

$$1 \quad \mathbf{a} = \frac{3x-1}{6(3x-1)} = \frac{1}{6} \quad \mathbf{b} = \frac{3(2x+5)}{4(2x+5)} = \frac{3}{4} \quad \mathbf{c} = \frac{3(y+1)}{(y+6)(y+1)} = \frac{3}{y+6} \quad \mathbf{d} = \frac{(x+5)(x-5)}{(x-2)(x-5)} = \frac{x+5}{x-2}$$

$$\mathbf{e} = \frac{(a+2)(a-3)}{(a+6)(a-3)} = \frac{a+2}{a+6} \quad \mathbf{f} = \frac{x(x+3)}{(2x-1)(x+3)} = \frac{x}{2x-1} \quad \mathbf{g} = \frac{(3t+1)(t-4)}{(t+4)(t-4)} = \frac{3t+1}{t+4} \quad \mathbf{h} = \frac{(3x-2)(2x-3)}{(4x+3)(3x-2)} = \frac{2x-3}{4x+3}$$

$$2 \quad \mathbf{a} = \frac{3x^2}{9(x-1)} \times \frac{4(x-1)}{2x} = \frac{2x}{3} \quad \mathbf{b} = \frac{(x+6)(x-6)}{(x+2)(x+5)} \times \frac{x+2}{x-6} = \frac{x+6}{x+5}$$

$$\mathbf{c} = \frac{n(n+2)}{(n+4)(n+2)} \times \frac{n+4}{n^2} = \frac{1}{n} \quad \mathbf{d} = \frac{4(x-3)}{(x+2)(x-2)} \times \frac{x(x+2)}{(x+1)(x-3)} = \frac{4x}{(x-2)(x+1)}$$

$$\mathbf{e} = \frac{4y^2}{y(2y+1)} \times \frac{(2y+1)(y+5)}{(y+5)(y-3)} = \frac{4y}{y-3} \quad \mathbf{f} = \frac{(x+1)(x-1)}{(2x-1)(x+4)} \times \frac{(3x-1)(2x-1)}{(3x-1)(x-1)} = \frac{x+1}{x+4}$$

$$\mathbf{g} = \frac{10(x-1)}{5(x+3)} \times \frac{(x+3)(x+4)}{(4+x)(1-x)} = -2 \quad \mathbf{h} = \frac{a^2(a-3)}{4a(2a-1)} \times \frac{(2a-1)(a+3)}{(a+3)(a-3)} = \frac{a}{4}$$

$$3 \quad \mathbf{a} = \frac{2(y+4)+7y}{y(y+4)} = \frac{9y+8}{y(y+4)} \quad \mathbf{b} = \frac{2x(x+3)-(x-5)}{(x-5)(x+3)} = \frac{2x^2+5x+5}{(x-5)(x+3)} \quad \mathbf{c} = \frac{7-3x^2}{x(x+2)}$$

$$\mathbf{d} = \frac{2x+5(x-3)}{2(x-3)(x-1)} = \frac{7x-15}{2(x-3)(x-1)} \quad \mathbf{e} = \frac{2}{q(q+3)} + \frac{5q}{4(q+3)} = \frac{8+5q^2}{4q(q+3)} \quad \mathbf{f} = \frac{4}{3(x-1)} + \frac{x+2}{x(x-1)} = \frac{4x+3(x+2)}{3x(x-1)} = \frac{7x+6}{3x(x-1)}$$

$$\mathbf{g} = \frac{4}{x+5} + \frac{x}{(x+3)(x+5)} = \frac{4(x+3)+x}{(x+3)(x+5)} = \frac{5x+12}{(x+3)(x+5)} \quad \mathbf{h} = \frac{6x}{(x+2)(x-2)} - \frac{3}{x+2} = \frac{6x-3(x-2)}{(x+2)(x-2)} = \frac{3x+6}{(x+2)(x-2)} = \frac{3(x+2)}{(x+2)(x-2)} = \frac{3}{x-2} \quad \mathbf{i} = \frac{5t+12}{(2t+1)(t+3)} - \frac{4}{2t+1} = \frac{5t+12-4(t+3)}{(2t+1)(t+3)} = \frac{t}{(2t+1)(t+3)}$$

$$4 \quad \mathbf{a} = \frac{x(x-5)}{6(x-5)} = \frac{x}{6} \quad \mathbf{b} = \frac{(4+x)(4-x)}{(x+4)(x-2)} = \frac{4-x}{x-2} \quad \mathbf{c} = \frac{2(x-3)(x+1)}{3(x-1)(x-3)} = \frac{2(x+1)}{3(x-1)} \quad \mathbf{d} = \frac{x(x+1)(x-1)}{(2x+1)(x-1)} = \frac{x(x+1)}{2x+1}$$

$$\mathbf{e} = \frac{x(3-x)}{2(x+3)(x-3)} = -\frac{x}{2(x+3)} \quad \mathbf{f} = \frac{x(x+2)(x-1)}{(3x-2)(x+2)} = \frac{x(x-1)}{3x-2} \quad \mathbf{g} = \frac{(2-x)(1+3x)}{(2x+5)(x-2)} = -\frac{3x+1}{2x+5} \quad \mathbf{h} = \frac{(x^2-1)(x^2-4)}{(x+1)(x-2)} = \frac{(x+1)(x-1)(x+2)(x-2)}{(x+1)(x-2)} = (x-1)(x+2)$$

$$5 \quad \mathbf{a} = \frac{10(x+1)(x-1)}{5(x+2)} \times \frac{(x+2)(x+4)}{(x+1)(x+4)} = 2(x-1) \quad \mathbf{b} = \frac{t(t-2)}{(2t+3)(t-2)} \times \frac{(3t+2)(2t+3)}{(3t+2)(3t-2)} = \frac{t}{3t-2}$$

$$\mathbf{c} = \frac{2(x+1)(x+5)}{(4x-3)(x-1)} \times \frac{x(4x-3)}{4x(x+5)} = \frac{x+1}{2(x-1)} \quad \mathbf{d} = \frac{(4x-3)(2x+3)}{(2x+3)^2} \times \frac{x(2x+3)}{2(3-4x)} = -\frac{x}{2}$$

$$\mathbf{e} = \frac{(x^2+1)(x^2+5)}{(x+3)(x-3)} \times \frac{2x(x-3)}{4(x^2+1)} = \frac{x(x^2+5)}{2(x+3)} \quad \mathbf{f} = \frac{(y^2+4)(y+2)(y-2)}{(5y-1)(y+2)} \times \frac{(5y-1)^2}{y^2+4} = (y-2)(5y-1)$$

$$6 \quad \mathbf{a} = \frac{5}{(x+1)(x-1)} - \frac{1}{2(x+1)}$$

$$= \frac{10 - (x-1)}{2(x+1)(x-1)}$$

$$= \frac{11-x}{2(x+1)(x-1)} \quad \mathbf{b} = \frac{3x}{(x+2)(x-2)} - \frac{4}{(2x-1)(x+2)}$$

$$= \frac{3x(2x-1) - 4(x-2)}{(x+2)(x-2)(2x-1)}$$

$$= \frac{6x^2 - 7x + 8}{(x+2)(x-2)(2x-1)}$$

$$\mathbf{c} = \frac{4}{(x+3)(x-1)} + \frac{1}{(x-1)(x-2)}$$

$$= \frac{4(x-2) + (x+3)}{(x+3)(x-1)(x-2)}$$

$$= \frac{5x-5}{(x+3)(x-1)(x-2)}$$

$$= \frac{5}{(x+3)(x-2)} \quad \mathbf{d} = \frac{x+1}{(x+5)(x-5)} + \frac{2}{x(x+5)}$$

$$= \frac{x(x+1) + 2(x-5)}{x(x+5)(x-5)}$$

$$= \frac{x^2 + 3x - 10}{x(x+5)(x-5)}$$

$$= \frac{(x+5)(x-2)}{x(x+5)(x-5)}$$

$$= \frac{x-2}{x(x-5)}$$

$$\mathbf{e} = \frac{2x-1}{(x+2)^2} + \frac{x}{3(x+2)}$$

$$= \frac{3(2x-1) + x(x+2)}{3(x+2)^2}$$

$$= \frac{x^2 + 8x - 3}{3(x+2)^2} \quad \mathbf{f} = \frac{1}{x-3} + \frac{3}{x(x-3)} + \frac{x}{(x-3)^2}$$

$$= \frac{x(x-3) + 3(x-3) + x^2}{x(x-3)^2}$$

$$= \frac{2x^2 - 9}{x(x-3)^2}$$

$$\mathbf{g} = \frac{x}{(x+2)(x-3)} + \frac{2}{(2x-1)(x+2)}$$

$$= \frac{x(2x-1) + 2(x-3)}{(x+2)(x-3)(2x-1)}$$

$$= \frac{2x^2 + x - 6}{(x+2)(x-3)(2x-1)}$$

$$= \frac{(2x-3)(x+2)}{(x+2)(x-3)(2x-1)}$$

$$= \frac{2x-3}{(x-3)(2x-1)} \quad \mathbf{h} = \frac{1}{x^2} - \frac{1}{x(3x-2)} + \frac{3}{2(3x-2)}$$

$$= \frac{2(3x-2) - 2x + 3x^2}{2x^2(3x-2)}$$

$$= \frac{3x^2 + 4x - 4}{2x^2(3x-2)}$$

$$= \frac{(3x-2)(x+2)}{2x^2(3x-2)}$$

$$= \frac{x+2}{2x^2}$$

7 a  $x(2x - 5) - 2(2x - 5) = 3x$

$$x^2 - 6x + 5 = 0$$

$$(x - 1)(x - 5) = 0$$

$$x = 1, 5$$

c  $\frac{20}{(2x+1)(x+2)} + 1 = \frac{10}{2x+1}$

$$20 + 2x^2 + 5x + 2 = 10(x + 2)$$

$$2x^2 - 5x + 2 = 0$$

$$(2x - 1)(x - 2) = 0$$

$$x = \frac{1}{2}, 2$$

e  $5 + \frac{1}{(x+3)(x+2)} = \frac{11}{x+3}$

$$5(x^2 + 5x + 6) + 1 = 11(x + 2)$$

$$5x^2 + 14x + 9 = 0$$

$$(5x + 9)(x + 1) = 0$$

$$x = -\frac{9}{5}, -1$$

8  $f(x) = \frac{7x-15}{x(x-5)} - \frac{4}{x-5}$   
 $= \frac{7x-15-4x}{x(x-5)} = \frac{3x-15}{x(x-5)}$   
 $= \frac{3(x-5)}{x(x-5)} = \frac{3}{x} \quad [k = 3]$

10  $f(x) - \frac{1}{f(x)} = \frac{x+2}{x-2} - \frac{x-2}{x+2}$   
 $= \frac{(x+2)^2 - (x-2)^2}{(x-2)(x+2)}$   
 $= \frac{x^2 + 4x + 4 - (x^2 - 4x + 4)}{x^2 - 4}$   
 $= \frac{8x}{x^2 - 4}$

12  $\frac{3}{2(2x+1)} - \frac{5}{(2x+1)^2} = 2$   
 $3(2x + 1) - 10 = 4(4x^2 + 4x + 1)$

$$16x^2 + 10x + 11 = 0$$

$$b^2 - 4ac = 100 - 704 = -604$$

$$b^2 - 4ac < 0 \therefore \text{no real roots}$$

b  $\frac{2}{(x+1)(x-1)} + \frac{3}{x+1} = 1$

$$2 + 3(x - 1) = x^2 - 1$$

$$x^2 - 3x = 0$$

$$x(x - 3) = 0$$

$$x = 0, 3$$

d  $2y(y + 3) - y(y + 5) = 2(y + 5)(2y - 1)$

$$2y^2 + 6y - y^2 - 5y = 4y^2 + 18y - 10$$

$$3y^2 + 17y - 10 = 0$$

$$y = \frac{-17 \pm \sqrt{289 + 120}}{6} = \frac{-17 \pm \sqrt{409}}{6}$$

$$y = -6.20, 0.537 \text{ (3sf)}$$

f  $\frac{3}{(1-2x)^2} - \frac{10}{(1+2x)(1-2x)} = \frac{5}{1+2x}$

$$3(1 + 2x) - 10(1 - 2x) = 5(1 - 4x + 4x^2)$$

$$10x^2 - 23x + 6 = 0$$

$$(10x - 3)(x - 2) = 0$$

$$x = \frac{3}{10}, 2$$

9  $f(x) = \frac{x-5}{(3x-1)(x+2)} + \frac{2}{3x-1}$   
 $= \frac{x-5+2(x+2)}{(3x-1)(x+2)} = \frac{3x-1}{(3x-1)(x+2)}$   
 $= \frac{1}{x+2}$

11 a  $= \frac{2(x+2)+3}{(x+2)(x+5)} = \frac{2x+7}{(x+2)(x+5)}$

b  $\frac{2x+7}{(x+2)(x+5)} = \frac{1}{3}$

$$3(2x + 7) = (x + 2)(x + 5)$$

$$x^2 + x - 11 = 0$$

$$x = \frac{-1 \pm \sqrt{1+44}}{2} = \frac{-1 \pm \sqrt{45}}{2} = -3.85, 2.85$$

13  $= \frac{6x - (x+5)}{x(x+5)} \div \frac{x-1}{(x+5)(x-5)}$

$$= \frac{5x-5}{x(x+5)} \times \frac{(x+5)(x-5)}{x-1}$$

$$= \frac{5(x-1)}{x(x+5)} \times \frac{(x+5)(x-5)}{x-1}$$

$$= \frac{5(x-5)}{x}$$