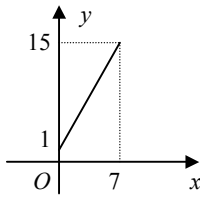


1    a 4            b 2            c 11            d -2            e -4            f -2  
       g  $\frac{2}{5}$           h -3            i  $\frac{5}{4}$             j -8            k -4            l  $\frac{12}{13}$

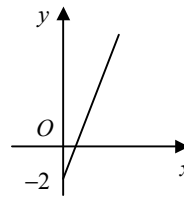
2    a =  $\sin \pi$       b =  $\ln 2$       c = 5            d =  $\sin \frac{2\pi}{3}$     e =  $3 + 2e^{-1}$     f =  $\ln \frac{9}{2}$   
       = 0            = 0.693                            =  $\frac{\sqrt{3}}{2}$  or 0.866    = 3.74            = 1.50  
       g =  $3 + 2e^{1.8}$     h =  $\ln 1$       i =  $\sin(0.6 + \frac{\pi}{3})$     j =  $3 + 2e^{\frac{1}{3}}$     k =  $\sin(\frac{\pi}{3} - 2)$     l =  $\ln \frac{23}{4}$   
       = 15.1            = 0            = 0.997            = 5.79            = -0.815            = 1.75

3 a



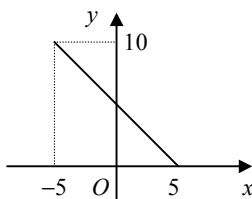
range:  
 $1 \leq f(x) \leq 15$

b



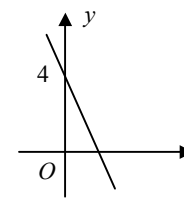
range:  
 $f(x) \geq -2$

c



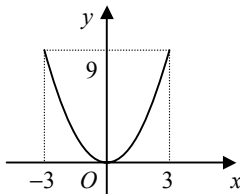
range:  
 $0 \leq f(x) \leq 10$

d



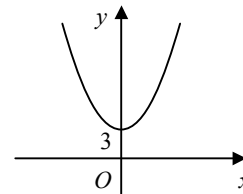
range:  
 $f(x) \in \mathbb{R}$

e



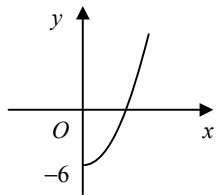
range:  
 $0 \leq f(x) < 9$

f



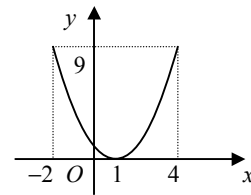
range:  
 $f(x) \geq 3$

g



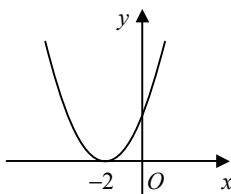
range:  
 $f(x) \geq -6$

h



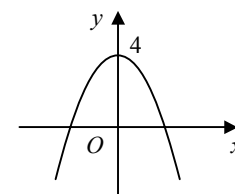
range:  
 $0 \leq f(x) \leq 9$

i



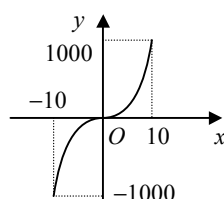
range:  
 $f(x) \geq 0$

j



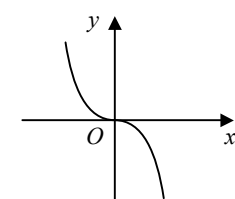
range:  
 $f(x) \leq 4$

k



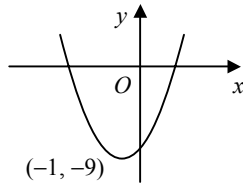
range:  
 $-1000 < f(x) \leq 1000$

l



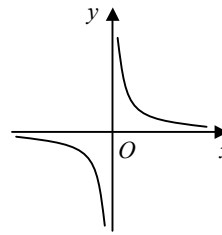
range:  
 $f(x) \in \mathbb{R}$

4 a  $f(x) = (x + 1)^2 - 9 \therefore (-1, -9)$



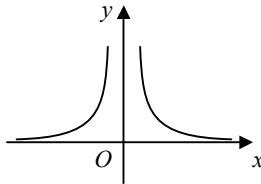
range:  
 $f(x) \geq -9$

b



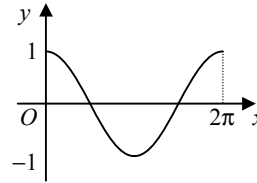
range:  
 $f(x) \in \mathbb{R}, f(x) \neq 0$

c



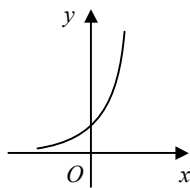
range:  
 $f(x) > 0$

d



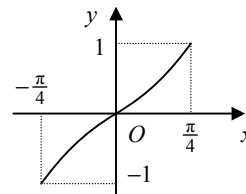
range:  
 $-1 \leq f(x) \leq 1$

e



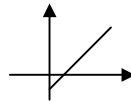
range:  
 $f(x) > 0$

f



range:  
 $-1 \leq f(x) \leq 1$

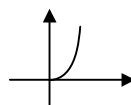
5 a  $f(0) = -1, f(7) = 6$   
 $\therefore 0 \leq x < 7$



b  $f(0) = 4$   
 $\therefore x \geq 0$



c  $f(0) = 0, f(5) = 125$   
 $\therefore 0 \leq x \leq 5$



d  $f(\frac{1}{2}) = 2, f(\frac{1}{10}) = 10$   
 $\therefore \frac{1}{10} < x < \frac{1}{2}$



6 a  $4x + 3 = 9$   
 $x = \frac{3}{2}$

b  $x^2 - 7 = 18$   
 $x^2 = 25$   
 $x = \pm 5$

c  $\frac{9}{x+2} = 6$   
 $6x + 12 = 9$   
 $x = -\frac{1}{2}$

d  $4x + 3 = \frac{9}{x+2}$   
 $(4x + 3)(x + 2) = 9$   
 $4x^2 + 11x - 3 = 0$   
 $(4x - 1)(x + 3) = 0$   
 $x = -3, \frac{1}{4}$

e  $x^2 - 7 - \frac{x+2}{9} = -\frac{19}{3}$   
 $9x^2 - 63 - x - 2 = -57$   
 $9x^2 - x - 8 = 0$   
 $(9x + 8)(x - 1) = 0$   
 $x = -\frac{8}{9}, 1$

f  $4x + 3 + x^2 - 7 = 0$   
 $x^2 + 4x - 4 = 0$   
 $x = \frac{-4 \pm \sqrt{16+16}}{2}$   
 $x = -2 \pm 2\sqrt{2}$   
or  $-4.83, 0.828$  (3sf)

7 a  $f(x) = (x + 2)^2 - 4 + 11 = (x + 2)^2 + 7$  range:  $f(x) \geq 7$

b  $f(x) = (x - 1)^2 - 1 - 6 = (x - 1)^2 - 7$  range:  $f(x) \geq -7$

c  $f(x) = (2x + 3)^2 - 9 + 3 = (2x + 3)^2 - 6$  range:  $f(x) \geq -6$

d  $f(x) = (3x - 1)^2 - 1 + 16 = (3x - 1)^2 + 15$  range:  $f(x) \geq 15$

e  $f(x) = 15 - [x^2 + 4x] = 15 - [(x + 2)^2 - 4] = 19 - (x + 2)^2$  range:  $f(x) \leq 19$



<b>6</b>	<b>a</b> $gh(x) = g(e^x)$ $= 3 + 2e^x$ $3 + 2e^x = 9$ $e^x = 3$ $x = \ln 3$ $x = 1.10$	<b>b</b> $fg(x) = f(3 + 2x)$ $= \ln(3 + 2x)$ $\ln(3 + 2x) = 3.6$ $3 + 2x = e^{3.6}$ $x = \frac{1}{2}(e^{3.6} - 3)$ $x = 16.80$	<b>c</b> $hg(x) = h(3 + 2x)$ $= e^{3+2x}$ $e^{3+2x} = 4$ $3 + 2x = \ln 4$ $x = \frac{1}{2}(\ln 4 - 3)$ $x = -0.81$	<b>d</b> $gf(x) = g(\ln x)$ $= 3 + 2 \ln x$ $3 + 2 \ln x = 10.4$ $\ln x = 3.7$ $x = e^{3.7}$ $x = 40.45$
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**7** **a**  $g(x) > 0$

**b**  $fg(x) = f(e^x) = \frac{e^x + 1}{5}$

$$\frac{e^x + 1}{5} = 17$$

$$e^x + 1 = 85$$

$$e^x = 84$$

$$x = \ln 84 = 4.43 \text{ (3sf)}$$

**8** **a**  $= f(4) = 7$

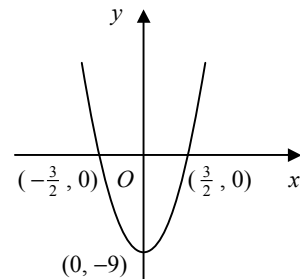
**b**  $gf(x) = g(4x - 9) = (4x - 9)^2$

$$\therefore (4x - 9)^2 = 25$$

$$4x - 9 = \pm 5$$

$$x = \frac{1}{4}(9 \pm 5) = 1, \frac{7}{2}$$

**c**  $fg(x) = f(x^2) = 4x^2 - 9$



**9** **a**  $= g(1) = 4$

**b**  $= h(2) = e^3$

**c**  $= g(e^{-3}) = 1$

**d**  $= f(\tan 1) = 74.7 \text{ (3sf)}$

**e**  $= h(\tan 0.2)$   
 $= 0.552 \text{ (3sf)}$

**f**  $= f(4 + \ln 7)$   
 $= -0.351 \text{ (3sf)}$

**g**  $= h(e^{-\frac{1}{2}})$   
 $= 1.24 \text{ (3sf)}$

**h**  $= f(4 + e)$   
 $= 0.465 \text{ (3sf)}$

**10** **a**  $= f(4x + 1)$   
 $= 3e^{4x+1} + 2$

**b**  $= g(3e^x + 2)$   
 $= 4(3e^x + 2) + 1$

**c**  $= h(3e^x + 2)$   
 $= \frac{1}{3e^x + 2 + 1}$

**d**  $= g(4x + 1)$   
 $= 4(4x + 1) + 1$

$fg : x \rightarrow 3e^{4x+1} + 2,$   
 $x \in \mathbb{R}$

$gf : x \rightarrow 12e^x + 9,$   
 $x \in \mathbb{R}$

$hf : x \rightarrow \frac{1}{3e^x + 3},$   
 $x \in \mathbb{R}$

$gg : x \rightarrow 16x + 5,$   
 $x \in \mathbb{R}$

**e**  $= h(4x + 1)$   
 $= \frac{1}{4x+1+1}$

**f**  $= g\left(\frac{1}{x+1}\right)$   
 $= \frac{4}{x+1} + 1$

**g**  $= h\left(\frac{1}{x+1}\right)$   
 $= \frac{1}{\frac{1}{x+1} + 1}$

**h**  $= g(16x + 5)$   
 $= 4(16x + 5) + 1$

$hg : x \rightarrow \frac{1}{4x+2},$   
 $x \in \mathbb{R}, x \neq -\frac{1}{2}$

$\frac{4+x+1}{x+1}$   
 $gh : x \rightarrow \frac{x+5}{x+1},$   
 $x \in \mathbb{R}, x \neq -1$

$\frac{x+1}{1+x+1}$   
 $hh : x \rightarrow \frac{x+1}{x+2},$   
 $x \in \mathbb{R}, x \neq -1, -2$

$ggg : x \rightarrow 64x + 21,$   
 $x \in \mathbb{R}$

11 a  $fh(x) = f\left(\frac{x+1}{3}\right)$       b  $fg(x) = f(e^{1+2x})$       c  $gh(x) = g\left(\frac{x+1}{3}\right)$       d  $hh(x) = h\left(\frac{x+1}{3}\right)$

$$= \sqrt{\frac{x+1}{3} + 4}$$

$$= \sqrt{\frac{x+13}{3}}$$

$$\sqrt{\frac{x+13}{3}} = 3$$

$$\frac{x+13}{3} = 9$$

$$x + 13 = 27$$

$$x = 14$$

$$= \sqrt{e^{1+2x} + 4}$$

$$\sqrt{e^{1+2x} + 4} = 7$$

$$e^{1+2x} + 4 = 49$$

$$e^{1+2x} = 45$$

$$1 + 2x = \ln 45$$

$$x = \frac{1}{2}(\ln 45 - 1)$$

$$x = 1.40 \text{ (3sf)}$$

$$= e^{1 + \frac{2(x+1)}{3}}$$

$$= e^{\frac{2x+5}{3}}$$

$$e^{\frac{2x+5}{3}} = 11$$

$$\frac{2x+5}{3} = \ln 11$$

$$2x + 5 = 3 \ln 11$$

$$x = \frac{1}{2}(3 \ln 11 - 5)$$

$$x = 1.10 \text{ (3sf)}$$

$$= \frac{\frac{x+1}{3} + 1}{3}$$

$$= \frac{x+1+3}{9}$$

$$= \frac{x+4}{9}$$

$$\frac{x+4}{9} = \frac{2}{3}$$

$$3x + 12 = 18$$

$$x = 2$$

e  $hg(x) = h(e^{1+2x})$       f  $hf(x) = h(\sqrt{x+4})$       g  $ff(x) = f(\sqrt{x+4})$       h  $ghh(x) = g\left(\frac{x+4}{9}\right)$

$$= \frac{e^{1+2x} + 1}{3}$$

$$\frac{e^{1+2x} + 1}{3} = 1.2$$

$$e^{1+2x} = 2.6$$

$$1 + 2x = \ln 2.6$$

$$x = \frac{1}{2}(\ln 2.6 - 1)$$

$$x = -0.0222 \text{ (3sf)}$$

$$= \frac{\sqrt{x+4} + 1}{3}$$

$$\frac{\sqrt{x+4} + 1}{3} = \frac{1}{2}$$

$$\sqrt{x+4} = \frac{1}{2}$$

$$x + 4 = \frac{1}{4}$$

$$x = -3\frac{3}{4}$$

$$= \sqrt{\sqrt{x+4} + 4}$$

$$\sqrt{\sqrt{x+4} + 4} = 3$$

$$\sqrt{x+4} = 5$$

$$x + 4 = 25$$

$$x = 21$$

$$= e^{1 + \frac{2(x+4)}{9}}$$

$$= e^{\frac{2x+17}{9}}$$

$$e^{\frac{2x+17}{9}} = \frac{1}{2}$$

$$\frac{2x+17}{9} = \ln \frac{1}{2}$$

$$x = \frac{1}{2}(9 \ln \frac{1}{2} - 17)$$

$$x = -11.6 \text{ (3sf)}$$

12 a  $h(x) = fg(x)$

b  $h(x) = gf(x)$

c  $h(x) = gg(x)$

d  $h(x) = ff(x)$

e  $h(x) = gff(x)$

f  $h(x) = gfg(x)$

13 a  $j(x) = fg(x)$

b  $j(x) = hf(x)$

c  $j(x) = gh(x)$

d  $j(x) = gg(x)$

e  $j(x) = fhg(x)$

f  $j(x) = hfg(x)$

14 a  $gf(x) = g(5^x - 7)$   
 $= 2(5^x - 7) + 3$   
 $= 2(5^x) - 11$   
 $gf : x \rightarrow 2(5^x) - 11, x \in \mathbb{R}$

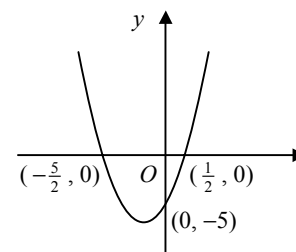
b  $2(5^x) - 11 = 10$

$$5^x = \frac{21}{2}$$

$$x = \frac{\ln \frac{21}{2}}{\ln 5} = 1.46 \text{ (3sf)}$$

15 a  $gf(x) = g[2(x+1)] = [2(x+1)]^2 - 9$   
 $gf : x \rightarrow 4x^2 + 8x - 5, x \in \mathbb{R}$   
 range:  $gf(x) \geq -9$

b  $gf(x) = (2x+5)(2x-1)$



c  $gf(x) - 2f(x) = a$

$$4x^2 + 8x - 5 - 2[2(x+1)] = a$$

$$4x^2 + 4x - (a+9) = 0$$

$$\text{no real roots } \therefore b^2 - 4ac < 0$$

$$16 + 16(a+9) < 0$$

$$1 + a + 9 < 0$$

$$a < -10$$

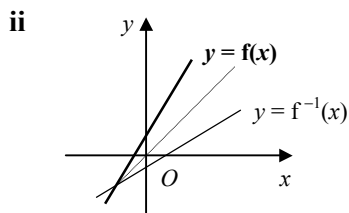
- 1**
- a**  $y = 10x + 3$   
 swap  $x = 10y + 3$   
 $y = \frac{x-3}{10}$   
 $f^{-1}(x) = \frac{x-3}{10}, x \in \mathbb{R}$
- b**  $y = 9 + 2x$   
 swap  $x = 9 + 2y$   
 $y = \frac{x-9}{2}$   
 $f^{-1}(x) = \frac{x-9}{2}, x \in \mathbb{R}$
- c**  $y = 5 - 6x$   
 swap  $x = 5 - 6y$   
 $y = \frac{5-x}{6}$   
 $f^{-1}(x) = \frac{5-x}{6}, x \in \mathbb{R}$
- d**  $y = \frac{x+3}{4}$   
 swap  $x = \frac{y+3}{4}$   
 $y = 4x - 3$   
 $f^{-1}(x) = 4x - 3, x \in \mathbb{R}$
- e**  $y = \frac{1}{3}(2x - 5)$   
 swap  $x = \frac{1}{3}(2y - 5)$   
 $y = \frac{3x+5}{2}$   
 $f^{-1}(x) = \frac{3x+5}{2}, x \in \mathbb{R}$
- f**  $y = 8 - \frac{3}{5}x$   
 swap  $x = 8 - \frac{3}{5}y$   
 $y = \frac{40-5x}{3}$   
 $f^{-1}(x) = \frac{40-5x}{3}, x \in \mathbb{R}$
- 2**
- a**  $y = \ln x$   
 swap  $x = \ln y$   
 $y = e^x$   
 $f^{-1}(x) = e^x, x \in \mathbb{R}$
- b**  $y = \frac{1}{x}$   
 swap  $x = \frac{1}{y}$   
 $y = \frac{1}{x}$   
 $f^{-1}(x) = \frac{1}{x}, x \in \mathbb{R}, x \neq 0$
- c**  $y = \sqrt[4]{x}$   
 swap  $x = \sqrt[4]{y}$   
 $y = x^4$   
 $f^{-1}(x) = x^4, x \in \mathbb{R}, x > 0$
- d**  $y = 3x - 4$   
 swap  $x = 3y - 4$   
 $y = \frac{x+4}{3}$   
 $f(0) = -4, f(3) = 5$   
 $f^{-1}(x) = \frac{x+4}{3}, x \in \mathbb{R}, -4 \leq x < 5$
- e**  $y = \frac{1}{x-5}$   
 swap  $x = \frac{1}{y-5}$   
 $y = \frac{1}{x} + 5$   
 $f^{-1}(x) = \frac{1}{x} + 5, x \in \mathbb{R}, x \neq 0$
- f**  $y = 2 + \frac{1}{x}$   
 swap  $x = 2 + \frac{1}{y}$   
 $y = \frac{1}{x-2}$   
 $f^{-1}(x) = \frac{1}{x-2}, x \in \mathbb{R}, x \neq 2$

3 a i  $y = 2x + 1$

swap  $x = 2y + 1$

$$y = \frac{x-1}{2}$$

$$f^{-1}: x \rightarrow \frac{x-1}{2}, x \in \mathbb{R}$$

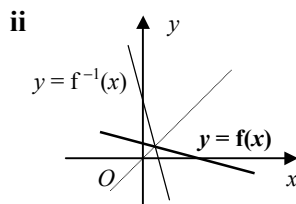


b i  $y = \frac{1-x}{5}$

swap  $x = \frac{1-y}{5}$

$$y = 1 - 5x$$

$$f^{-1}: x \rightarrow 1 - 5x, x \in \mathbb{R}$$

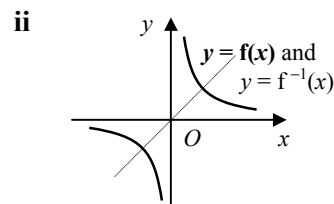


c i  $y = \frac{10}{x}$

swap  $x = \frac{10}{y}$

$$y = \frac{10}{x}$$

$$f^{-1}: x \rightarrow \frac{10}{x}, x \in \mathbb{R}, x \neq 0$$



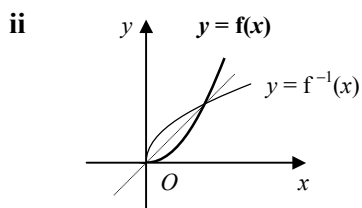
d i  $y = x^2$

swap  $x = y^2$

$$y = \pm\sqrt{x}$$

(domain of  $f \Rightarrow +$ )

$$f^{-1}: x \rightarrow \sqrt{x}, x \in \mathbb{R}, x > 0$$

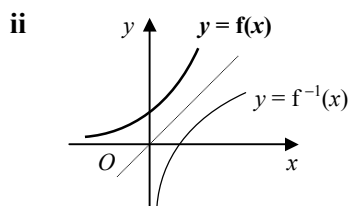


e i  $y = e^x$

swap  $x = e^y$

$$y = \ln x$$

$$f^{-1}: x \rightarrow \ln x, x \in \mathbb{R}, x > 0$$

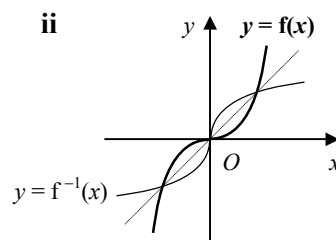


f i  $y = x^3$

swap  $x = y^3$

$$y = \sqrt[3]{x}$$

$$f^{-1}: x \rightarrow \sqrt[3]{x}, x \in \mathbb{R}$$



4 a  $y = 5x + 1$

swap  $x = 5y + 1$

$$f^{-1}(x) = y = \frac{x-1}{5}$$

$$\frac{x-1}{5} = 2$$

$$x - 1 = 10$$

$$x = 11$$

d  $y = \sqrt{x+2}$

swap  $x = \sqrt{y+2}$

$$f^{-1}(x) = y = x^2 - 2$$

$$x^2 - 2 = 3x - 4$$

$$x^2 - 3x + 2 = 0$$

$$(x-1)(x-2) = 0$$

$$x = 1, 2$$

b  $y = \frac{2x-4}{3}$

swap  $x = \frac{2y-4}{3}$

$$f^{-1}(x) = y = \frac{3x+4}{2}$$

$$\frac{3x+4}{2} = 7 - x$$

$$3x + 4 = 14 - 2x$$

$$x = 2$$

e  $y = \frac{4}{x+3}$

swap  $x = \frac{4}{y+3}$

$$f^{-1}(x) = y = \frac{4}{x} - 3$$

$$\frac{4}{x} - 3 = 5(x+1)$$

$$4 - 3x = 5x(x+1)$$

$$5x^2 + 8x - 4 = 0$$

$$(5x-2)(x+2) = 0$$

$$x = -2, \frac{2}{5}$$

c  $y = e^x + 2$

swap  $x = e^y + 2$

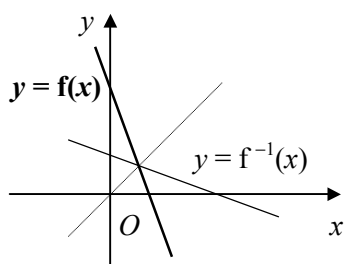
$$f^{-1}(x) = y = \ln(x-2)$$

$$\ln(x-2) = \ln(3x-8)$$

$$x-2 = 3x-8$$

$$x = 3$$

5 a



b  $4 - 2x = x$

$x = \frac{4}{3}$

$\therefore \left(\frac{4}{3}, \frac{4}{3}\right)$

6 a  $g \Rightarrow y = \frac{1}{2x+4}$

swap  $x = \frac{1}{2y+4}$

$y = \frac{1}{2} \left( \frac{1}{x} - 4 \right) = \frac{1}{2x} - 2$

$g^{-1}(x) = \frac{1}{2x} - 2, x \in \mathbb{R}, x \neq 0$

range:  $g^{-1}(x) \in \mathbb{R}, g^{-1}(x) \neq -2$

b  $= g(3 - 2x)$

$= \frac{1}{2(3-2x)+4} = \frac{1}{10-4x}$

$gf(x) = \frac{1}{10-4x}, x \in \mathbb{R}, x \neq \frac{5}{2}$

c  $f \Rightarrow y = 3 - 2x$

swap  $x = 3 - 2y$

$f^{-1}(x) = y = \frac{3-x}{2}$

$\therefore \frac{1}{10-4x} = \frac{3-x}{2}$

$2 = (3-x)(10-4x)$

$2x^2 - 11x + 14 = 0$

$(2x-7)(x-2) = 0$

$x = 2, \frac{7}{2}$

7 a i  $y = 5x + 2$

swap  $x = 5y + 2$

$y = \frac{x-2}{5}$

$f^{-1}: x \rightarrow \frac{x-2}{5}, x \in \mathbb{R}$

ii  $= f\left(\frac{1}{x}\right)$

$= \frac{5}{x} + 2$

$fg: x \rightarrow \frac{5}{x} + 2, x \in \mathbb{R}, x \neq 0$

iii  $y = \frac{5}{x} + 2$

swap  $x = \frac{5}{y} + 2$

$y = \frac{5}{x-2}$

$(fg)^{-1}: x \rightarrow \frac{5}{x-2}, x \in \mathbb{R}, x \neq 2$

b  $\frac{x-2}{5} = \frac{5}{x} + 2$

$x(x-2) = 25 + 10x$

$x^2 - 12x - 25 = 0$

$x = \frac{12 \pm \sqrt{144+100}}{2} = 6 \pm \sqrt{61} = -1.81, 13.81$



8 a  $y = \frac{1}{2} \ln(4x - 9)$

swap  $x = \frac{1}{2} \ln(4y - 9)$

$$4y - 9 = e^{2x}$$

$$y = \frac{1}{4}(e^{2x} + 9)$$

$$f^{-1}: x \rightarrow \frac{1}{4}(e^{2x} + 9), x \in \mathbb{R}$$

b  $y = \frac{x-2}{x+5}$

swap  $x = \frac{y-2}{y+5}$

$$xy + 5x = y - 2$$

$$y(1-x) = 5x + 2$$

$$y = \frac{5x+2}{1-x}$$

$$f^{-1}: x \rightarrow \frac{5x+2}{1-x}, x \in \mathbb{R}, x \neq 1$$

c  $y = e^{0.4x-2}$

swap  $x = e^{0.4y-2}$

$$0.4y - 2 = \ln x$$

$$y = \frac{5}{2}(2 + \ln x)$$

$$f^{-1}: x \rightarrow 5 + \frac{5}{2} \ln x, x \in \mathbb{R}, x > 0$$

d  $y = \sqrt[3]{x^5 - 3}$

swap  $x = \sqrt[3]{y^5 - 3}$

$$y^5 - 3 = x^3$$

$$y = \sqrt[5]{x^3 + 3}$$

$$f^{-1}: x \rightarrow \sqrt[5]{x^3 + 3}, x \in \mathbb{R}$$

e  $y = \log_{10}(2 - 7x)$

swap  $x = \log_{10}(2 - 7y)$

$$2 - 7y = 10^x$$

$$y = \frac{1}{7}(2 - 10^x)$$

$$f^{-1}: x \rightarrow \frac{1}{7}(2 - 10^x), x \in \mathbb{R}$$

f  $y = \frac{4-x}{3x+2}$

swap  $x = \frac{4-y}{3y+2}$

$$3xy + 2x = 4 - y$$

$$y(3x+1) = 4 - 2x$$

$$y = \frac{4-2x}{3x+1}$$

$$f^{-1}: x \rightarrow \frac{4-2x}{3x+1}, x \in \mathbb{R}, x \neq -\frac{1}{3}$$

9 a i  $y = e^{2x}$

swap  $x = e^{2y}$

$$2y = \ln x$$

$$y = \frac{1}{2} \ln x$$

$$f^{-1}: x \rightarrow \frac{1}{2} \ln x, x \in \mathbb{R}, x > 0$$

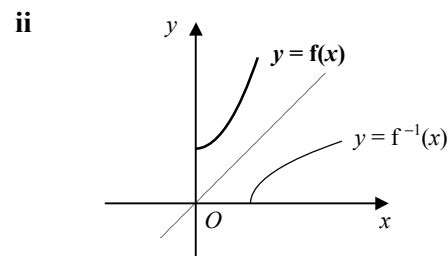
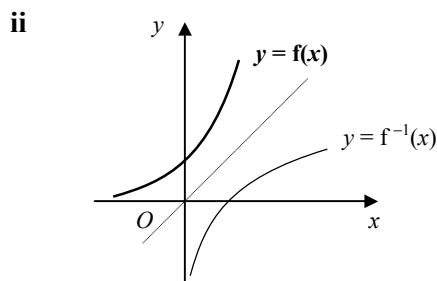
b i  $y = x^2 + 4$

swap  $x = y^2 + 4$

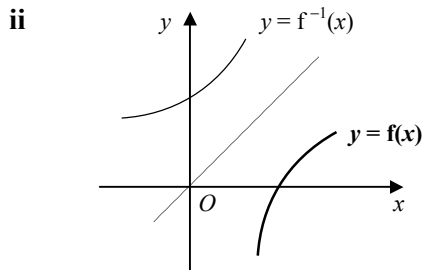
$$y = \pm \sqrt{x-4}$$

(domain of  $f \Rightarrow +$ )

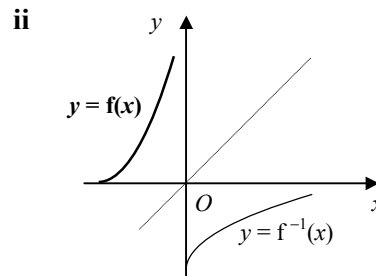
$$f^{-1}: x \rightarrow \sqrt{x-4}, x \in \mathbb{R}, x > 4$$



**c i**  $y = \ln(x - 3)$   
 swap  $x = \ln(y - 3)$   
 $y - 3 = e^x$   
 $y = e^x + 3$   
 $f^{-1} : x \rightarrow e^x + 3, x \in \mathbb{R}$



**d i**  $y = x^2 + 6x + 9 = (x + 3)^2$   
 swap  $x = (y + 3)^2$   
 $y = -3 \pm \sqrt{x}$   
 (domain of  $f \Rightarrow +$ )  
 $f^{-1} : x \rightarrow -3 + \sqrt{x}, x \in \mathbb{R}, x > 0$



**10 a i**  $f(x) = (x + 3)^2 - 6$   
 $x < -3 \therefore$  range:  $f(x) > -6$   
**ii**  $y = (x + 3)^2 - 6$   
 swap  $x = (y + 3)^2 - 6$   
 $y = -3 \pm \sqrt{x + 6}$   
 (domain of  $f \Rightarrow -$ )  
 $f^{-1}(x) = -3 - \sqrt{x + 6}, x \in \mathbb{R}, x > -6$

**b i**  $f(x) = (x - 2)^2 + 1$   
 $x \geq 2 \therefore$  range:  $f(x) \geq 1$   
**ii**  $y = (x - 2)^2 + 1$   
 swap  $x = (y - 2)^2 + 1$   
 $y = 2 \pm \sqrt{x - 1}$   
 (domain of  $f \Rightarrow +$ )  
 $f^{-1}(x) = 2 + \sqrt{x - 1}, x \in \mathbb{R}, x \geq 1$

**c i**  $f(x) = (x + \frac{5}{2})^2 - \frac{33}{4}$   
 $x < -\frac{5}{2} \therefore$  range:  $f(x) > -8\frac{1}{4}$   
**ii**  $y = (x + \frac{5}{2})^2 - \frac{33}{4}$   
 swap  $x = (y + \frac{5}{2})^2 - \frac{33}{4}$   
 $y = -\frac{5}{2} \pm \sqrt{x + \frac{33}{4}}$   
 (domain of  $f \Rightarrow -$ )  
 $f^{-1}(x) = -\frac{5}{2} - \sqrt{x + \frac{33}{4}}, x \in \mathbb{R}, x > -8\frac{1}{4}$

**d i**  $f(x) = (x - \frac{3}{2})^2 + \frac{11}{4}$   
 $2 < x < 4, f(2) = 3, f(4) = 9$   
 $\therefore$  range:  $3 < f(x) < 9$   
**ii**  $y = (x - \frac{3}{2})^2 + \frac{11}{4}$   
 swap  $x = (y - \frac{3}{2})^2 + \frac{11}{4}$   
 $y = \frac{3}{2} \pm \sqrt{x - \frac{11}{4}}$   
 (domain of  $f \Rightarrow +$ )  
 $f^{-1}(x) = \frac{3}{2} + \sqrt{x - \frac{11}{4}}, x \in \mathbb{R}, 3 < x < 9$

**e i**  $f(x) = 8 - 2x - x^2 = 9 - (x + 1)^2$   
 $x \geq -1 \therefore$  range:  $f(x) \leq 9$   
**ii**  $y = 9 - (x + 1)^2$   
 swap  $x = 9 - (y + 1)^2$   
 $y = -1 \pm \sqrt{9 - x}$   
 (domain of  $f \Rightarrow +$ )  
 $f^{-1}(x) = -1 + \sqrt{9 - x}, x \in \mathbb{R}, x \leq 9$

**f i**  $f(x) = -5(x^2 - 4x) = 20 - 5(x - 2)^2$   
 $x > 2 \therefore$  range:  $f(x) < 20$   
**ii**  $y = 20 - 5(x - 2)^2$   
 swap  $x = 20 - 5(y - 2)^2$   
 $y = 2 \pm \sqrt{\frac{20 - x}{5}}$   
 (domain of  $f \Rightarrow +$ )  
 $f^{-1}(x) = 2 + \sqrt{\frac{20 - x}{5}}, x \in \mathbb{R}, x < 20$

11 a  $y = \frac{1}{3}(2x - 5)$

swap  $x = \frac{1}{3}(2y - 5)$

$$f^{-1}(x) = y = \frac{3x+5}{2}$$

$$\therefore \frac{3x+5}{2} = \frac{4}{2-x}$$

$$(3x+5)(2-x) = 8$$

$$3x^2 - x - 2 = 0$$

$$(3x+2)(x-1) = 0$$

$$x = -\frac{2}{3}, 1$$

b  $y = \ln \frac{x+3}{5}$

swap  $x = \ln \frac{y+3}{5}$

$$f^{-1}(x) = y = 5e^x - 3$$

$$\therefore 5e^x - 3 = 10 - 6e^{-x}$$

$$5e^{2x} - 13e^x + 6 = 0$$

$$(5e^x - 3)(e^x - 2) = 0$$

$$e^x = \frac{3}{5}, 2$$

$$x = \ln \frac{3}{5}, \ln 2$$

$$x = -0.511, 0.693 \text{ (3sf)}$$

c  $y = x^2 - 4$

swap  $x = y^2 - 4$

$$y = \pm\sqrt{x+4}$$

(domain of  $f \Rightarrow +$ )

$$f^{-1}(x) = \sqrt{x+4}$$

$$\therefore \sqrt{x+4} = \frac{x+6}{3}$$

$$x+4 = \frac{(x+6)^2}{9}$$

$$9(x+4) = x^2 + 12x + 36$$

$$x^2 + 3x = 0$$

$$x(x+3) = 0$$

$$x = -3, 0$$

12 a  $-2$

b  $\frac{6+b}{6-2} = 4$

$$6+b = 16$$

$$b = 10$$

c  $y = \frac{x+10}{x-2}$

swap  $x = \frac{y+10}{y-2}$

$$xy - 2x = y + 10$$

$$y(x-1) = 2x+10$$

$$y = \frac{2x+10}{x-1}$$

$$f^{-1}(x) = \frac{2x+10}{x-1}, x \in \mathbb{R}, x \neq 1$$

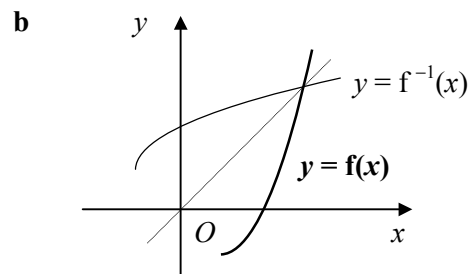
13 a  $y = x^2 - 3x = (x - \frac{3}{2})^2 - \frac{9}{4}$

swap  $x = (y - \frac{3}{2})^2 - \frac{9}{4}$

$$y = \frac{3}{2} \pm \sqrt{x + \frac{9}{4}}$$

(domain of  $f \Rightarrow +$ )

$$f^{-1}: x \rightarrow \frac{3}{2} + \sqrt{x + \frac{9}{4}}, x \in \mathbb{R}, x \geq -\frac{9}{4}$$



c  $g \Rightarrow y = 2x + 3$

swap  $x = 2y + 3$

$$g^{-1}(x) = y = \frac{x-3}{2}$$

$$g^{-1}(12) = \frac{9}{2}$$

$$f^{-1}g^{-1}(12) = f^{-1}\left(\frac{9}{2}\right)$$

$$= \frac{3}{2} + \sqrt{\frac{27}{4}}$$

$$= \frac{3}{2} + \frac{3}{2}\sqrt{3}$$

$$= \frac{3}{2}(1 + \sqrt{3})$$

$$\therefore a = 1\frac{1}{2}$$