C3 DIFFERENTIATION — Worksheet C

1 Differentiate with respect to x

a $(x + 3)^5$ **b** $(2x - 1)^3$ **c** $(8 - x)^7$ **d** $2(3x + 4)^6$

e $(6 - 5x)^4$ **f** $\frac{1}{x-2}$ **g** $\frac{4}{(2x+3)^3}$ **h** $\frac{1}{(7-3x)^2}$

2 Differentiate with respect to t

a $2e^{3t}$ **b** $\sqrt{4t-1}$ **c** $5 \ln 2t$ **d** $(8-3t)^{\frac{3}{2}}$

e $3 \ln (6t + 1)$ **f** $\frac{1}{2}e^{5t+4}$ **g** $\frac{6}{\sqrt[3]{2t-5}}$ **h** $2 \ln (3 - \frac{1}{4}t)$

3 Find $\frac{d^2y}{dx^2}$ for each of the following.

a $y = (3x - 1)^4$ **b** $y = 4 \ln (1 + 2x)$ **c** $y = \sqrt{5-2x}$

4 Find the value of $f'(x)$ at the value of x indicated in each case.

a $f(x) = x^2 - 6 \ln 2x$, $x = 3$ **b** $f(x) = 3 + 2x - e^{x-2}$, $x = 2$

c $f(x) = (2 - 5x)^4$, $x = \frac{1}{2}$ **d** $f(x) = \frac{4}{x+5}$, $x = -1$

5 Find the value of x for which $f'(x)$ takes the value indicated in each case.

a $f(x) = 4\sqrt{3x+15}$, $f'(x) = 2$ **b** $f(x) = x^2 - \ln (x - 2)$, $f'(x) = 5$

6 Differentiate with respect to x

a $(x^2 - 4)^3$ **b** $2(3x^2 + 1)^6$ **c** $\ln (3 + 2x^2)$ **d** $(2 + x)^3(2 - x)^3$

e $\left(\frac{x^4+6}{2}\right)^8$ **f** $\frac{1}{\sqrt{3-x^2}}$ **g** $4 + 7e^{x^2}$ **h** $(1 - 5x + x^3)^4$

i $3 \ln (4 - \sqrt{x})$ **j** $(e^{4x} + 2)^7$ **k** $\frac{1}{5+4\sqrt{x}}$ **l** $(\frac{2}{x} - x)^5$

7 Find the coordinates of any stationary points on each curve.

a $y = (2x - 3)^5$ **b** $y = (x^2 - 4)^3$ **c** $y = 8x - e^{2x}$

d $y = \sqrt{1+2x^2}$ **e** $y = 2 \ln (x - x^2)$ **f** $y = 4x + \frac{1}{x-3}$

8 Find an equation for the tangent to each curve at the point on the curve with the given x-coordinate.

a $y = (3x - 7)^4$, $x = 2$ **b** $y = 2 + \ln (1 + 4x)$, $x = 0$

c $y = \frac{9}{x^2+2}$, $x = 1$ **d** $y = \sqrt{5x-1}$, $x = \frac{1}{4}$

9 Find an equation for the normal to each curve at the point on the curve with the given x-coordinate.

a $y = e^{4-x^2} - 10$, $x = -2$ **b** $y = (1 - 2x^2)^3$, $x = \frac{1}{2}$

c $y = \frac{1}{2-\ln x}$, $x = 1$ **d** $y = 6e^{\frac{x}{3}}$, $x = 3$

1 Find an equation for the tangent to the curve with equation $y = x^2 + \ln(4x - 1)$ at the point on the curve where $x = \frac{1}{2}$.

2 A curve has the equation $y = \sqrt{8 - e^{2x}}$.

The point P on the curve has y-coordinate 2.

a Find the x-coordinate of P.

b Show that the tangent to the curve at P has equation

$$2x + y = 2 + \ln 4.$$

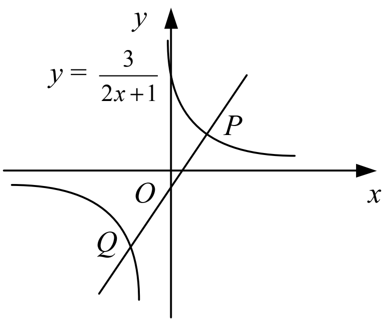
3 A curve has the equation $y = 2x + 1 + \ln(4 - 2x)$, $x < 2$.

a Find and simplify expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

b Find the coordinates of the stationary point of the curve.

c Determine the nature of this stationary point.

4



The diagram shows the curve with equation $y = \frac{3}{2x+1}$.

a Find an equation for the normal to the curve at the point $P(1, 1)$.

The normal to the curve at P intersects the curve again at the point Q.

b Find the exact coordinates of Q.

5 A quantity N is increasing such that at time t seconds,

$$N = ae^{kt}.$$

Given that at time $t = 0$, $N = 20$ and that at time $t = 8$, $N = 60$, find

a the values of the constants a and k,

b the value of N when $t = 12$,

c the rate at which N is increasing when $t = 12$.

6 $$f(x) \equiv (5 - 2x^2)^3.$$

a Find $f'(x)$.

b Find the coordinates of the stationary points of the curve $y = f(x)$.

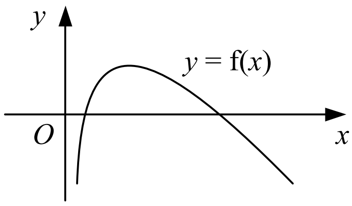
c Find the equation for the tangent to the curve $y = f(x)$ at the point with x-coordinate $\frac{3}{2}$, giving your answer in the form $ax + by + c = 0$, where a, b and c are integers.

7 A curve has the equation $y = 4x - \frac{1}{2}e^{2x}$.

a Find the coordinates of the stationary point of the curve, giving your answers in terms of natural logarithms.

b Determine the nature of the stationary point.

8



The diagram shows the curve $y = f(x)$ where $f(x) = 3 \ln 5x - 2x$, $x > 0$.

a Find $f'(x)$.

b Find the x-coordinate of the point on the curve at which the gradient of the normal to the curve is $-\frac{1}{4}$.

c Find the coordinates of the maximum turning point of the curve.

d Write down the set of values of x for which $f(x)$ is a decreasing function.

9 The curve C has the equation $y = \sqrt{x^2 + 3}$.

a Find an equation for the tangent to C at the point A $(-1, 2)$.

b Find an equation for the normal to C at the point B $(1, 2)$.

c Find the x-coordinate of the point where the tangent to C at A meets the normal to C at B.

10 A bucket of hot water is placed outside and allowed to cool. The surface temperature of the water, T °C, after t minutes is given by

$$T = 20 + 60e^{-kt},$$

where k is a positive constant.

a State the initial surface temperature of the water.

b State, with a reason, the air temperature around the bucket.

Given that $T = 30$ when $t = 25$,

c find the value of k,

d find the rate at which the surface temperature of the water is decreasing when $t = 40$.

11

$$f(x) \equiv x^2 - 7x + 4 \ln \left(\frac{x}{2}\right), \quad x > 0.$$

a Solve the equation $f'(x) = 0$, giving your answers correct to 2 decimal places.

b Find an equation for the tangent to the curve $y = f(x)$ at the point on the curve where $x = 2$.

12 A curve has the equation $y = x^2 - \dfrac{8}{x-1}$.

a Show that the x-coordinate of any stationary point of the curve satisfies the equation

$$x^3 - 2x^2 + x + 4 = 0.$$

b Hence, show that the curve has exactly one stationary point and find its coordinates.

c Determine the nature of this stationary point.