of potatoes cost £1.83 for a 2.5 kg bag at a supermarket.

Where are the potatoes the better value, at the farm shop or at the supermarket?
You must show your working.

Farm Shop £9 for 12.5kg

Supermarket £1.83 for 2.5kg

×5 ×5

£9.15 for 12.5kg

Farm Shop £9 < £9.15

(Total for Question 7 is 3 marks)

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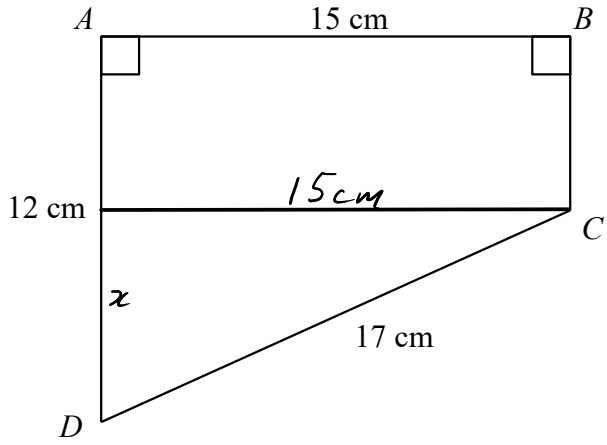
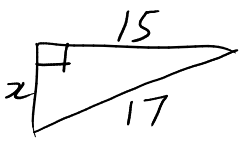
8 *ABCD* is a trapezium.



Calculate the area of *ABCD*.



$x^2 + 15^2 = 17^2$

$x^2 = 17^2 - 15^2$

$x = \sqrt{17^2 - 15^2}$

$= 8\text{cm}$

$12 - 8 = 4$ $BC = 4\text{cm}$

Area $= \frac{1}{2}(a+b)h$

$= \frac{1}{2}(4+12)(15)$

$= 120$

…….120…….cm^2

(Total for Question 8 is 4 marks)

9 Given that $81 \times \sqrt{3} = 3^n$
Find the value n.

$81 = 3^4$

$\sqrt{3} = 3^{\frac{1}{2}}$

$3^4 \times 3^{\frac{1}{2}} = 3^{\frac{9}{2}}$ [OR $3^{4.5}$]

$\frac{9}{2}$

.....

(Total for Question 9 is 2 marks)

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10 There are 5 starters, 6 main courses and x desserts in a restaurant.

Riley says there are 130 different ways of choosing a starter, a main course and a dessert.

Could Riley be correct?
You must show your working.

$5 \times 6 \times x = 130$

$30x = 130$

$x = \frac{130}{30}$

$= 4.\dot{3}$

No. The number of desserts has to be a whole no.

(Total for Question 10 is 2 marks)

11 The line AB passes through the points A(-2, k) and (4, 8).

$x_1, y_1 \quad x_2 y_2$

The gradient of AB is -2.

m

Work out the value of k.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$-2 = \frac{8 - k}{4 - -2}$$

$$-2 = \frac{8 - k}{6}$$

$$-12 = 8 - k$$

$$k - 12 = 8$$

$$k = 20$$

$k =$ .....20.....

(Total for Question 11 is 3 marks)

12 Expand and Simplify $(x + 5)(x - 3)(2x - 1)$

$$(x^2 - 3x + 5x - 15)(2x - 1)$$

$$(x^2 + 2x - 15)(2x - 1)$$

$$2x^3 - x^2 + 4x^2 - 2x - 30x + 15$$

$$2x^3 + 3x^2 - 32x + 15$$

$2x^3 + 3x^2 - 32x + 15$

(Total for Question 12 is 3 marks)

13 $\boldsymbol{a} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 5 \\ -1 \end{pmatrix}$

(a) Write down as a column vector

(i) **a** + **b**

$\begin{pmatrix} -2 \\ 3 \end{pmatrix} + \begin{pmatrix} 5 \\ -1 \end{pmatrix}$

$\begin{pmatrix} 3 \\ 2 \end{pmatrix}$

(1)

(ii) 2**a** – **b**

$\begin{pmatrix} -4 \\ 6 \end{pmatrix} - \begin{pmatrix} 5 \\ -1 \end{pmatrix}$

$\begin{pmatrix} -9 \\ 7 \end{pmatrix}$

(1)

(Total for Question 13 is 2 marks)

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14 A car's value is decreasing by x% each year.

The car's value will decrease by 60% in 6 years, work out the value of x.
Give your answer to 2 decimal places.

decrease by 60%

40% left

$100 \times y^6 = 40$

$y^6 = 0.4$

$y = \sqrt[6]{0.4}$

$= 0.85837\ldots$

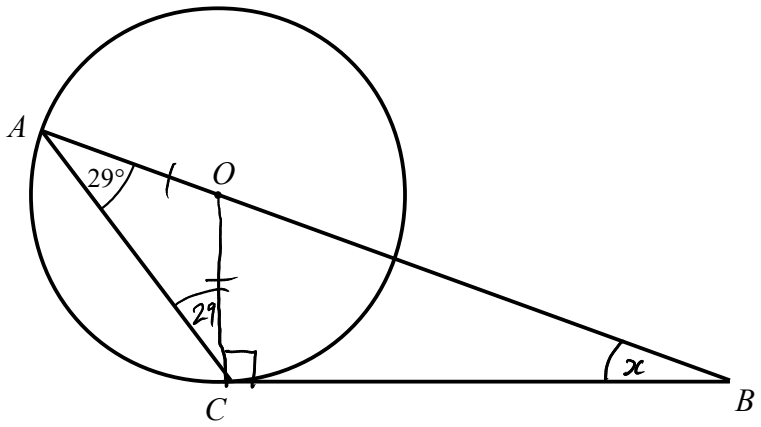
$1 - 0.85837 = 0.1416$

$0.1416 = 14.16\%$

14.16%

(Total for Question 14 is 3 marks)

15



A and *C* are points on the circumference of a circle, centre *O*.
BC is a tangent to the circle.

Angle *CAB* = 29°

Find the size of angle *ABC*.
You must show all your working.

OCB = 90° Tangent meets radius at 90°

OCA = 29° Angles at base of isosceles triangle are equal

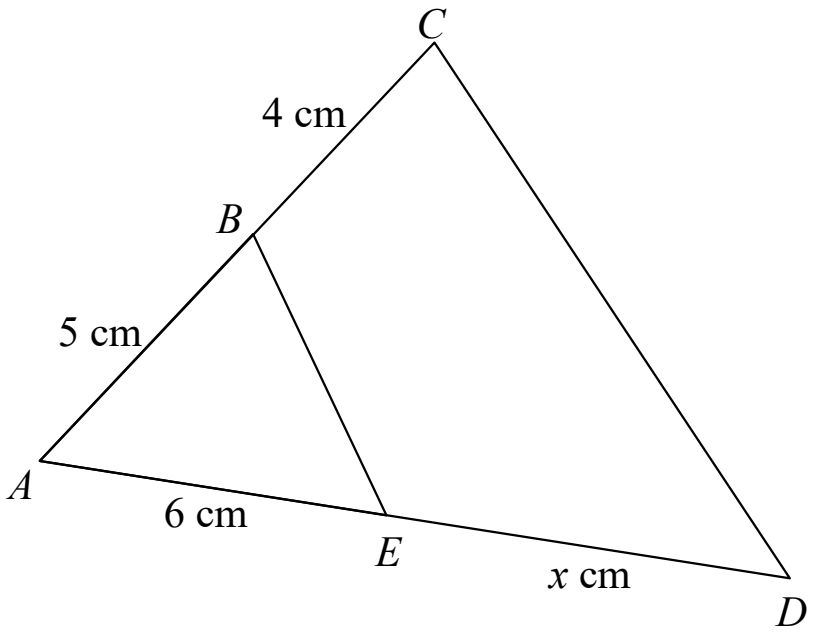
ABC is a triangle 180 − 29 − 29 − 90 = 32°

Angles in a triangle add to 180

32 °

(Total for Question 15 is 4 marks)

16 The two triangles in the diagram are similar.



There are two possible values of x.

Work out each of these values.
State any assumptions you make in your working

The scale factor is $\times \frac{9}{5}$ (if AB corresponds to AC)

$6 \times \frac{9}{5} = 10.8$ $\quad x = 10.8 - 6 = \underline{4.8}$

The scale factor is $\times \frac{9}{6}$ (if AE corresponds to AC)

$5 \times \frac{9}{6} = 7.5$ $\quad x = 7.5 - 6 = \underline{1.5}$

(Total for Question 16 is 5 marks)

17 Show that $3 + \left[(x+4) \div \frac{x^2 - 16}{x - 5} \right]$ simplifies to $\frac{ax - b}{cx - d}$ where a, b, c and d are integers.

$$3 + \left[(x+4) \times \frac{x-5}{(x+4)(x-4)} \right]$$

$$3 + \left[\frac{x+4}{1} \times \frac{x-5}{(x+4)(x-4)} \right]$$

$$3 + \frac{(x+4)(x-5)}{(x+4)(x-4)}$$

$$\frac{3(x-4)}{(x-4)} + \frac{(x-5)}{(x-4)}$$

$$\frac{3(x-4)+(x-5)}{(x-4)}$$

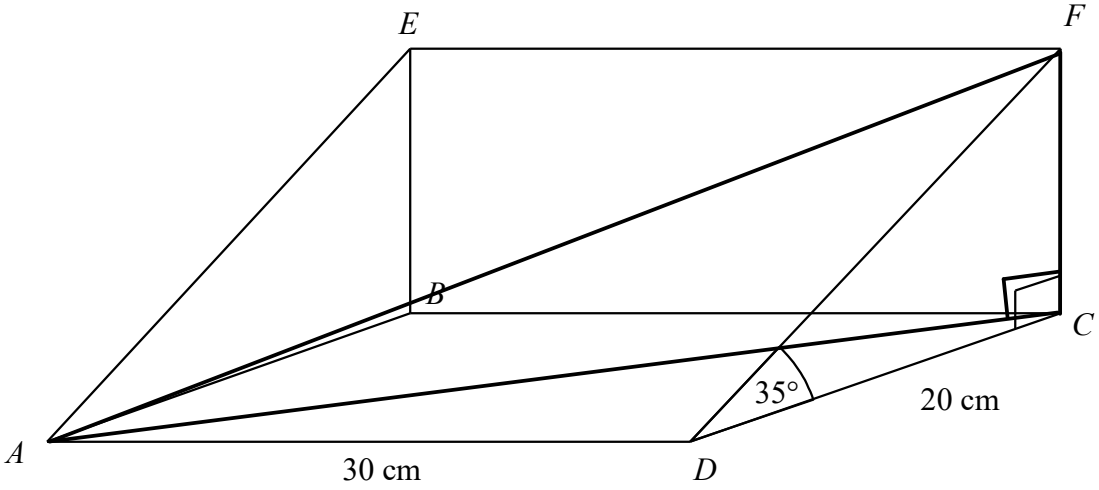
$$\frac{3x-12+x-5}{(x-4)}$$

$$\frac{4x-17}{x-4}$$

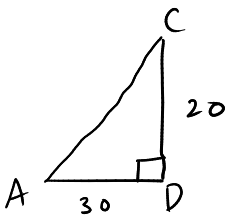
(Total for Question 17 is 4 marks)

18 The diagram shows a triangular prism.

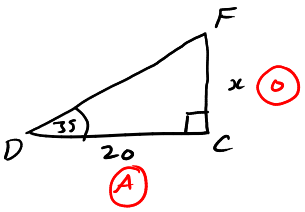
$CD = 20$ cm
$AD = 30$ cm
Angle $FDC = 35°$



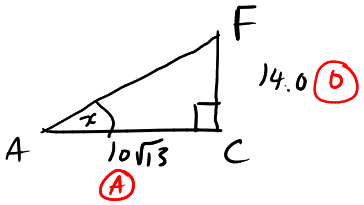
Calculate the size of the angle the line AF makes with the plane $ABCD$.
Give your answer correct to 3 significant figures.



$AC^2 = 20^2 + 30^2$
$AC = \sqrt{20^2 + 30^2}$
$= 10\sqrt{13}$



$\tan(35) = \frac{x}{20}$
$20\tan(35) = x$
$x = 14.0$



$\tan x = \frac{14.0}{10\sqrt{13}}$
$x = \tan^{-1}\left(\frac{14.0}{10\sqrt{13}}\right)$
$= 21.2$

21.2 °

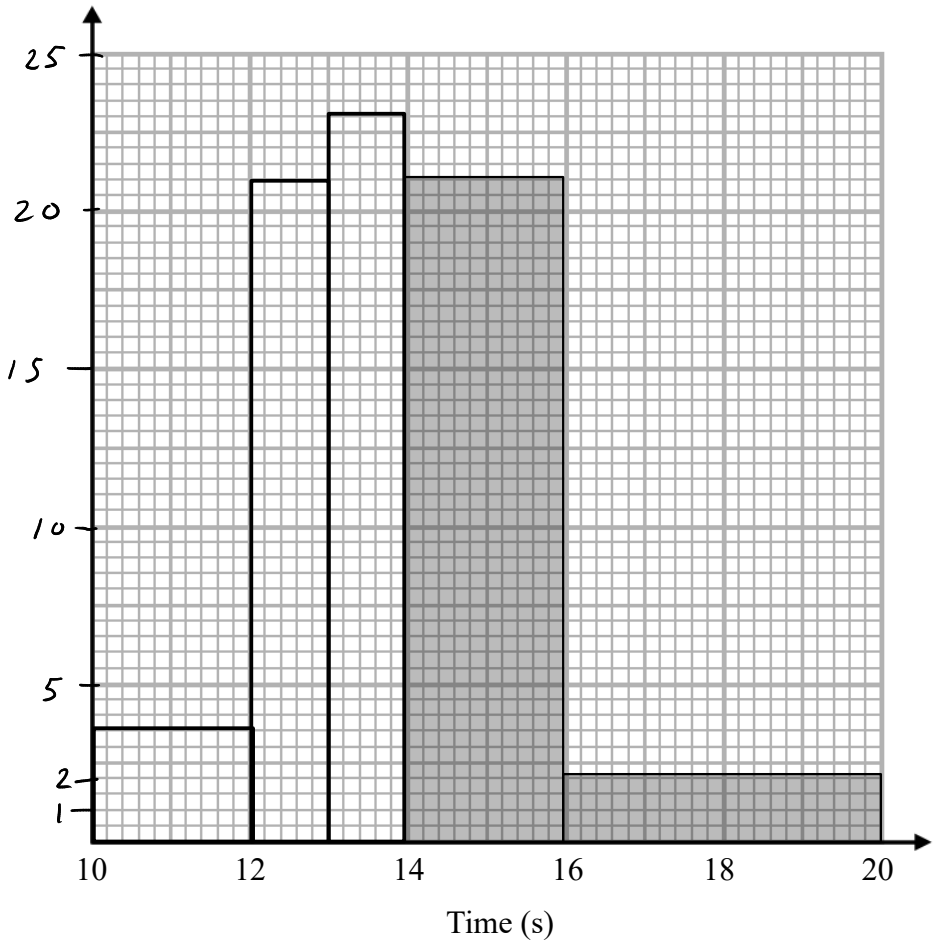
(Total for Question 18 is 4 marks)

19 The table shows information about the time, in seconds, taken for some people to run a 100m race.

Time (s)	Frequency
$10 < t \leqslant 12$	6
$12 < t \leqslant 13$	21
$13 < t \leqslant 14$	23
$14 < t \leqslant 16$	42
$16 < t \leqslant 20$	8

F.d
3
21
23
21
2

(a) Use the information on the table to complete the histogram.

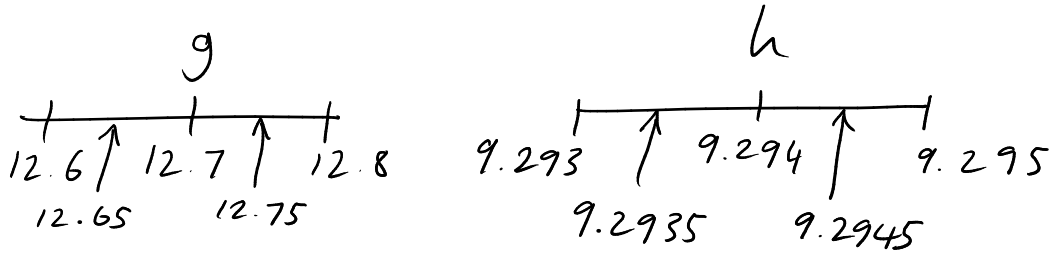


(b) Use the histogram to complete the table.

(Total for Question 19 is 4 marks)

20 $f = \dfrac{\sqrt{g}}{h}$ $g = 12.7$ correct to 3 significant figures
$h = 9.294$ correct to 3 decimal places

By considering bounds, work out the value of f to a suitable degree of accuracy.
Give a reason for your answer.



upper $f = \dfrac{\sqrt{\text{upper } g}}{\text{lower } h}$ lower $f = \dfrac{\sqrt{\text{lower } g}}{\text{upper } h}$

$= \dfrac{\sqrt{12.75}}{9.2935}$ $= \dfrac{\sqrt{12.65}}{9.2945}$

$= 0.3842\ldots$ $= 0.3826\ldots$

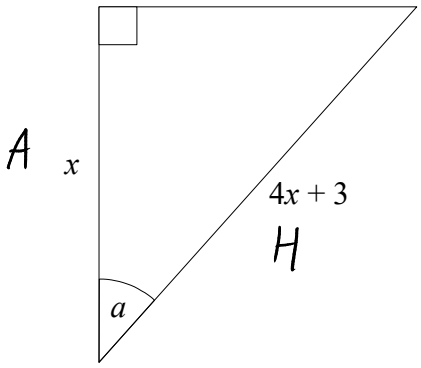
upper and lower bound both round to 0.38

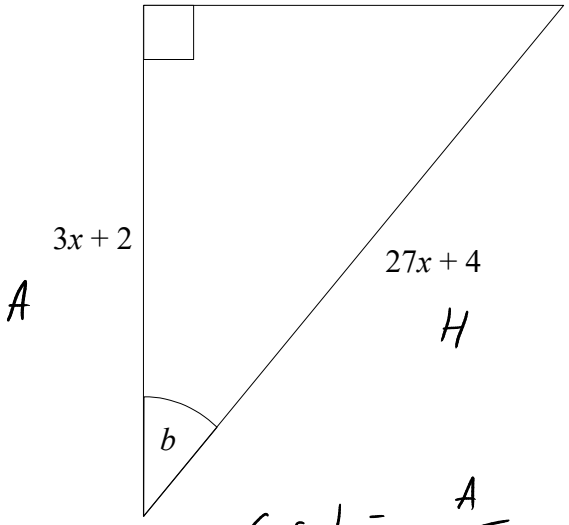
(2sf/2dp)

0.38

(Total for Question 20 is 5 marks)

21 Here are two right angled triangles.





$\cos a = \frac{A}{H}$ $\cos b = \frac{A}{H}$

Given that

$\cos a = \cos b$

find the value of x.

You must show all your working.

$$\frac{x}{4x+3} = \frac{3x+2}{27x+4}$$

$$x(27x+4) = (3x+2)(4x+3)$$

$$27x^2 + 4x = 12x^2 + 9x + 8x + 6$$

$$15x^2 - 13x - 6 = 0$$

$$x = \frac{-(-13) \pm \sqrt{(-13)^2 - 4(15)(-6)}}{2(15)}$$

$$x = \frac{6}{5} \quad \text{or} \quad x = -\frac{1}{3}$$

x cannot be negative

$\frac{6}{5}$

.....

(Total for Question 21 is 5 marks)

22 Solve algebraically the simultaneous equations

$$x^2 - 3y^2 = 13$$
$$2x + 3y = 4$$

$2x = 4 - 3y$

$x = \frac{4 - 3y}{2}$

$\left(\frac{4-3y}{2}\right)^2 - 3y^2 = 13$

$\left(\frac{(4-3y)}{2}\left(\frac{4-3y}{2}\right)\right) - 3y^2 = 13$

$\frac{16 - 12y - 12y + 9y^2}{4} - 3y^2 = 13$

$16 - 24y + 9y^2 - 12y^2 = 52$

$16 - 24y - 3y^2 = 52$

$0 = 3y^2 + 24y + 36$

$0 = y^2 + 8y + 12$

$0 = (y + 6)(y + 2)$

$y = -6 \quad y = -2$

$x = \frac{4 - 3y}{2}$

$x = \frac{4 - 3(-6)}{2}$ $\qquad$ $x = \frac{4 - 3(-2)}{2}$

$= 11$ $\qquad$ $= 5$

$x = 11$ and $y = -6$ or $x = 5$ and $y = -2$

(Total for Question 22 is 5 marks)

23 There are some red counters and some blue counters in a bag.

The ratio of red counters to blue counters is 3:1. $\frac{3}{4}$ $\frac{1}{4}$

Two counters are removed at random. $3x$ Red x Blue $4x$ total

The probability that both the counters taken are blue is $\frac{1}{20}$

Work how many counters were in the bag before any counters were removed.

First Counter $P(\text{Blue}) = \frac{1}{4}$

Second Counter $P(\text{Blue}) = \frac{x-1}{4x-1}$

$$\frac{1}{4} \times \frac{x-1}{4x-1} = \frac{1}{20}$$

$$\frac{x-1}{4(4x-1)} = \frac{1}{20}$$

$$20(x-1) = 4(4x-1)$$

$$20x - 20 = 16x - 4$$

$$4x - 20 = -4$$

$$4x = 16$$

$$x = 4$$

Total counters $= 4x$

$= 4(4)$

16

(Total for Question 23 is 5 marks)

Extra Question

The number of people living in a town t years from now is P_t where

$P_0 = 55000$
$P_{t+1} = 1.03(P_t - 800)$

Work out the number of people in the town 3 years from now.

$P_1 = 1.03(55000 - 800)$
$= 55826$
$P_2 = 1.03(55826 - 800)$
$= 56676.78$
$P_3 = 1.03(\text{Ans} - 800)$
$= 57553$

57553

(nearest whole number) **(Total for Extra Question is 3 marks)**

* Extra question added because the original paper did not cover Iteration. With this question the paper is out of 83.