

**1.** In an arithmetic series

• the first term is 16

• the 21st term is 24

(*a*)Find the common difference of the series.

**(2)**

(*b*)Hence find the sum of the first 500 terms of the series.

**(2)**

**(Total for Question 1 is 4 marks)**

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**2.** The functions f and g are defined by

 f (*x*) = 7 – 2*x*2

 

(*a*)State the range of f

**(1)**

(*b*)Find gf (1.8)

**(2)**

(*c*)Find g–1(*x*)

**(2)**

**(Total for Question 2 is 5 marks)**

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**3.** Using the laws of logarithms, solve the equation

log3 (12*y* + 5) – log3 (1 – 3*y*) = 2

**(3)**

**(Total for Question 3 is 3 marks)**

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**4.** Given that *θ* is small and measured in radians, use the small angle approximations to show that



where *a*, *b* and *c* are integers to be found.

**(3)**

**(Total for Question 4 is 3 marks)**

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**5.** The curve *C* has equation

*y* = 5*x*4 – 24*x*3 + 42*x*2 – 32*x* + 11 *x* ∈ ℝ

(*a*)Find

(i) 

(ii) 

**(3)**

(*b*)(i) Verify that *C* has a stationary point at *x* = 1

(ii) Show that this stationary point is a point of inflection, giving reasons for

your answer.

**(4)**

**(Total for Question 5 is 7 marks)**

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**6.**

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The shape *OABCDEFO* shown in Figure 1 is a design for a logo.

In the design

• *OAB* is a sector of a circle centre *O* and radius *r*

• sector *OFE* is congruent to sector *OAB*

• *ODC* is a sector of a circle centre *O* and radius 2*r*

• *AOF* is a straight line

Given that the size of angle *COD* is *θ* radians,

(*a*)write down, in terms of *θ*, the size of angle *AOB*

**(1)**

(*b*)Show that the area of the logo is



**(2)**

(*c*)Find the perimeter of the logo, giving your answer in simplest form in terms of

*r*, *θ* and *π*.

**(2)**

**(Total for Question 6 is 5 marks)**

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**7. In this question you should show all stages of your working.**

**Solutions relying entirely on calculator technology are not acceptable.**



Figure 2 shows a sketch of part of the curve *C* with equation

*y* = *x*3 – 10*x*2 + 27*x* – 23

The point *P*(5, –13) lies on *C*

The line *l* is the tangent to *C* at *P*

(*a*)Use differentiation to find the equation of *l*, giving your answer in the form *y* = *mx* + *c*

where *m* and *c* are integers to be found.

**(4)**

(*b*)Hence verify that *l* meets *C* again on the *y*-axis.

**(1)**

The finite region *R*, shown shaded in Figure 2, is bounded by the curve *C* and the line *l*.

(*c*)Use algebraic integration to find the exact area of *R*.

**(4)**

**(Total for Question 7 is 9 marks)**

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**8.** The curve *C* has equation

*px*3 + *qxy* + 3*y*2 = 26

where *p* and *q* are constants.

(*a*)Show that



where *a*, *b* and *c* are integers to be found.

**(4)**

Given that

• the point *P* (–1, – 4) lies on *C*

• the normal to *C* at *P* has equation 19*x* + 26*y* + 123 = 0

(*b*)find the value of *p* and the value of *q*.

**(Total for Question 8 is 9 marks)**

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**9.** Show that



**(3)**

**(Total for Question 9 is 3 marks)**

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**10.** The time, *T* seconds, that a pendulum takes to complete one swing is modelled by the formula

*T* = *al b*

where *l* metres is the length of the pendulum and *a* and *b* are constants.

(*a*)Show that this relationship can be written in the form

log10 *T* = *b* log10 *l* + log10 *a*

**(2)**



A student carried out an experiment to find the values of the constants *a* and *b*.

The student recorded the value of *T* for different values of *l*.

Figure 3 shows the linear relationship between log10 *l* and log10 *T* for the student’s data.

The straight line passes through the points (– 0.7, 0) and (0.21, 0.45)

Using this information,

(*b*)find a complete equation for the model in the form

*T* = *al b*

giving the value of *a* and the value of *b*, each to 3 significant figures.

**(3)**

(*c*)With reference to the model, interpret the value of the constant *a*.

**(1)**

**(Total for Question 10 is 6 marks)**

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**11.**

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Figure 4 shows a sketch of the graph with equation

*y* = | 2*x* – 3*k* |

where *k* is a positive constant.

(*a*)Sketch the graph with equation *y* = f (*x*) where

f (*x*) = *k* – | 2*x* – 3*k* |

stating

• the coordinates of the maximum point

• the coordinates of any points where the graph cuts the coordinate axes

**(4)**

(*b*)Find, in terms of *k*, the set of values of *x* for which

*k* – | 2*x* – 3*k* | > *x* – *k*

giving your answer in set notation.

**(4)**

(*c*)Find, in terms of *k*, the coordinates of the minimum point of the graph with equation



**(2)**

**(Total for Question 11 is 10 marks)**

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**12.** (*a*)Use the substitution *u* = 1 + ** to show that



where *p* and *q* are constants to be found.

**(3)**

(*b*)Hence show that



where *A* and *B* are constants to be found.

**(4)**

**(Total for Question 12 is 7 marks)**

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**13.** The curve *C* has parametric equations

*x* = sin 2*θ y* = cosec3 *θ* 0 < *θ* < 

(*a*)Find an expression for in terms of *θ*

**(3)**

(*b*)Hence find the exact value of the gradient of the tangent to *C* at the point where *y* = 8

**(3)**

**(Total for Question 13 is 6 marks)**

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**14.**

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Water flows at a constant rate into a large tank.

The tank is a cuboid, with all sides of negligible thickness.

The base of the tank measures 8 m by 3 m and the height of the tank is 5 m.

There is a tap at a point *T* at the bottom of the tank, as shown in Figure 5.

At time *t* minutes after the tap has been opened

• the depth of water in the tank is *h* metres

• water is flowing into the tank at a constant rate of 0.48 m3 per minute

• water is modelled as leaving the tank through the tap at a rate of 0.1*h* m3 per minute

(*a*)Show that, according to the model,



**(4)**

Given that when the tap was opened, the depth of water in the tank was 2 m,

(*b*)show that, according to the model,

*h* = *A* + *B* e*–kt*

where *A*, *B* and *k* are constants to be found.

**(6)**

Given that the tap remains open,

(*c*)determine, according to the model, whether the tank will ever become full, giving a

reason for your answer.

**(2)**

**(Total for Question 14 is 12 marks)**

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**15.** (*a*)Express 2cos *θ* – sin *θ* in the form *R* cos (*θ* + *α*), where *R* > 0 and 0 < *α* < 

Give the exact value of *R* and the value of *α* in radians to 3 decimal places.

**(3)**



Figure 6 shows the cross-section of a water wheel.

The wheel is free to rotate about a fixed axis through the point *C*.

The point *P* is at the end of one of the paddles of the wheel, as shown in Figure 6.

The water level is assumed to be horizontal and of constant height.

The vertical height, *H* metres, of *P* above the water level is modelled by the equation

*H* = 3 + 4 cos (0.5*t*) – 2 sin (0.5*t*)

where *t* is the time in seconds after the wheel starts rotating.

Using the model, find

(*b*)(i) the maximum height of *P* above the water level,

(ii) the value of *t* when this maximum height first occurs, giving your answer to one

decimal place.

**(3)**

In a single revolution of the wheel, *P* is below the water level for a total of *T* seconds.

According to the model,

(*c*)find the value of *T* giving your answer to 3 significant figures.

(*Solutions based entirely on calculator technology are not acceptable*.)

**(4)**

In reality, the water level may not be of constant height.

(*d*)Explain how the equation of the model should be refined to take this into account.

**(1)**

**(Total for Question 15 is 11 marks)**

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**TOTAL FOR PAPER IS 100 MARKS**