C3 Numerical Methods

1a) $x_{n+1} = \frac{2}{(x_n)^2} + 2$

$x_0 = 2.5$

$x_1 = 2.32$

$x_2 = 2.372$

$x_3 = 2.356$

$x_4 = 2.360$

b) Upper Bound: 2.3595

Lower Bound: 2.3585

$-(2.3595)^3 + 2(2.3595)^2 + 2 = -1.42 \times 10^{-3}$

$-(2.3585)^3 + 2(2.3585)^2 + 2 = 5.84 \times 10^{-3}$

Change of sign $\therefore$ x=2.359 is correct to 3dp

2a) $f(x) = \ln(x+2) - x + 1$

$f(2) = \ln(4) - 2 + 1 = 0.386$ (3dp)

$f(3) = \ln(5) - 3 + 1 = -0.391$ (3dp)

change of sign $\therefore$ there is a root of $f(x)=0$ in the interval $2 < x < 3$

b) $x_{n+1} = \ln(x_n + 2) + 1$

$x_0 = 2.5$

$x_1 = 2.50408$

$x_2 = 2.50498$

$x_3 = 2.50518$

c) $f(x) = \ln(x+2) - x + 1$

upper bound = 2.5055

lower bound = 2.5045

$f(2.5055) = \ln(4.5055) - 2.5055 + 1 = -2.01 \times 10^{-4}$ (2dp)

$f(2.5045) = \ln(4.5045) - 2.5045 + 1 = 5.77 \times 10^{-4}$ (2dp)

change of sign $\therefore$ $x = 2.505$ is a root of $f(x) = 0$ correct to 3dp.

3a) $f(x) = 3xe^x - 1$

turning point where $f'(x) = 0$

$f'(x) = 3e^x + 3xe^x$ $\quad u = 3x \quad v = e^x$

$\frac{du}{dx} = 3 \quad \frac{dv}{dx} = e^x$

$f'(x) = 3e^x(1 + 3x)$

$0 = 3e^x(1 + 3x)$

$x = -1/3$

$y = 3(-1/3)e^{-1/3} - 1$

$= -e^{-1/3} - 1$

$(-1/3, -e^{-1/3} - 1)$

b) $x_{n+1} = \frac{1}{3}e^{-x_n}$

$x_0 = 0.25$

$x_1 = 0.2596$

$x_2 = 0.2571$

$x_3 = 0.2578$

c) upper bound = 0.25765 lower bound = 0.25755

$f(0.25765) = 3(0.25765)e^{(0.25765)} - 1 = 1.09 \times 10^{-4}$

$f(0.25755) = 3(0.25755)e^{(0.25755)} - 1 = -3.79 \times 10^{-4}$

change of sign $\therefore$ $x = 0.2576$ is correct to 4dp

4a) $f(x) = 3x^3 - 2x - 6$

$f(1.4) = 3(1.4)^3 - 2(1.4) - 6 = -0.568$

$f(1.45) = 3(1.45)^3 - 2(1.45) - 6 = 0.245875$

change of sign $\therefore$ a lies between 1.4 and 1.45

b) $3x^3 - 2x - 6 = 0$

$3x^3 - 2x = 6$

$x(3x^2 - 2) = 6$

$3x^2 - 2 = \frac{6}{x}$

$3x^2 = \frac{6}{x} + 2$

$x^2 = \frac{2}{x} + \frac{2}{3}$

$x = \sqrt{\frac{2}{x} + \frac{2}{3}}$

c) $x_{n+1} = \sqrt{\frac{2}{x_n} + \frac{2}{3}}$

$x_0 = 1.43$

$x_1 = 1.4371$

$x_2 = 1.4347$

$x_3 = 1.4355$

d) upper bound = 1.4355 lower bound = 1.4345

$f(1.4355) = 3(1.4355)^3 - 2(1.4355) - 6 = 3.23 \times 10^{-3}$ (2dp)

$f(1.4345) = 3(1.4345)^3 - 2(1.4345) - 6 = -0.01$ (2dp)

change of sign $\therefore a = 1.435$ (3dp)

5a) $f(x) = -x^3 + 3x^2 - 1$

$0 = -x^3 + 3x^2 - 1$

$0 = x^2(-x+3) - 1$

$1 = x^2(3-x)$

$\frac{1}{3-x} = x^2$

$x = \sqrt{\frac{1}{3-x}}$

b) $x_{n+1} = \sqrt{\frac{1}{3-x_n}}$

$x_1 = 0.6$

$x_2 = 0.6455$

$x_3 = 0.6517$

$x_4 = 0.6526$

c) upper bound = 0.6535

lower bound = 0.6525

(2dp)

$f(0.6535) = -(0.6535)^3 + 3(0.6535)^2 - 1 = 2.10 \times 10^{-3}$

$f(0.6525) = -(0.6525)^3 + 3(0.6525)^2 - 1 = -5.37 \times 10^{-4}$

Change of sign $\therefore$ $x = 0.653$ is a root of $f(x) = 0$ (to 3dp)

6a) $f(x) = 2x^3 - x - 4$

$0 = 2x^3 - x - 4$

$0 = x(2x^2 - 1) - 4$

$4 = x(2x^2 - 1)$

$\frac{4}{x} = 2x^2 - 1$

$\frac{4}{x} + 1 = 2x^2$

$\frac{2}{x} + \frac{1}{2} = x^2$

$x = \sqrt{\frac{2}{x} + \frac{1}{2}}$

b) $x_{n+1} = \sqrt{\frac{2}{x_n} + \frac{1}{2}}$

$x_0 = 1.35$

$x_1 = 1.41$

$x_2 = 1.39$

$x_3 = 1.39$

c) $\alpha = 1.392$

upper bound = 1.3925

Lower bound = 1.3915

$f(1.3925) = 2(1.3925)^3 - (1.3925) - 4 = 7.77 \times 10^{-3}$ (2dp)

$f(1.3915) = 2(1.3915)^3 - (1.3915) - 4 = -2.85 \times 10^{-3}$ (2dp)

Change of sign $\therefore \alpha = 1.392$ to 3dp.

7a) $y = \ln(4 - 2x)$

$e^y = 4 - 2x$

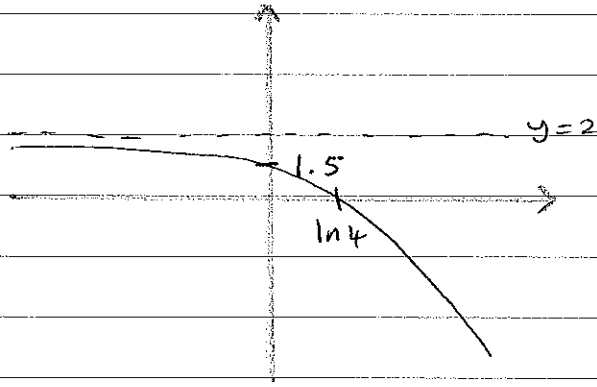
$2x = 4 - e^y$

$x = 2 - \frac{1}{2}e^y$

$f^{-1}(x) = 2 - \frac{1}{2}e^x \qquad x \in \mathbb{R}$

b) $f^{-1}(x) < 2$

c)



crosses x when $y = 0$

$0 = 2 - \frac{1}{2}e^x$

$\frac{1}{2}e^x = 2$

$e^x = 4$

$x = \ln 4$

d) $x_{n+1} = -\frac{1}{2}e^{x_n}$

$x_0 = -0.3$

$x_1 = \cancel{-0.6749}\ -0.3704$

$x_2 = \cancel{-0.2546}\ -0.3452$

e) $x_3 = -0.3540$

$x_4 = -0.3509$

$x_5 = -0.3520$

$x_6 = -0.3516$

$x_7 = -0.3518$

$x_8 = -0.3517$

$k = -0.352$

8a) $y = (2x-1)\tan 2x$

$u = 2x-1 \qquad v = \tan 2x$

$\frac{du}{dx} = 2 \qquad \frac{dv}{dx} = 2\sec^2 2x$

$\frac{dy}{dx} = 2\tan 2x + (2x-1)(2)\sec^2 2x$

$0 = \cancel{2}\tan 2k + (2k-1)(\cancel{2})\sec^2 2k$

$0 = \frac{\sin 2k}{\cos 2k} + (2k-1)\left(\frac{1}{\cos^2 2k}\right)$

$0 = \sin 2k \cos 2k + 2k - 1$

$0 = 2\sin 2k \cos 2k + 4k - 2$

$0 = \sin 4k + 4k - 2$

$4k + \sin 4k - 2 = 0$

b) $x_{n+1} = \frac{1}{4}(2 - \sin 4x_n)$

$x_0 = 0.3$

$x_1 = \cancel{0.266}\ 0.2670$

$x_2 = 0.2809$

$x_3 = 0.2746$

$x_4 = 0.2774$

c) $k = 0.277$ upper bound = 0.2775

lower bound = 0.2765

$4(0.2775) + \sin(4\times 0.2775) - 2 = 5.699\times 10^{-3}$ 3dp

$4(0.2765) + \sin(4\times 0.2765) - 2 = -8.712\times 10^{-5}$ 3dp

change of sign $\therefore$ $k = 0.277$ to (3 sf)

9a) $3e^{\alpha} - \frac{1}{2\alpha} = 0$

$$3e^{\alpha} = \frac{1}{2\alpha}$$

$$6\alpha e^{\alpha} = 1$$

$$6\alpha = \frac{1}{e^{\alpha}}$$

$$\alpha = \frac{1}{6}e^{-\alpha}$$

b) $x_{n+1} = \frac{1}{6}e^{-x_n}$

$x_0 = 1$

$x_1 = 0.0613$

$x_2 = 0.1568$

$x_3 = 0.1425$

$x_4 = 0.1445$

c) $\alpha = 0.1443$ 4dp

upper bound = 0.14435

lower bound = 0.14425

$$3e^{0.14435} - \frac{1}{2(0.14435)} = 2.06 \times 10^{-3} \quad 2dp$$

$$3e^{0.14425} - \frac{1}{2(0.14425)} = -6.86 \times 10^{-4} \quad 2dp$$

change of sign $\therefore$ $\alpha = 0.1443$ correct to 4dp.