1 **a** $x = 30, 180 - 30$ **b** $x = 60, 180 + 60$ **c** $x = 90°, 270°$ **d** $x = 270°$
　　　 $x = 30°, 150°$　　　　　 $x = 60°, 240°$

　　e $x = 30, 360 - 30$ **f** $x = 45, 180 - 45$ **g** $x = 180–45, 360–45$ **h** $x = 180 - 60, 180 + 60$
　　　 $x = 30°, 330°$　　　　　 $x = 45°, 135°$　　　　　 $x = 135°, 315°$　　　　 $x = 120°, 240°$

　　i $x = 180+60, 360–60$ **j** $x = 30, 180 + 30$ **k** $x = 180–45, 180+45$ **l** $x = 180 - 60, 360 - 60$
　　　 $x = 240°, 300°$　　　　 $x = 30°, 210°$　　　　　 $x = 135°, 225°$　　　　 $x = 120°, 300°$

2 **a** $\theta = 66.4, 360 - 66.4$ **b** $\theta = 15.7, 180 - 15.7$ **c** $\theta = 58.0, 180 + 58.0$ **d** $\theta = 54.4, 180 - 54.4$
　　　 $\theta = 66.4°, 293.6°$　　 $\theta = 15.7°, 164.3°$　　 $\theta = 58.0°, 238.0°$　　 $\theta = 54.4°, 125.6°$

　　e $\theta = 5.7, 180 + 5.7$ **f** $\theta = 79.3, 360 - 79.3$ **g** $\theta = 180 + 36.9,$ **h** $\theta = 180 - 35.0,$
　　　 $\theta = 5.7°, 185.7°$　　　 $\theta = 79.3°, 280.7°$　　　　 $360 - 36.9$　　　　　　 $360 - 35.0$
　　　　　　　　　　　　　　　　　　　　　　　　　　 $\theta = 216.9°, 323.1°$　 $\theta = 145.0°, 325.0°$

　　i $\theta = 180 - 67.0,$ **j** $\theta = 180 - 73.6,$ **k** $\theta = 180 - 50.5,$ **l** $\theta = 180 + 11.7,$
　　　　 $180 + 67.0$　　　　　　 $360 - 73.6$　　　　　　 $180 + 50.5$　　　　　　 $360 - 11.7$
　　　 $\theta = 113.0°, 247.0°$　 $\theta = 106.4°, 286.4°$　 $\theta = 129.5°, 230.5°$　 $\theta = 191.7°, 348.3°$

3 **a** $x - 60 = 30, 180 - 30$　　**b** $x + 30 = 45, 180 + 45$　　**c** $x - 45 = 78.5, 360 - 78.5$
　　　　　　 $= 30, 150$　　　　　　　　 $= 45, 225$　　　　　　　　 $= 78.5, 281.5$
　　　 $x = 90, 210$　　　　　　 $x = 15, 195$　　　　　　 $x = 123.5, 326.5$

　　d $x + 30 = 38.0, 180 + 38.0$　**e** $x + 45 = 180 - 60, 180 + 60$ **f** $x - 60 = 180 + 62.9, 360 - 62.9$
　　　　　　 $= 38.0, 218.0$　　　　　　　 $= 120, 240$　　　　　　　 $= 242.9, 297.1$
　　　 $x = 8.0, 188.0$　　　　　 $x = 75, 195$　　　　　　 $x = 302.9, 357.1$

　　g $x + 45 = 360 - 25.8,$　　**h** $x + 30 = 180 - 8.0,$　　**i** $x - 60 = -53.1, 53.1$
　　　　　　　 $360 + 25.8$　　　　　　　　 $360 + 8.0$　　　　　 $x = 6.9, 113.1$
　　　　　　　 $= 334.2, 385.8$　　　　　　 $= 172.0, 368.0$
　　　 $x = 289.2, 340.8$　　　　 $x = 142.0, 338.0$

　　j $x - 30 = -17.5, 180 + 17.5$ **k** $x - 60 = -51.6, 180 - 51.6$ **l** $2x = 30, 180 - 30,$
　　　　　　　 $= -17.5, 197.5$　　　　　　 $= -51.6, 128.4$　　　　　 $360 + 30, 540 - 30$
　　　 $x = 12.5, 227.5$　　　　 $x = 8.4, 188.4$　　　　　　 $= 30, 150, 390, 510$
　　　　　　　　　　　　　　　　　　　　　　　　　　　　 $x = 15, 75, 195, 255$

　　m $2x = 50.208,$　　　　　　**n** $2x = 180 + 10.370,$　　　**o** $2x = 180 - 69.950,$
　　　　　　 $360 - 50.208,$　　　　　　　 $360 - 10.370,$　　　　　　 $360 - 69.950,$
　　　　　　 $360 + 50.208,$　　　　　　　 $540 + 10.370,$　　　　　　 $540 - 69.950,$
　　　　　　 $720 - 50.208$　　　　　　　 $720 - 10.370$　　　　　　 $720 - 69.950$
　　　　　 $= 50.208, 309.792,$　　　　　 $= 190.370, 349.630,$　　　　 $= 110.050, 290.050,$
　　　　　　 $410.208, 669.792$　　　　　　 $550.370, 709.630$　　　　　 $470.050, 650.050$
　　　 $x = 25.1, 154.9, 205.1, 334.9$　 $x = 95.2, 174.8, 275.2, 354.8$　 $x = 55.0, 145.0, 235.0, 325.0$

　　p $\frac{1}{2}x = 44.668, 180 - 44.668$ **q** $3x = 30.583, 180 + 30.583,$ **r** $2x = 180 - 65.481,$
　　　　　　 $= 44.668, 135.332$　　　　　 $360 + 30.583,$　　　　　　 $180 + 65.481,$
　　　 $x = 89.3, 270.7$　　　　　　 $540 + 30.583,$　　　　　　 $540 - 65.481,$
　　　　　　　　　　　　　　　　　 $720 + 30.583,$　　　　　　 $540 + 65.481$
　　　　　　　　　　　　　　　　　 $900 + 30.583$　　　　　 $= 114.519, 245.481,$
　　　　　　　　　　　　　　　　 $= 30.583, 210.583,$　　　　　 $474.519, 605.481$
　　　　　　　　　　　　　　　　　 $390.583, 570.583,$　　　 $x = 57.3, 122.7, 237.3, 302.7$
　　　　　　　　　　　　　　　　　 $750.583, 930.583$
　　　　　　　　　　　　　　　 $x = 10.2, 70.2, 130.2$
　　　　　　　　　　　　　　　　　 $190.2, 250.2, 310.2$

4 **a** $x = 0, \pi, 2\pi$ **b** $x = \frac{\pi}{3}, 2\pi - \frac{\pi}{3}$ **c** $x = \frac{\pi}{4}, \pi + \frac{\pi}{4}$

$\qquad\qquad\qquad\qquad\qquad\quad x = \frac{\pi}{3}, \frac{5\pi}{3} \qquad\qquad\qquad\qquad x = \frac{\pi}{4}, \frac{5\pi}{4}$

d $x = \pi$ **e** $x = \pi - \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$ **f** $x = \pi + \frac{\pi}{4}, 2\pi - \frac{\pi}{4}$

$\qquad\qquad\qquad\qquad\qquad\quad x = \frac{5\pi}{6}, \frac{11\pi}{6} \qquad\qquad\qquad\qquad x = \frac{5\pi}{4}, \frac{7\pi}{4}$

g $x + \frac{\pi}{6} = \frac{\pi}{3}, \pi + \frac{\pi}{3}$ **h** $x - \frac{\pi}{4} = \frac{\pi}{6}, \pi - \frac{\pi}{6}$ **i** $x + \frac{\pi}{3} = \pi - \frac{\pi}{6}, \pi + \frac{\pi}{6}$

$\qquad\quad = \frac{\pi}{3}, \frac{4\pi}{3} \qquad\qquad\qquad = \frac{\pi}{6}, \frac{5\pi}{6} \qquad\qquad\qquad = \frac{5\pi}{6}, \frac{7\pi}{6}$

$\quad x = \frac{\pi}{6}, \frac{7\pi}{6} \qquad\qquad\quad x = \frac{5\pi}{12}, \frac{13\pi}{12} \qquad\qquad\quad x = \frac{\pi}{2}, \frac{5\pi}{6}$

j $x + \frac{\pi}{3} = \pi - \frac{\pi}{4}, 2\pi + \frac{\pi}{4}$ **k** $2x = \pi - \frac{\pi}{4}, \pi + \frac{\pi}{4},$ **l** $3x = \frac{\pi}{6}, \pi + \frac{\pi}{6}, 2\pi + \frac{\pi}{6},$

$\qquad\quad = \frac{3\pi}{4}, \frac{9\pi}{4} \qquad\qquad\quad 3\pi - \frac{\pi}{4}, 3\pi + \frac{\pi}{4} \qquad\qquad 3\pi + \frac{\pi}{6}, 4\pi + \frac{\pi}{6}, 5\pi + \frac{\pi}{6}$

$\quad x = \frac{5\pi}{12}, \frac{23\pi}{12} \qquad\qquad\quad = \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{11\pi}{4}, \frac{13\pi}{4} \qquad\quad = \frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}, \frac{19\pi}{6}, \frac{25\pi}{6}, \frac{31\pi}{6}$

$\qquad\qquad\qquad\qquad\qquad\quad x = \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8} \qquad x = \frac{\pi}{18}, \frac{7\pi}{18}, \frac{13\pi}{18}, \frac{19\pi}{18}, \frac{25\pi}{18}, \frac{31\pi}{18}$

5 **a** $\theta = -90°, 90°$ **b** $\tan 2\theta = -1$ **c** $\theta + 60 = 16.9, 180 - 16.9$

$\qquad\qquad\qquad\qquad\quad 2\theta = 180 - 45, 360 - 45 \qquad\qquad\quad = 16.9, 163.1$

$\qquad\qquad\qquad\qquad\qquad\quad -45, -45 - 180 \qquad\qquad\quad \theta = -43.1°, 103.1°$

$\qquad\qquad\qquad\qquad\qquad\quad = -225, -45, 135, 315$

$\qquad\qquad\qquad\qquad\quad \theta = -112.5°, -22.5°,$

$\qquad\qquad\qquad\qquad\qquad\quad 67.5°, 157.5°$

d $\tan(\theta - 15) = 1.85$ **e** $\sin 2\theta = 0.3$ **f** $\cos 3\theta = 0.5$

$\quad \theta - 15 = 61.6, 61.6 - 180 \qquad 2\theta = 17.458, 180 - 17.458, \qquad 3\theta = 60, 360 - 60, 360 + 60,$

$\qquad\qquad = -118.4, 61.6 \qquad\qquad\quad 17.458 - 360, \qquad\qquad\quad -60, 60 - 360, -60 - 360$

$\quad \theta = -103.4°, 76.6° \qquad\qquad\quad -17.458 - 180 \qquad\qquad\quad = -420, -300, -60,$

$\qquad\qquad\qquad\qquad\qquad\quad = -342.542, -197.458, \qquad\qquad 60, 300, 420$

$\qquad\qquad\qquad\qquad\qquad\quad 17.458, 162.542 \qquad\qquad \theta = -140°, -100°, -20°$

$\qquad\qquad\qquad\qquad\quad \theta = -171.3°, -98.7° \qquad\qquad\quad 20°, 100°, 140°$

$\qquad\qquad\qquad\qquad\qquad\quad 8.7°, 81.3°$

g $\sin(\theta + 110) = -1$ **h** $\cos(\theta - 27) = 0.6$ **i** $\tan\theta = \frac{7}{3}$

$\quad \theta + 110 = 270 \qquad\qquad\quad \theta - 27 = 53.1, -53.1 \qquad\quad \theta = 66.8, 66.8 - 180$

$\quad \theta = 160° \qquad\qquad\qquad\quad \theta = -26.1°, 80.1° \qquad\qquad \theta = -113.2°, 66.8°$

j $\cos 2\theta = -0.375$ **k** $\tan(\theta + 92) = -\frac{1}{3}$ **l** $\sin\frac{1}{3}\theta = 0.25$

$\quad 2\theta = 180 - 67.976, \qquad\qquad \theta + 92 = 180 - 18.4, -18.4 \qquad \frac{1}{3}\theta = 14.478$

$\qquad\qquad 180 + 67.976, \qquad\qquad\qquad = -18.4, 161.6 \qquad\qquad \theta = 43.4°$

$\qquad\qquad 67.976 - 180, \qquad\qquad \theta = -110.4°, 69.6°$

$\qquad\qquad -67.976 - 180$

$\qquad\quad = -247.976, -112.024,$

$\qquad\qquad 112.024, 247.976$

$\quad \theta = -124.0°, -56.0°,$

$\qquad\qquad 56.0°, 124.0°$

6 **a** $2x + 30 = 45, 180 + 45$
$= 45, 225$
$2x = 15, 195$
$x = 7.5°, 97.5°$

b $2x - 15 = 0, 180$
$2x = 15, 195$
$x = 7.5°, 97.5°$

c $2x + 70 = 360 - 60, 360 + 60$
$= 300, 420$
$2x = 230, 350$
$x = 115°, 175°$

d $2x + 210 = 360 + 15.070,$
$540 - 15.070$
$= 375.070, 524.930$
$2x = 165.070, 314.930$
$x = 82.5°, 157.5°$

e $2x - 38 = 180 - 50.208,$
$180 + 50.208$
$= 129.792, 230.208$
$2x = 167.792, 268.208$
$x = 83.9°, 134.1°$

f $2x - 56 = 180 - 17.745,$
-17.745
$= -17.745, 162.256$
$2x = 38.256, 218.256$
$x = 19.1°, 109.1°$

g $3x - 24 = 42.862,$
$360 - 42.862,$
$360 + 42.862$
$= 42.862, 317.138,$
402.862
$3x = 66.862, 341.138,$
426.862
$x = 22.3°, 113.7°, 142.3°$

h $3x + 60 = 180 - 62.241,$
$360 - 62.241,$
$540 - 62.241$
$= 117.759, 297.759,$
477.759
$3x = 57.759, 237.759,$
417.759
$x = 19.3°, 79.3°, 139.3°$

i $\frac{1}{2}x + 18 = 34.890$
$\frac{1}{2}x = 16.890$
$x = 33.8°$

7 **a** $x = 0.48, \pi + 0.4795$
$x = 0.48^c, 3.62^c$

b $2x = 1.2503, 2\pi - 1.2503,$
$2\pi+1.2503, 4\pi-1.25032$
$= 1.2503, 5.0328,$
$7.5335, 11.3160$
$x = 0.63^c, 2.52^c, 3.77^c, 5.66^c$

c $x + \frac{\pi}{4} = \pi - 0.7754,$
$2\pi + 0.7754$
$= 2.3662, 7.0586$
$x = 1.58^c, 6.27^c$

d $\cos x = -\frac{1}{3}$
$x = \pi - 1.2310, \pi + 1.2310$
$= 1.91^c, 4.37^c$

e $\frac{1}{2}x = 0.0901, \pi - 0.0901$
$= 0.0901, 3.0515$
$x = 0.18^c, 6.10^c$

f $2x = \pi - 0.2213, 2\pi - 0.2213$
$3\pi - 0.2213, 4\pi - 0.2213$
$= 2.9203, 6.0619,$
$9.2035, 12.3451$
$x = 1.46^c, 3.03^c, 4.60^c, 6.17^c$

g $\sin (x - \frac{\pi}{3}) = 0.75$
$x - \frac{\pi}{3} = 0.8481, \pi - 0.8481$
$= 0.8481, 2.2935$
$x = 1.90^c, 3.34^c$

h $2x + \frac{\pi}{6} = 1.1071, \pi + 1.1071,$
$2\pi + 1.1071, 3\pi + 1.1071$
$= 1.1071, 4.2487,$
$7.3903, 10.5319$
$2x = 0.5835, 3.7251,$
$6.8667, 10.0083$
$x = 0.29^c, 1.86^c, 3.43^c, 5.00^c$

i $3x = \pi - 0.6266, \pi + 0.6266,$
$3\pi - 0.6266, 3\pi + 0.6266,$
$5\pi - 0.6266, 5\pi + 0.6266$
$= 2.5149, 3.7682, 8.7981,$
$10.0514, 15.0813, 16.3346$
$x = 0.84^c, 1.26^c, 2.93^c,$
$3.35^c, 5.03^c, 5.44^c$

j $\tan x = -\frac{5}{3}$
$x = \pi - 1.0304, 2\pi - 1.0304$
$x = 2.11^c, 5.25^c$

k $2x - \frac{\pi}{2} = \pi - 1.2239, \pi + 1.2239,$
$3\pi - 1.2239, 3\pi + 1.2239$
$= 1.9177, 4.3655,$
$8.2009, 10.6487$
$2x = 3.4885, 5.9363,$
$9.7717, 12.2195$
$x = 1.74^c, 2.97^c, 4.89^c, 6.11^c$

l $\sin 2x = -\frac{1}{6}$
$2x = \pi + 0.1674, 2\pi - 0.1674,$
$3\pi + 0.1674, 4\pi - 0.1674$
$= 3.3090, 6.1157,$
$9.5922, 12.3989$
$x = 1.65^c, 3.06^c, 4.80^c, 6.20^c$

8 **a** $(2y - 1)(y - 1) = 0$

$y = \frac{1}{2}, 1$

b $\sin x = \frac{1}{2}$ or 1

$x = 30, 180 - 30$ or 90

$x = 30°, 90°, 150°$

9 **a** $\sin \theta = \pm\frac{\sqrt{3}}{2}$

$\theta = 60, 180 - 60$ or $180 + 60, 360 - 60$

$\theta = 60, 120, 240, 300$

b $\tan \theta = \pm 1$

$\theta = 45, 180 + 45$ or $180 - 45, 360 - 45$

$\theta = 45, 135, 225, 315$

c $\cos \theta (2 \cos \theta + 1) = 0$

$\cos \theta = 0$ or -0.5

$\theta = 90, 270$ or $180 - 60, 180 + 60$

$\theta = 90, 120, 240, 270$

d $\sin \theta = 0$ or $\cos \theta = 0.25$

$\theta = 0, 180, 360$ or $75.5, 360 - 75.5$

$\theta = 0, 75.5, 180, 284.5, 360$

e $\sin \theta (4 - \tan \theta) = 0$

$\sin \theta = 0$ or $\tan \theta = 4$

$\theta = 0, 180, 360$ or $76.0, 180 + 76.0$

$\theta = 0, 76.0, 180, 256.0, 360$

f $\cos \theta = -1$ or 0.5

$\theta = 180$ or $60, 360 - 60$

$\theta = 60, 180, 300$

g $(\tan \theta - 1)(\tan \theta - 2) = 0$

$\tan \theta = 1$ or 2

$\theta = 45, 180 + 45$ or $63.4, 180 + 63.4$

$\theta = 45, 63.4, 225, 243.4$

h $(3 \sin \theta - 1)(\sin \theta - 2) = 0$

$\sin \theta = \frac{1}{3}$ or 2 [no solutions]

$\theta = 19.5, 180 - 19.5$

$\theta = 19.5, 160.5$

i $\tan^2 \theta - \tan \theta - 6 = 0$

$(\tan \theta + 2)(\tan \theta - 3) = 0$

$\tan \theta = -2$ or 3

$\theta = 180{-}63.4, 360{-}63.4$ or $71.6, 180{+}71.6$

$\theta = 71.6, 116.6, 251.6, 296.6$

j $(3 \cos \theta - 2)(2 \cos \theta + 1) = 0$

$\cos \theta = -0.5$ or $\frac{2}{3}$

$\theta = 180 - 60, 180 + 60$ or $48.2, 360 - 48.2$

$\theta = 48.2, 120, 240, 311.8$

k $4 \sin^2 \theta - 8 \sin \theta + 3 = 0$

$(2 \sin \theta - 1)(2 \sin \theta - 3) = 0$

$\sin \theta = 0.5$ or 1.5 [no solutions]

$\theta = 30, 180 - 30$

$\theta = 30, 150$

l $\cos \theta = \frac{-2 \pm \sqrt{4 + 4}}{2}$

$\cos \theta = -1 + \sqrt{2}$ or $-1 - \sqrt{2}$ [no solutions]

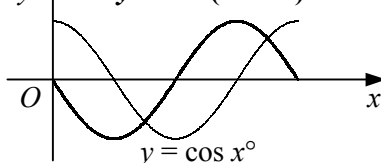$\theta = 65.5, 360 - 65.5$

$\theta = 65.5, 294.5$

m $\tan \theta = \frac{-3 \pm \sqrt{9 + 4}}{2}$

$\tan \theta = \frac{1}{2}(-3 \pm \sqrt{13})$

$\theta = 180{-}73.2, 360{-}73.2$ or $16.8, 180{+}16.8$

$\theta = 16.8, 106.8, 196.8, 286.8$

n $3 \sin^2 \theta + \sin \theta - 1 = 0$

$\sin \theta = \frac{-1 \pm \sqrt{1 + 12}}{6} = \frac{1}{6}(-1 \pm \sqrt{13})$

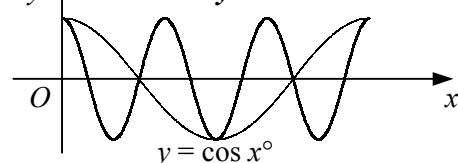$\theta = 180{+}50.1, 360{-}50.1$ or $25.7, 180{-}25.7$

$\theta = 25.7, 154.3, 230.1, 309.9$

10 **a**, **b**

c $x = 135, 315$

11 **a**

b $x = 0, 90, 180, 270, 360$

c $x = 0, 45, 90, 135, 180$