**Candidates may use any calculator allowed by the regulations of the**

**Joint Council for Qualifications. Calculators must not have the facility**

**for symbolic algebra manipulation, differentiation and integration, or**

**have retrievable mathematical formulae stored in them.**

**Instructions**

* Use **black** ink or ball-point pen.
* If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).

 Coloured pencils and highlighter pens must not be used.

* **Fill in the boxes** at the top of this page with your name,
centre number and candidate number.
* Answer **all** questions and ensure that your answers to parts of questions are
clearly labelled.
* Answer the questions in the spaces provided
*– there may be more space than you need.*
* You should show sufficient working to make your methods clear. Answers
without working may not gain full credit.
* When a calculator is used, the answer should be given to an appropriate
degree of accuracy.

**Information**

* The total mark for this paper is 75.
* The marks for each question are shown in brackets
*– use this as a guide as to how much time to spend on each question.*

**Advice**

* Read each question carefully before you start to answer it.
* Try to answer every question.
* Check your answers if you have time at the end.

**1.** Use the binomial series to find the e*x*pansion of

,

in ascending powers of *x*, up to and including the term in *x*3.

Give each coefficient as a fraction in its simplest form.

**(Total 6 marks)**

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**2.**

Figure 1 shows a sketch of part of the curve with equation *y* = *x*2 ln *x*, *x* ≥ 1.

The finite region *R*, shown shaded in Figure 1, is bounded by the curve, the *x*-a*x*is and the line *x* = 2.

The table below shows corresponding values of *x* and *y* for *y* = *x*2 ln *x*.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| *x* | 1 | 1.2 | 1.4 | 1.6 | 1.8 | 2 |
| *y* | 0 | 0.2625 |  | 1.2032 | 1.9044 | 2.7726 |

(*a*)Complete the table above, giving the missing value of *y* to 4 decimal places.

**(1)**

(*b*)Use the trapezium rule with all the values of *y* in the completed table to obtain an estimate for the area of *R*, giving your answer to 3 decimal places.

**(3)**

(*c*)Use integration to find the e*x*act value for the area of *R*.

**(5)**

**(Total 9 marks)**

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**3.** The curve *C* has equation

2*x*2*y* + 2*x* + 4*y* – cos (*πy*) = 17.

(*a*)Use implicit differentiation to find  in terms of *x* and *y*.

**(5)**

The point *P* with coordinates  lies on *C*.

The normal to *C* at *P* meets the *x*-a*x*is at the point *A*.

(*b*)Find the *x* coordinate of *A*, giving your answer in the form , where *a*, *b*, *c* and *d* are integers to be determined.

**(4)**

**(Total 9 marks)**

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**4.** The rate of decay of the mass of a particular substance is modelled b*y* the differential equation

,

where *x* is the mass of the substance measured in grams and *t* is the time measured in days.

Given that *x* = 60 when *t* = 0,

(*a*)solve the differential equation, giving *x* in terms of *t*. You should show all steps in your working and give your answer in its simplest form.

**(4)**

(*b*)Find the time taken for the mass of the substance to decay from 60 grams to 20 grams.

 Give your answer to the nearest minute.

**(3)**

**(Total 7 marks)**

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**5.**

Figure 2 shows a sketch of the curve *C* with parametric equations

*x* = 4 tan *t,* *y* = 5sin2*t,* 0 ≤ *t* < .

The point *P* lies on *C* and has coordinates .

(*a*)Find the e*x*act value of  at the point *P*.

 Give your answer as a simplified surd.

**(4)**

The point *Q* lies on the curve *C*, where  = 0.

(*b*)Find the e*x*act coordinates of the point *Q*.

**(2)**

**(Total 6 marks)**

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**6.** (i) Given that *y* > 0, find

.

**(6)**

(ii) (*a*)Use the substitution *x* = 4sin2*θ* to show that

 = *λ*,

 where *λ* is a constant to be determined.

**(5)**

 (*b*)Hence use integration to find

,

 giving your answer in the form *aπ* + *b*, where *a* and *b* are e*x*act constants.

**(4)**

**(Total 15 marks)**

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**7.** (*a*)Find

,

 giving your answer in its simplest form.

**(2)**

Figure 3 shows a sketch of part of the curve *C* with equation

.

The curve *C* cuts the line *y* = 8 at the point *P* with coordinates (*k*, 8), where *k* is a constant.

(*b*)Find the value of *k*.

**(2)**

The finite region S, shown shaded in Figure 3, is bounded b*y* the curve *C*, the *x*-a*x*is, the *y‑*a*x*is and the line *y* = 8. This region is rotated through 2*π* radians about the *x*-a*x*is to form a solid of revolution.

(*c*)Find the e*x*act value of the volume of the solid generated.

**(4)**

**(Total 8 marks)**

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**8.** With respect to a fi*x*ed origin *O*, the line *l*1 is given b*y* the equation

**r** = ,

where *μ* is a scalar parameter.

The point *A* lies on *l*1 where *μ* = 1.

(*a*)Find the coordinates of *A*.

**(1)**

The point *P* has position vector .

The line *l*2 passes through the point *P* and is parallel to the line *l*1.

(*b*)Write down a vector equation for the line *l*2.

**(2)**

(*c*)Find the e*x*act value of the distance *AP*.

 Give your answer in the form *k*, where *k* is a constant to be determined.

**(2)**

The acute angle between *AP* and *l*2 is *θ*.

(*d*)Find the value of cos *θ*.

**(3)**

A point E lies on the line *l*2.

Given that *AP* = *PE*,

(*e*)find the area of triangle *APE*,

**(2)**

(*f*)find the coordinates of the two possible positions of *E*.

**(5)**

**(Total 15 marks)**

**TOTAL FOR PAPER: 75 MARKS**

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