1 **a** $\frac{1}{3}x^3 + c$ **b** $\frac{1}{7}x^7 + c$ **c** $\frac{1}{2}x^2 + c$ **d** $-\frac{1}{3}x^{-3} + c$ **e** $5x + c$ **f** $x^3 + c$

g $\frac{1}{2}x^8 + c$ **h** $-6x^{-1} + c$ **i** $\frac{4}{3}x^6 + c$ **j** $\frac{1}{6}x^2 + c$ **k** $-\frac{1}{4}x^{-8} + c$ **l** $-\frac{3}{8}x^{-2} + c$

2 **a** $= x^2 + 3x + c$ **b** $= 3x^4 - 2x^2 + c$ **c** $= 7x - \frac{1}{3}x^3 + c$ **d** $= \frac{1}{3}x^3 + \frac{1}{2}x^2 + x + c$

e $= \frac{1}{5}x^5 + \frac{5}{3}x^3 + c$ **f** $= \int (x^3 - 3x)\,\mathrm{d}x$ $= \frac{1}{4}x^4 - \frac{3}{2}x^2 + c$ **g** $= \int (x^2 - 4x + 4)\,\mathrm{d}x$ $= \frac{1}{3}x^3 - 2x^2 + 4x + c$ **h** $= \frac{3}{5}x^5 + \frac{1}{3}x^3 - 6x + c$

i $= \int (2 + x^{-2})\,\mathrm{d}x$ $= 2x - x^{-1} + c$ **j** $= \int (x - x^{-3})\,\mathrm{d}x$ $= \frac{1}{2}x^2 + \frac{1}{2}x^{-2} + c$ **k** $= \int (2x^{-2} - 3x^2)\,\mathrm{d}x$ $= -2x^{-1} - x^3 + c$ **l** $= \int (x^2 - 8 + 16x^{-2})\,\mathrm{d}x$ $= \frac{1}{3}x^3 - 8x - 16x^{-1} + c$

3 **a** $= \frac{2}{3}y^{\frac{3}{2}} + c$ **b** $= \frac{2}{7}y^{\frac{7}{2}} + c$ **c** $= 2y^{\frac{1}{2}} + c$

d $= 3y^{\frac{4}{3}} + c$ **e** $= \frac{4}{7}y^{\frac{7}{4}} + c$ **f** $= 15y^{\frac{1}{3}} + c$

g $= \int y^{\frac{1}{4}}\,\mathrm{d}x$ $= \frac{4}{5}y^{\frac{5}{4}} + c$ **h** $= \int 7y^{-\frac{1}{2}}\,\mathrm{d}x$ $= 14y^{\frac{1}{2}} + c$ **i** $= \int \frac{1}{2}y^{-2}\,\mathrm{d}x$ $= -\frac{1}{2}y^{-1} + c$

j $= \int y^{\frac{3}{2}}\,\mathrm{d}x$ $= \frac{2}{5}y^{\frac{5}{2}} + c$ **k** $= \int \frac{5}{2}y^{-4}\,\mathrm{d}x$ $= -\frac{5}{6}y^{-3} + c$ **l** $= \int \frac{1}{3}y^{-\frac{1}{2}}\,\mathrm{d}x$ $= \frac{2}{3}y^{\frac{1}{2}} + c$

4 **a** $= 2t^{\frac{3}{2}} - t + c$ **b** $= \int (2r + r^{\frac{1}{2}})\,\mathrm{d}r$ $= r^2 + \frac{2}{3}r^{\frac{3}{2}} + c$ **c** $= \int (9p^2 - 6p + 1)\,\mathrm{d}p$ $= 3p^3 - 3p^2 + p + c$ **d** $= 2x^2 + \frac{3}{4}x^{\frac{4}{3}} + c$

e $= \int (y^{-3} + y)\,\mathrm{d}y$ $= -\frac{1}{2}y^{-2} + \frac{1}{2}y^2 + c$ **f** $= \frac{1}{6}x^3 - \frac{2}{5}x^{\frac{5}{2}} + c$ **g** $= \int (t^2 + 2)\,\mathrm{d}t$ $= \frac{1}{3}t^3 + 2t + c$ **h** $= \frac{3}{8}r^{\frac{8}{3}} - \frac{3}{5}r^{\frac{5}{3}} + c$

i $= \int (2p^3 - \frac{1}{2}p)\,\mathrm{d}p$ $= \frac{1}{2}p^4 - \frac{1}{4}p^2 + c$ **j** $= 4y - \frac{4}{11}y^{\frac{11}{4}} + c$ **k** $= \int (\frac{1}{3}x^{-2} + 2)\,\mathrm{d}x$ $= -\frac{1}{3}x^{-1} + 2x + c$ **l** $= \int (2t^{\frac{1}{2}} + 3t^{-\frac{1}{2}})\,\mathrm{d}t$ $= \frac{4}{3}t^{\frac{3}{2}} + 6t^{\frac{1}{2}} + c$

5 **a** $= x^3 - \frac{1}{2}x^2 + 6x + c$ **b** $= \frac{1}{7}x^7 - \frac{1}{4}x^4 + x^2 - 5x + c$ **c** $= \int (x^3 - x^2 - 2x)\ dx$
$= \frac{1}{4}x^4 - \frac{1}{3}x^3 - x^2 + c$

d $= \int (x + 4x^{\frac{1}{2}} + 4)\ dx$
$= \frac{1}{2}x^2 + \frac{8}{3}x^{\frac{3}{2}} + 4x + c$

e $= \int (2x^3 + 3x^2 - 8x - 12)\ dx$
$= \frac{1}{2}x^4 + x^3 - 4x^2 - 12x + c$

f $= \int (x^3 - 2x^{\frac{4}{3}} + 7x^{-2})\ dx$
$= \frac{1}{4}x^4 - \frac{6}{7}x^{\frac{7}{3}} - 7x^{-1} + c$

g $= \int (\frac{1}{4}x^{-3} - \frac{2}{3}x^{-2})\ dx$
$= -\frac{1}{8}x^{-2} + \frac{2}{3}x^{-1} + c$

h $= \int (1 - 4x^{-2} + 4x^{-4})\ dx$
$= x + 4x^{-1} - \frac{4}{3}x^{-3} + c$

i $= \int (x^4 + x^{\frac{5}{2}} - x^{\frac{3}{2}} - 1)\ dx$
$= \frac{1}{5}x^5 + \frac{2}{7}x^{\frac{7}{2}} - \frac{2}{5}x^{\frac{5}{2}} - x + c$

6 **a** $y = \int (8x + 3)\ dx$
$y = 4x^2 + 3x + c$

b $y = \int (\frac{1}{2}x^3 - x^2)\ dx$
$y = \frac{1}{8}x^4 - \frac{1}{3}x^3 + c$

c $y = \int \frac{4}{3}x^{-3}\ dx$
$y = -\frac{2}{3}x^{-2} + c$

d $y = \int (x^3 + 3x^2 + 3x + 1)\ dx$
$y = \frac{1}{4}x^4 + x^3 + \frac{3}{2}x^2 + x + c$

e $y = \int (2x - 3x^{-\frac{1}{2}})\ dx$
$y = x^2 - 6x^{\frac{1}{2}} + c$

f $y = \int (x^{\frac{3}{2}} - 2x^{-\frac{3}{2}})\ dx$
$y = \frac{2}{5}x^{\frac{5}{2}} + 4x^{-\frac{1}{2}} + c$

g $y = \int (\frac{3}{2}x^{-2} - \frac{1}{2})\ dx$
$y = -\frac{3}{2}x^{-1} - \frac{1}{2}x + c$

h $y = \int (10x^{-3} - 2x^{-2})\ dx$
$y = -5x^{-2} + 2x^{-1} + c$

i $y = \int (3x^{\frac{1}{2}} - \frac{2}{3}x^{-\frac{1}{2}})\ dx$
$y = 2x^{\frac{3}{2}} - \frac{4}{3}x^{\frac{1}{2}} + c$

1 **a** $x^2 + x + c$

b $y = x^2 + x + c$

$(1, 5) \Rightarrow 5 = 1 + 1 + c$

$\therefore c = 3$

$y = x^2 + x + 3$

2 **a** $y = \int (3 - 6x)\ dx$

$y = 3x - 3x^2 + c$

$(2, 1) \Rightarrow 1 = 6 - 12 + c$

$\therefore c = 7$

$y = 3x - 3x^2 + 7$

b $y = \int (3x^2 - x)\ dx$

$y = x^3 - \frac{1}{2}x^2 + c$

$(4, 41) \Rightarrow 41 = 64 - 8 + c$

$\therefore c = -15$

$y = x^3 - \frac{1}{2}x^2 - 15$

c $y = \int (x^2 + 4x + 1)\ dx$

$y = \frac{1}{3}x^3 + 2x^2 + x + c$

$(-3, 4) \Rightarrow 4 = -9 + 18 - 3 + c$

$\therefore c = -2$

$y = \frac{1}{3}x^3 + 2x^2 + x - 2$

d $y = \int (7 - 5x - x^3)\ dx$

$y = 7x - \frac{5}{2}x^2 - \frac{1}{4}x^4 + c$

$(2, 0) \Rightarrow 0 = 14 - 10 - 4 + c$

$\therefore c = 0$

$y = 7x - \frac{5}{2}x^2 - \frac{1}{4}x^4$

e $y = \int (8x - 2x^{-2})\ dx$

$y = 4x^2 + 2x^{-1} + c$

$(\frac{1}{2}, -1) \Rightarrow -1 = 1 + 4 + c$

$\therefore c = -6$

$y = 4x^2 + 2x^{-1} - 6$

f $y = \int (3 - x^{\frac{1}{2}})\ dx$

$y = 3x - \frac{2}{3}x^{\frac{3}{2}} + c$

$(4, 8) \Rightarrow 8 = 12 - \frac{16}{3} + c$

$\therefore c = \frac{4}{3}$

$y = 3x - \frac{2}{3}x^{\frac{3}{2}} + \frac{4}{3}$

3 $f(x) = \int (3 + 2x - x^2)\ dx$

$f(x) = 3x + x^2 - \frac{1}{3}x^3 + c$

$(3, 5) \Rightarrow 5 = 9 + 9 - 9 + c$

$\therefore c = -4$

$f(x) = 3x + x^2 - \frac{1}{3}x^3 - 4$

4 $y = \int (10x^{\frac{3}{2}} - 2x^{-\frac{1}{2}})\ dx$

$y = 4x^{\frac{5}{2}} - 4x^{\frac{1}{2}} + c$

$y = 0$ when $x = 7$

$\therefore 7 = 0 + 0 + c$

$c = 7$

$\therefore y = 4x^{\frac{5}{2}} - 4x^{\frac{1}{2}} + 7$

when $x = 4$

$y = 4(32) - 4(2) + 7$

$y = 127$

5 **a** $f(x) = \int (2x^3 - x - 8)\ dx$

$f(x) = \frac{1}{2}x^4 - \frac{1}{2}x^2 - 8x + c$

$(-1, 4) \Rightarrow 4 = \frac{1}{2} - \frac{1}{2} + 8 + c$

$\therefore\ c = -4$

$f(x) = \frac{1}{2}x^4 - \frac{1}{2}x^2 - 8x - 4$

b at $x = 2$, $y = 8 - 2 - 16 - 4 = -14$

grad $= 16 - 2 - 8 = 6$

$\therefore\ y + 14 = 6(x - 2)$

$[y = 6x - 26]$

6 $f(x) = \int (3x^2 - 8x - 5)\ dx$

$f(x) = x^3 - 4x^2 - 5x + c$

$(0, 0) \Rightarrow 0 = 0 + c$

$\therefore\ c = 0$

$f(x) = x^3 - 4x^2 - 5x$

$= x(x^2 - 4x - 5)$

$= x(x + 1)(x - 5)$

crosses x-axis when $f(x) = 0$

$\therefore\ (-1, 0)$ and $(5, 0)$

7 **a** $y = \int (3x + 2x^{-2})\ dx$

$y = \frac{3}{2}x^2 - 2x^{-1} + c$

b $y = 8$ when $x = 2$

$\therefore\ 8 = 6 - 1 + c$

$c = 3$

$\therefore\ y = \frac{3}{2}x^2 - 2x^{-1} + 3$

when $x = \frac{1}{2}$

$y = \frac{3}{8} - 4 + 3$

$y = -\frac{5}{8}$

8 **a** $y = \int (3x^2 + kx)\ dx$

$y = x^3 + \frac{1}{2}kx^2 + c$

$(1, 6) \Rightarrow 6 = 1 + \frac{1}{2}k + c$

$5 = \frac{1}{2}k + c \qquad (1)$

$(2, 1) \Rightarrow 1 = 8 + 2k + c$

$-7 = 2k + c \qquad (2)$

$(2) - (1) \qquad -12 = \frac{3}{2}k$

$k = -8$

b sub. $-7 = -16 + c$

$c = 9$

$\therefore\ y = x^3 - 4x^2 + 9$