Write your name here

Surname	Other Names

Mathematics

Paper 1 (Non-Calculator)
Higher Tier

Time: 1 hour 30 minutes

You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided
  – there may be more space than you need.
- **Calculators may not be used.**
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.
- You must **show all your working out.**

Information

- The total mark for this paper is 80
- The marks for **each** question are shown in brackets
  – use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

Higher Tier Formulae Sheet

Perimeter, area and volume

Where a and b are the lengths of the parallel sides and h is their perpendicular separation:

$$\text{Area of a trapezium} = \frac{1}{2}(a+b)\,h$$

Volume of a prism = area of cross section × length

Where r is the radius and d is the diameter:

$$\text{Circumference of a circle} = 2\pi r = \pi d$$

$$\text{Area of a circle} = \pi r^2$$

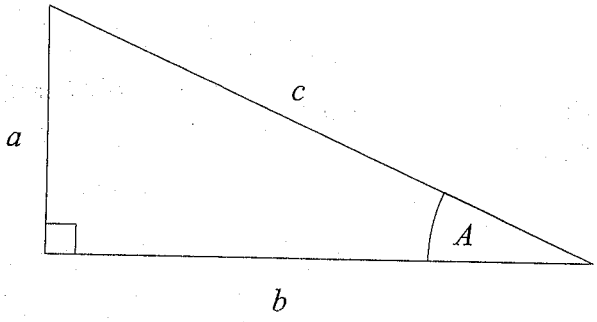
Quadratic formula

The solution of $ax^2 + bx + c = 0$

where $a \neq 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Pythagoras' Theorem and Trigonometry

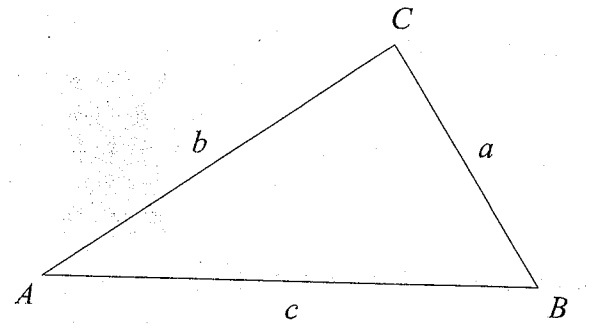


In any right-angled triangle where a, b and c are the length of the sides and c is the hypotenuse:

$$a^2 + b^2 = c^2$$

In any right-angled triangle ABC where a, b and c are the length of the sides and c is the hypotenuse:

$$\sin A = \frac{a}{c} \qquad \cos A = \frac{b}{c} \qquad \tan A = \frac{a}{b}$$



In any triangle ABC where a, b and c are the length of the sides:

sine rule: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$

Area of triangle = $\dfrac{1}{2}ab\sin C$

Compound Interest

Where P is the principal amount, r is the interest rate over a given period and n is number of times that the interest is compounded:

$$\text{Total accrued} = P\left(1 + \frac{r}{100}\right)^n$$

Probability

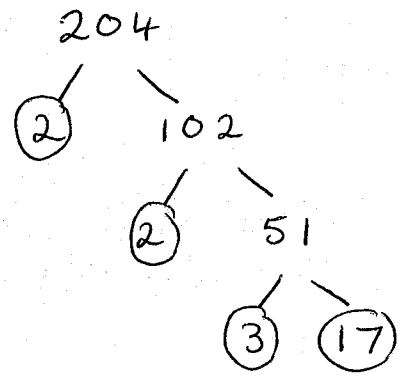
Where P (A) is the probability of outcome A and P (B) is the probability of outcome B:

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A \text{ and } B) = P(A \text{ given } B)\,P(B)$$

END OF EXAM AID

1 Write 204 as a product of its prime factors.



$2 \times 2 \times 3 \times 17$

(Total for Question 1 is 2 marks)

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2 Show that $1\frac{2}{3} \times 3\frac{1}{5} = 5\frac{1}{3}$

$\frac{5}{3} \times \frac{16}{5} = \frac{16}{3} = 5\frac{1}{3}$

(Total for Question 2 is 3 marks)

3 Abbie is 9 years older than Ben.
Charlotte is twice as old as Abbie.
The sum of their three ages is 67

Find the ratio of Abbie's age to Bens's age to Charlotte's age

$A - 9 = B$

$C = 2A$

$A - 9 + 2A + A = 67$

$4A - 9 = 67$

$4A = 76$

$A = 19$

$B = 19 - 9$

$= 10$

$C = 2(19)$

$= 38$

19 : 10 : 38

(Total for Question 3 is 4 marks)

4 A shop sells packs of black pens, packs of red pens and packs of green pens.

There are

5 pens in each pack of black pens
4 pens in each pack of red pens
3 pens in each pack of green pens

On Monday,

number of packs of black pens sold : number of packs of red pens sold : number of packs of green pens sold = 8 : 5 : 2

A total of 264 pens were sold.

Work out the number of green pens sold.

Number of pens sold

$8 \times 5 : 5 \times 4 : 2 \times 3$

$40 : 20 : 6$ (66 parts)

$\frac{264}{66} = \frac{132}{33} = \frac{12}{3} = \underline{\underline{4}}$ each part is worth 4 pens

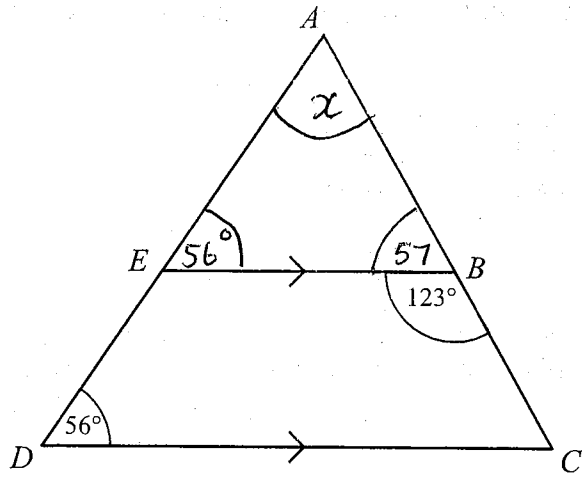
Green pens sold $= 6 \times 4$

$= \underline{\underline{24}}$

24

(Total for Question 4 is 4 marks)

5 *ADC* is a triangle.



AED and *ABC* are straight lines.
EB is parallel to *DC*.

Angle *EBC* = 123°
Angle *ADC* = 56°

Work out the size of angle *EAB*.
You must give a reason for each stage of your working.

Angle ABE = 57° Angles on a straight line add to 180° ~~are equal~~

Angle AEB = 56° Corresponding angles are equal

$x = 180 - 56 - 57$

$= 67°$

Angles in a triangle add to 180°

(Total for Question 5 is 5 marks)

6 A car travels for 42 minutes at an average speed of 90 km/h.

Speed = $\frac{\text{distance}}{\text{time}}$

(a) How far will the car travel in these 42 minutes?

distance = speed x time

42 minutes = $\frac{42}{60} = \frac{21}{30} = \frac{7}{10}$ of an hour

$90 \times \frac{7}{10} = 63$ km

63 km

(2)

David says,

"90 kilometres per hour is faster than 25 metres per second."

(b) Is David correct?
You must show how you get your answer

90 km per hour

90000 m per hour

$\frac{9000\cancel{0}}{6\cancel{0}} = \frac{4500}{3} = 1500$ m per minute

$\frac{150\cancel{0}}{6\cancel{0}} = \frac{75}{3} = 25$ m per second

No. They are the same.

(2)

(Total for Question 6 is 4 marks)

7 The table shows some information about the profit made each day at a cricket club on 100 days.

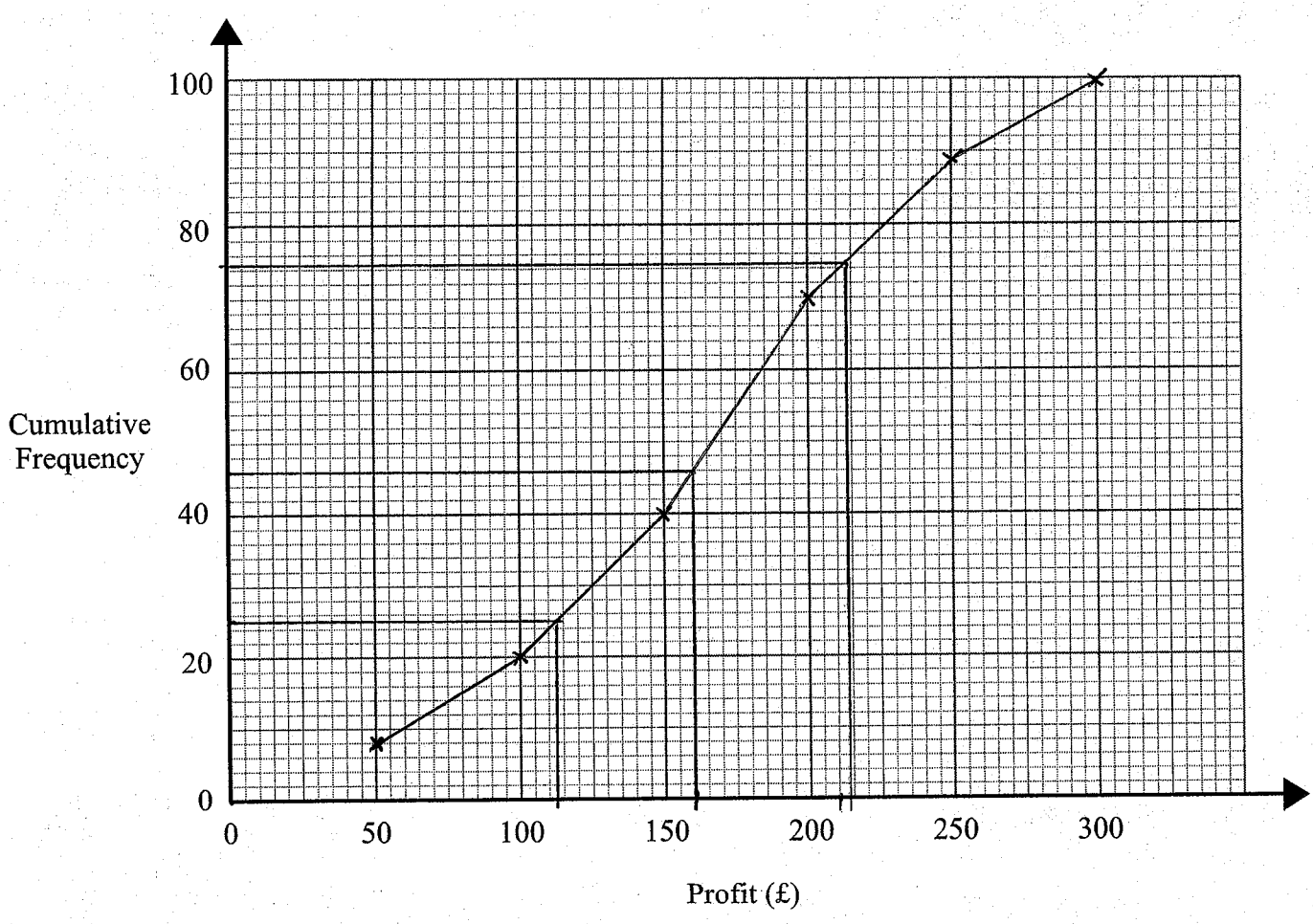
Profit (£x)	Frequency
$0 \leqslant x < 50$	8
$50 \leqslant x < 100$	12
$100 \leqslant x < 150$	20
$150 \leqslant x < 200$	30
$200 \leqslant x < 250$	19
$250 \leqslant x < 300$	11

(a) Complete the cumulative frequency table.

Profit (£x)	Cumulative Frequency
$0 \leqslant x < 50$	8
$0 \leqslant x < 100$	20
$0 \leqslant x < 150$	40
$0 \leqslant x < 200$	70
$0 \leqslant x < 250$	89
$0 \leqslant x < 300$	100

(1)

(b) On the grid, draw a cumulative frequency graph for this information.



(2)

(c) Use your graph to find an estimate for the number of days the profit was less than £160

46 days

(1)

(d) Use your graph to find an estimate for the interquartile range.

214 − 112 = 102

£ 102

(2)

(Total for Question 7 is 6 marks)

8 (a) Write down the value of 27^0

1

(1)

(b) Find the value of $3^1 \times 3^4 \times 3^{-5}$

$3^5 \times 3^{-5} = 3^0$

1

(1)

(c) Find the value of 5^{-3}

$\frac{1}{5^3}$

$\frac{1}{125}$

(1)

(d) Find the value of $64^{\frac{1}{3}}$

$\sqrt[3]{64}$

$4 \times 4 \times 4 = 64$

4

(1)

(Total for Question 8 is 4 marks)

9 Given that $\frac{a}{b} = \frac{3}{4}$ and $\frac{b}{c} = \frac{3}{7}$

Find $a : b : c$

$a : b$ $\quad$ $b : c$ $\quad$ Make b the same

$\times 3$ $\quad$ $3 : 4$ $\quad$ $3 : 7$ $\quad$ $\times 4$

$9 : 12$ $\quad$ $12 : 28$

$a : b : c$

$9 : 12 : 28$

$9 : 12 : 28$

(Total for Question 9 is 3 marks)

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10 Express $0.2\dot{6}\dot{5}$ as a fraction.
You must show all your working.

$0.2\dot{6}\dot{5} = x$

$2.\dot{6}\dot{5} = 10x$

$265.\dot{6}\dot{5} = 1000x$

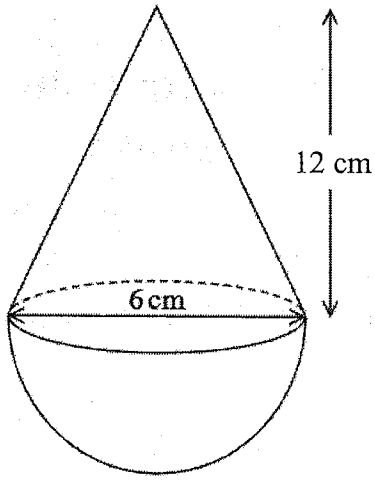
$263 = 990x$

$x = \frac{263}{990}$

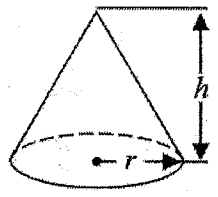
$\frac{263}{990}$

(Total for Question 10 is 3 marks)

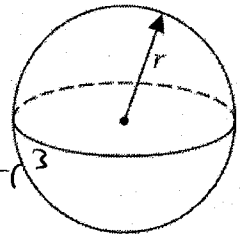
11 The diagram shows a solid shape.
The shape is a cone on top of a hemisphere.



Volume of a cone = $\frac{1}{3}\pi r^2 h$



Volume of a sphere = $\frac{4}{3}\pi r^3$



Hemisphere = $\frac{2}{3}\pi r^3$

The height of the cone is 12 cm.
The base of the cone has a diameter of 6 cm. $r = 3$
The hemisphere has a diameter of 6 cm.

The total volume of the shape is $k\pi$ cm^3, where k is an integer.

Work out the value of k.

volume $= \frac{1}{3}\pi(3)^2(12) + \frac{2}{3}\pi(3)^3$

$= \frac{1}{3}\pi(9)(12) + \frac{2}{3}\pi(27)$

$= \pi(3)(12) + \pi(18)$

$= 36\pi + 18\pi$

$= 54\pi$ cm^3

$k =$ 54

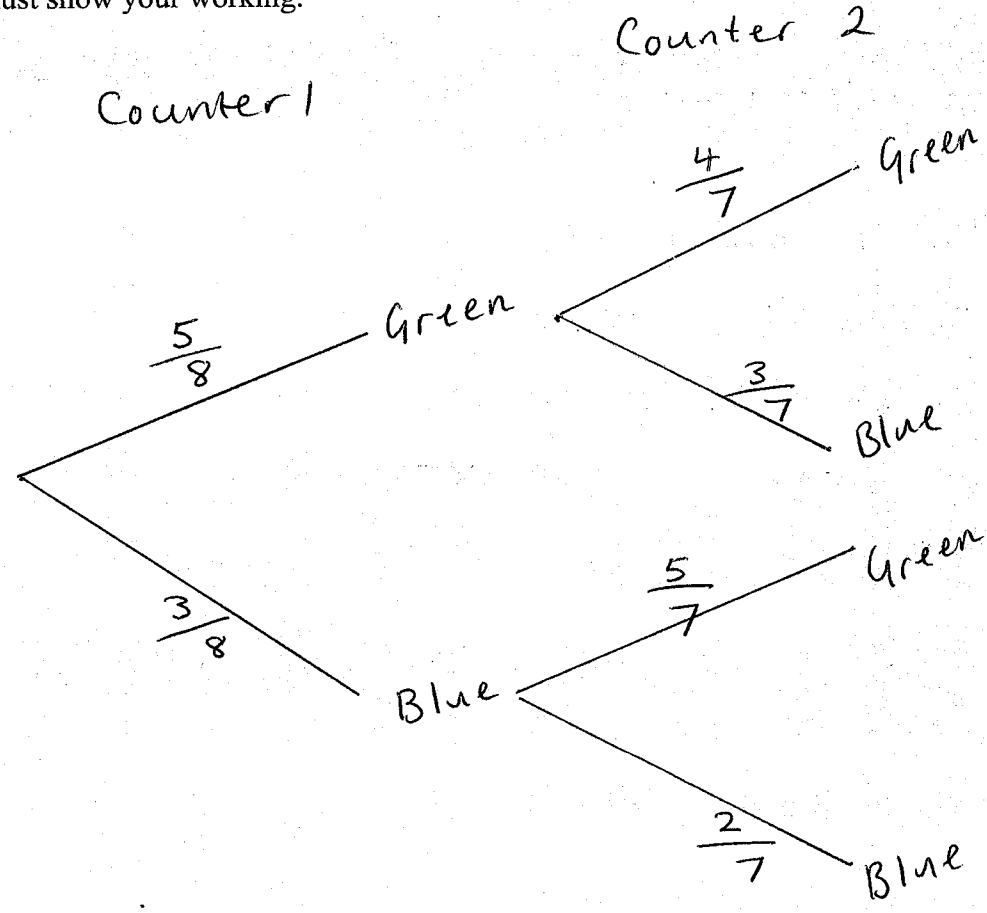
(Total for Question 11 is 4 marks)

12 There are 8 counters in a bag.

5 of the counters are green.
3 of the counters are blue.

Riley takes at random two counters from the bag.

Work out the probability that Riley takes one counter of each colour.
You must show your working.



$P(\text{Green, Blue}) = \frac{5}{8} \times \frac{3}{7} = \frac{15}{56}$

$P(\text{Blue, Green}) = \frac{3}{8} \times \frac{5}{7} = \frac{15}{56}$

$\frac{15}{56} + \frac{15}{56} = \frac{30}{56}$

$\frac{30}{56}$

(Total for Question 12 is 4 marks)

13 Prove that the sum of the squares of two consecutive odd numbers is always 2 more than a multiple of 8

$(2n+1)^2 + (2n+3)^2$

$(2n+1)(2n+1) + (2n+3)(2n+3)$

$4n^2 + 2n + 2n + 1 + 4n^2 + 6n + 6n + 9$

$8n^2 + 16n + 10$

$8n^2 + 16n + 8 + 2$

$8(n^2 + 2n + 1) + 2$

multiple of 8 — 2 more

(Total for Question 13 is 4 marks)

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14 Show that $\frac{8+\sqrt{18}}{3+\sqrt{2}}$ can be written in the form $\frac{a+\sqrt{2}}{b}$, where a and b are integers.

$\sqrt{18} = \sqrt{9}\sqrt{2} = 3\sqrt{2}$

$\frac{(8+3\sqrt{2})(3-\sqrt{2})}{(3+\sqrt{2})(3-\sqrt{2})}$

$\frac{24 - 8\sqrt{2} + 9\sqrt{2} - 6}{9 - 3\sqrt{2} + 3\sqrt{2} - 2}$

$\frac{18 + \sqrt{2}}{7}$

(Total for Question 14 is 4 marks)

15 f and g are functions such that

$$f(x) = \frac{20}{\sqrt{x}} \quad \text{and} \quad g(x) = 4(3x + 2)$$

(a) Find g(3)

$$g(3) = 4(3(3) + 2)$$
$$= 4(11)$$
$$= 44$$

44

(1)

(b) Find gf(16)

$$f(16) = \frac{20}{\sqrt{16}} = \frac{20}{4} = 5$$

$$g(5) = 4(3(5) + 2)$$
$$= 4(17)$$
$$= 68$$

68

(2)

(c) Find $g^{-1}(2)$

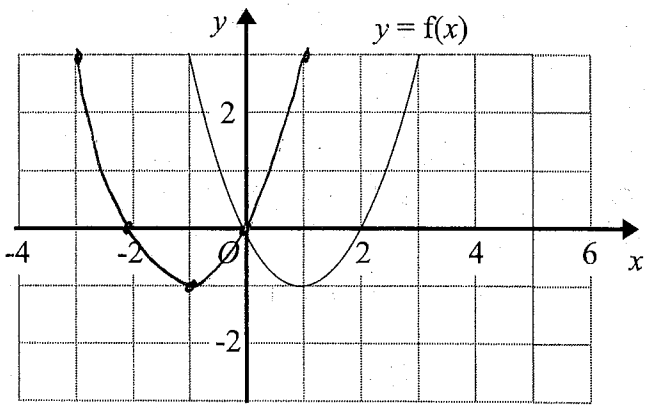
$$2 = 4(3x + 2)$$
$$2 = 12x + 8$$
$$-6 = 12x$$
$$x = \frac{-6}{12} = -\frac{1}{2}$$

−0.5

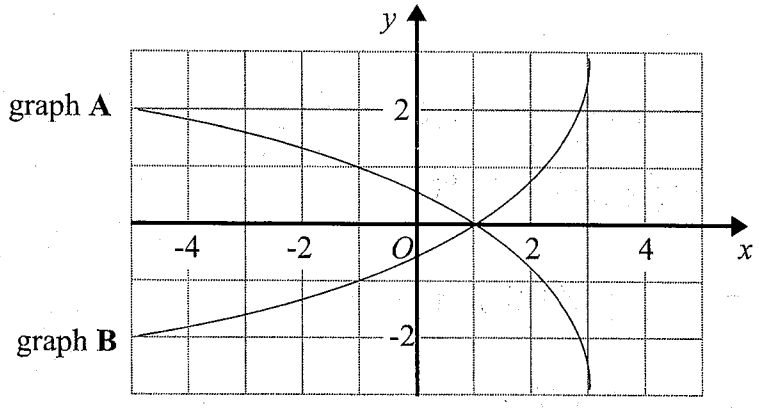
(2)

(Total for Question 15 is 5 marks)

16 The graph of $y = f(x)$ is shown on the grid below.



(a) On the grid above, sketch the graph of $y = f(x + 2)$ Two left



On the grid, graph **A** has been reflected to give graph **B**.

The equation of graph **A** is $y = g(x)$

(b) Write down the equation of graph **B**.

$y = -g(x)$

(1)

(Total for Question 16 is 2 marks)

17 The point P has coordinates (5, 2)
The point Q has coordinates (a, b)

A line perpendicular to PQ is given by the equation $2x - 3y = 10$

Find an expression for b in terms of a.

$2x - 3y = 10$

$2x = 3y + 10$

$2x - 10 = 3y$

$\frac{2}{3}x - \frac{10}{3} = y$

$y = \frac{2}{3}x - \frac{10}{3}$ $\quad m = \frac{2}{3}$

$\therefore$ perpendicular gradient $= -\frac{3}{2}$

$(5, 2)$ (a, b)
x_1 y_1 x_2 y_2

$$-\frac{3}{2} = \frac{b-2}{a-5}$$

$-3(a-5) = 2(b-2)$

$-3a + 15 = 2b - 4$

$-3a + 19 = 2b$

$$b = \frac{-3a+19}{2}$$

$$b = \frac{19-3a}{2}$$

(Total for Question 17 is 5 marks)

18 x is proportional to $\sqrt{y}$ where $y > 0$

y is increased by 21%

Work out the percentage increase in x.

$x = k\sqrt{y}$

→ To increase by 21% x by 1.21

$\therefore y_2 = 1.21y$

$x_2 = k\sqrt{1.21y}$

$= k\sqrt{1.21}\sqrt{y}$

$= 1.1k\sqrt{y}$

$= 1.1x$

Increase of 10%

10 ..... %

(Total for Question 18 is 3 marks)

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19 Given that $x^2 - 8x + 5 = (x - a)^2 - b$ for all values of x,

(i) find the value of a and the value of b.

$(x-4)^2 - 16 + 5$

$(x-4)^2 - 11$

$a =$ 4

$b =$ 11

(2)

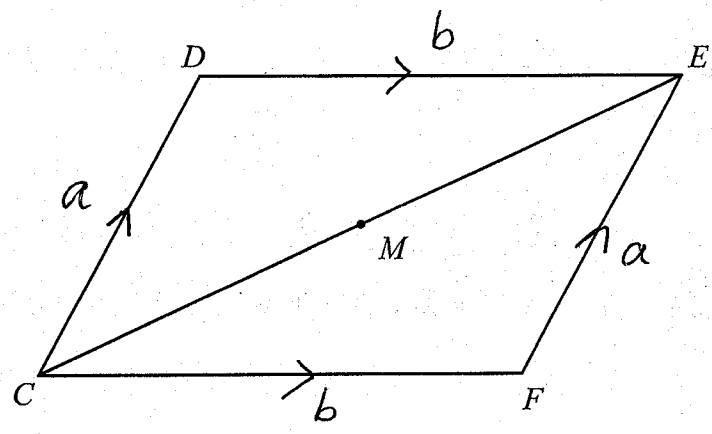
(ii) Hence write down the coordinates of the turning point on the graph of $y = x^2 - 8x + 5$

(4 , −11)

(1)

(Total for Question 19 is 3 marks)

20 *CDEF* is a parallelogram.



M is the midpoint of *CE*.

$\overrightarrow{CD} = \mathbf{a}$ and $\overrightarrow{CF} = \mathbf{b}$

Use a vector method to prove that *M* is the midpoint of the line *DF*.

$\overrightarrow{CE} = a + b$

$\therefore \overrightarrow{CM} = \frac{1}{2}a + \frac{1}{2}b$

$\overrightarrow{DM} = \overrightarrow{DC} + \overrightarrow{CM}$

$= -a + \frac{1}{2}a + \frac{1}{2}b$

$= -\frac{1}{2}a + \frac{1}{2}b$

$\overrightarrow{DF} = \overrightarrow{DC} + \overrightarrow{CF}$

$= -a + b$

$\overrightarrow{DM} = \frac{1}{2}\overrightarrow{DF}$ $\therefore$ M is the midpoint

(Total for Question 20 is 4 marks)

21 Solve $\frac{3}{2x-1} + \frac{2}{x+1} = 1$

Give your answer in the form $\frac{p \pm \sqrt{q}}{2}$ where p and q are integers.

$$\frac{3(x+1)}{(2x-1)(x+1)} + \frac{2(2x-1)}{(2x+1)(x+1)} = 1$$

$$\frac{3(x+1) + 2(2x-1)}{(2x-1)(x+1)} = 1$$

$$3(x+1) + 2(2x-1) = (2x-1)(x+1)$$

$$3x + 3 + 4x - 2 = 2x^2 + 2x - x - 1$$

$$7x + 1 = 2x^2 + x - 1$$

$$0 = 2x^2 - 6x - 2$$

$$0 = x^2 - 3x - 1$$

$$a = 1 \quad b = -3 \quad c = -1$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(-1)}}{2(1)}$$

$$= \frac{3 \pm \sqrt{9 + 4}}{2}$$

$$\frac{3 \pm \sqrt{13}}{2}$$

(Total for Question 21 is 4 marks)

TOTAL FOR PAPER IS 80 MARKS