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| **Pearson Edexcel Level 3** |
| **GCE Mathematics** **Advanced Level** **Paper 2: Pure Mathematics** |
| **Sample assessment material for first teaching September 2017** **Time: 2 hours** | **Paper Reference(s)** |
| **9MA0/02** |
| **You must have:** **Mathematical Formulae and Statistical Tables, calculator** |

**Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for algebraic manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

**Instructions**

• Use black ink or ball-point pen.

• If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).

• Answer all the questions and ensure that your answers to parts of questions are clearly labelled.

• Answer the questions in the spaces provided – there may be more space than you need.

• You should show sufficient working to make your methods clear. Answers without working may not gain full credit.

• Inexact answers should be given to three significant figures unless otherwise stated.

**Information**

• A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.

• There are 16 questions in this question paper. The total mark for this paper is 100.

• The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.

**Advice**

• Read each question carefully before you start to answer it.

• Try to answer every question.

• Check your answers if you have time at the end.

 • If you change your mind about an answer, cross it out and put your new answer and any working underneath.

**Answer ALL questions.**

**1.** f(*x*) = 2*x*3 – 5*x*2 + *ax* + *a*.

Given that (*x* + 2) is a factor of f (*x*), find the value of the constant *a*.

 **(Total for Question 1 is 3 marks)**

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**2.** Some A level students were given the following question.

Solve, for −90° < *θ* < 90°, the equation

cos *θ* = 2 sin *θ*.

 The attempts of two of the students are shown below.

|  |  |  |
| --- | --- | --- |
| Student A |  | Student B |
| cos *θ* = 2 sin *θ* |  | cos *θ* = 2 sin *θ* |
| tan *θ* = 2 |  | cos2 *θ* = 4 sin2 *θ* |
|  *θ* = 63.4° |  | 1 – sin2 *θ* = 4 sin2 *θ* |
|  |  | sin2 *θ* =  |
|  |  | sin *θ* = ± |
|  |  | *θ* = ±26.6° |

(*a*) Identify an error made by student A.

**(1)**

Student B gives *θ* = −26.6° as one of the answers to cos *θ* = 2 sin *θ*.

(*b*) (i) Explain why this answer is incorrect.

 (ii) Explain how this incorrect answer arose.

**(2)**

**(Total for Question 2 is 3 marks)**

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**3.** Given *y* = *x*(2*x* + 1)4, show that

 = (2*x* + 1)*n* (*Ax* + *B*)

 where *n*, *A* and *B* are constants to be found.

**(Total for Question 3 is 4 marks)**

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**4.** Given

 f(*x*) = e*x*, *x* ∈ ℝ,

 g(*x*) = 3 ln *x*, *x* > 0, *x* ∈ ℝ,

 (*a*) find an expression for gf (*x*), simplifying your answer.

**(2)**

(*b*) Show that there is only one real value of *x* for which gf(*x*) = fg(*x*).

**(3)**

**(Total for Question 4 is 5 marks)**

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**5.** The mass, *m* grams, of a radioactive substance, *t* years after first being observed, is modelled by the equation

*m* = 25e–0.05*t*.

 According to the model,

 (*a*) find the mass of the radioactive substance six months after it was first observed,

**(2)**

(*b*) show that  = *km*, where *k* is a constant to be found.

**(2)**

**(Total for Question 5 is 4 marks)**

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**6.** Complete the table below. The first one has been done for you.

For each statement below you must state if it is always true, sometimes true or never true, giving a reason in each case.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Statement | Always True | Sometimes True | NeverTrue | Reason |
| The quadratic equation*ax*2 + *bx* + *c* = 0 (*a* ≠ 0) has 2 real roots. |  | ✓ |  | It only has 2 real roots when*b*2 – 4*ac* > 0When *b*2 – 4*ac* = 0 it has 1 realroot and when *b*2 – 4*ac* < 0 it has0 real roots. |
| (i)When a real value of *x* is substituted into*x*2 – 6*x* + 10 the resultis positive.**(2)** |  |  |  |  |
| (ii)If *ax* > *b* then *x* > **(2)**  |  |  |  |  |
| (iii)The difference between consecutive square numbers is odd.**(2)** **(2)** |  |  |  |  |

**(Total for Question 6 is 6 marks)**

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**7.** (*a*) Use the binomial expansion, in ascending powers of x, to show that

 = 2 –  + …

 where *k* is a rational constant to be found.

**(4)**

A student attempts to substitute *x* =1 into both sides of this equation to find an
approximate value for √3.

(*b*) State, giving a reason, if the expansion is valid for this value of *x*.

**(1)**

**(Total for Question 7 is 5 marks)**

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**8.**

**Figure 1**

Figure 1 shows a rectangle *ABCD*.

The point *A* lies on the *y*-axis and the points *B* and *D* lie on the *x*-axis as shown in Figure 1.

Given that the straight line through the points *A* and *B* has equation 5*y* + 2*x* = 10,

(*a*) show that the straight line through the points *A* and *D* has equation 2*y* − 5*x* = 4,

**(4)**

(*b*) find the area of the rectangle *ABCD*.

**(3)**

**(Total for Question 8 is 7 marks)**

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**9.** Given that *A* is constant and

 = 2*A*2,

 show that there are exactly two possible values for *A*.

**(Total for Question 9 is 5 marks)**

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**10.** In a geometric series the common ratio is *r* and sum to *n* terms is *Sn*

 Given

*S*∞= × *S*6

 show that *r* = ±, where *k* is an integer to be found.

**(Total for Question 10 is 4 marks)**

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**11.**

**Figure 2**

Figure 2 shows a sketch of part of the graph *y* = f(*x*) where

f(*x*) = 2⏐3 – *x*⏐ + 5, *x* ≥ 0

(*a*) State the range of f.

**(1)**

(*b*) Solve the equation

f(*x*) = *x* + 30

**(3)**

Given that the equation f(*x*) = *k*, where *k* is a constant, has two distinct roots,

(*c*) state the set of possible values for *k*.

**(2)**

**(Total for Question 11 is 6 marks)**

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**12.** (*a*) Solve, for –180° ≤ *x* < 180°, the equation

3 sin2 *x* + sin *x* + 8 = 9 cos2 *x*

 giving your answers to 2 decimal places.

**(6)**

(*b*) Hence find the smallest positive solution of the equation

3 sin2 (2*θ* – 30°) + sin (2*θ* – 30°) + 8 = 9 cos2 (2*θ* – 30°)

 giving your answer to 2 decimal places.

**(2)**

**(Total for Question 12 is 8 marks)**

\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

**13.** (*a*) Express 10 cos *θ* – 3 sin *θ* in the form *R* cos (*θ* + 𝛼), where *R* > 0 and 0 < 𝛼 < 90°.

Give the exact value of *R* and give the value of 𝛼, in degrees, to 2 decimal places.

**(3)**

**Figure 3**

The height above the ground, *H* metres, of a passenger on a Ferris wheel *t* minutes after the wheel starts turning, is modelled by the equation

*H* = *a* – 10 cos (80*t*)° + 3 sin (80*t*)°

where *a* is a constant.

Figure 3 shows the graph of *H* against *t* for two complete cycles of the wheel.

Given that the initial height of the passenger above the ground is 1 metre,

(*b*) (i) find a complete equation for the model.

 (ii) Hence find the maximum height of the passenger above the ground.

**(2)**

(*c*) Find the time taken, to the nearest second, for the passenger to reach the maximum height on the second cycle.

 (*Solutions based entirely on graphical or numerical methods are not acceptable.*)

**(3)**

It is decided that, to increase profits, the speed of the wheel is to be increased.

(*d*) How would you adapt the equation of the model to reflect this increase in speed?

**(1)**

**(Total for Question 13 is 9 marks)**

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**14.** A company decides to manufacture a soft drinks can with a capacity of 500 ml.

 The company models the can in the shape of a right circular cylinder with radius *r* cm and height *h* cm. In the model they assume that the can is made from a metal of negligible thickness.

(*a*) Prove that the total surface area, *S* cm2, of the can is given by

*S* = 2*π r* 2 + 

**(3)**

Given that *r* can vary,

(*b*) find the dimensions of a can that has minimum surface area.

**(5)**

(*c*) With reference to the shape of the can, suggest a reason why the company may choose not to manufacture a can with minimum surface area.

**(1)**

**(Total for Question 14 is 9 marks)**

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**15.**

**Figure 4**

Figure 4 shows a sketch of the curve *C* with equation

*y* =  – 9*x* + 11, *x* ≥ 0

The point *P* with coordinates (4, 15) lies on *C*.

The line *l* is the tangent to *C* at the point *P*.

The region *R*, shown shaded in Figure 4, is bounded by the curve *C*, the line *l* and the *y*-axis.

Show that the area of *R* is 24, making your method clear.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

**(Total for Question 15 is 10 marks)**

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**16.** (*a*) Express  in partial fractions.

**(3)**

A population of meerkats is being studied.

The population is modelled by the differential equation

 = *P*(11 – 2*P*), *t* ≥ 0, 0 < *P* < 5.5,

where *P*, in thousands, is the population of meerkats and *t* is the time measured in years

since the study began.

Given that there were 1000 meerkats in the population when the study began,

(*b*) determine the time taken, in years, for this population of meerkats to double,

**(6)**

(*c*) show that

*P* = 

 where *A*, *B* and *C* are integers to be found.

**(3)**

**(Total for Question 16 is 12 marks)**

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**TOTAL FOR PAPER IS 100 MARKS**