1 **a** e^x **b** $3e^x$ **c** $\frac{1}{x}$ **d** $\frac{1}{2x}$

2 **a** $-2e^t$ **b** $6t + \frac{1}{t}$ **c** $e^t + 5t^4$ **d** $\frac{3}{2}t^{\frac{1}{2}} + 2e^t$

e $\frac{2}{t} + \frac{1}{2}t^{-\frac{1}{2}}$ or $\frac{2}{t} + \frac{1}{2\sqrt{t}}$ **f** $2.5e^t - \frac{7}{2t}$ **g** $-t^{-2} + \frac{8}{t}$ or $\frac{8}{t} - \frac{1}{t^2}$ **h** $14t - 2 + 4e^t$

3 **a** $\frac{dy}{dx} = 12x^2 + e^x$
$\frac{d^2y}{dx^2} = 24x + e^x$

b $\frac{dy}{dx} = 7e^x - 10x + 3$
$\frac{d^2y}{dx^2} = 7e^x - 10$

c $\frac{dy}{dx} = \frac{1}{x} + \frac{5}{2}x^{\frac{3}{2}}$
$\frac{d^2y}{dx^2} = -x^{-2} + \frac{15}{4}x^{\frac{1}{2}}$

d $\frac{dy}{dx} = 5e^x + \frac{6}{x}$
$\frac{d^2y}{dx^2} = 5e^x - 6x^{-2}$

e $\frac{dy}{dx} = -3x^{-2} + \frac{3}{x}$
$\frac{d^2y}{dx^2} = 6x^{-3} - 3x^{-2}$

f $\frac{dy}{dx} = 2x^{-\frac{1}{2}} + \frac{1}{4x}$
$\frac{d^2y}{dx^2} = -x^{-\frac{3}{2}} - \frac{1}{4}x^{-2}$

4 **a** $f'(x) = 3 + e^x$
$f'(0) = 3 + 1 = 4$

b $f'(x) = \frac{1}{x} - 2x$
$f'(4) = \frac{1}{4} - 8 = -7\frac{3}{4}$

c $f'(x) = \frac{1}{2}x^{-\frac{1}{2}} + \frac{2}{x}$
$f'(9) = \frac{1}{6} + \frac{2}{9} = \frac{7}{18}$

d $f'(x) = 5e^x - 2x^{-3}$
$f'(-\frac{1}{2}) = 5e^{-\frac{1}{2}} + 16$ [19.0 (3sf)]

5 **a** $\frac{dy}{dx} = \frac{5}{x} - 8 = 0$
$5 = 8x$
$x = \frac{5}{8}$

b $\frac{dy}{dx} = 2.4e^x - 3.6 = 0$
$e^x = 1.5$
$x = \ln 1.5$
[0.405 (3sf)]

c $\frac{dy}{dx} = 6x - 14 + \frac{4}{x} = 0$
$3x^2 - 7x + 2 = 0$
$(3x - 1)(x - 2) = 0$
$x = \frac{1}{3}, 2$

6 **a** $f'(x) = 2e^x - 3 = 7$
$e^x = 5$
$x = \ln 5$ [1.61 (3sf)]

b $f'(x) = 15 + \frac{1}{x} = 23$
$\frac{1}{x} = 8$
$x = \frac{1}{8}$

c $f'(x) = \frac{1}{4}x - 2 + \frac{1}{x} = -1$
$x^2 - 4x + 4 = 0$
$(x - 2)^2 = 0$
$x = 2$

d $f'(x) = \frac{30}{x} - 2x = 4$
$x^2 + 2x - 15 = 0$
$(x + 5)(x - 3) = 0$
$\ln x$ only real for $x > 0$ $\therefore x = 3$

7 **a** $\frac{dy}{dx} = e^x - 2$

SP: $e^x - 2 = 0$

$x = \ln 2$

$\frac{d^2y}{dx^2} = e^x$

$x = \ln 2$: $\frac{d^2y}{dx^2} = 2$

$\therefore$ (ln 2, 2 – 2 ln 2), min

b $\frac{dy}{dx} = \frac{1}{x} - 10$

SP: $\frac{1}{x} - 10 = 0$

$x = \frac{1}{10}$

$\frac{d^2y}{dx^2} = -x^{-2}$

$x = \frac{1}{10}$: $\frac{d^2y}{dx^2} = -100$

$\therefore$ ($\frac{1}{10}$, –1 – ln 10), max

c $\frac{dy}{dx} = \frac{2}{x} - \frac{1}{2}x^{-\frac{1}{2}}$

SP: $\frac{2}{x} - \frac{1}{2}x^{-\frac{1}{2}} = 0$

$4 - x^{\frac{1}{2}} = 0$

$x^{\frac{1}{2}} = 4,\ x = 16$

$\frac{d^2y}{dx^2} = -2x^{-2} + \frac{1}{4}x^{-\frac{3}{2}}$

$x = 16$: $\frac{d^2y}{dx^2} = -\frac{1}{256}$

$\therefore$ (16, 8 ln 2 – 4), max

d $\frac{dy}{dx} = 4 - 5e^x$

SP: $4 - 5e^x = 0$

$x = \ln \frac{4}{5}$

$\frac{d^2y}{dx^2} = -5e^x$

$x = \ln \frac{4}{5}$: $\frac{d^2y}{dx^2} = -4$

$\therefore$ ($\ln \frac{4}{5}$, $4 \ln \frac{4}{5} - 4$), max

e $\frac{dy}{dx} = 2 - \frac{4}{x}$

SP: $2 - \frac{4}{x} = 0$

$x = 2$

$\frac{d^2y}{dx^2} = 4x^{-2}$

$x = 2$: $\frac{d^2y}{dx^2} = 1$

$\therefore$ (2, 11 – 4 ln 2), min

f $\frac{dy}{dx} = 2x - 26 + \frac{72}{x}$

SP: $2x - 26 + \frac{72}{x} = 0$

$x^2 - 13x + 36 = 0$

$(x - 4)(x - 9) = 0$

$x = 4, 9$

$\frac{d^2y}{dx^2} = 2 - 72x^{-2}$

$x = 4$: $\frac{d^2y}{dx^2} = -\frac{5}{2}$

$x = 9$: $\frac{d^2y}{dx^2} = \frac{10}{9}$

$\therefore$ (4, 144 ln 2 – 88), max
(9, 144 ln 3 – 153), min

8 $\frac{dy}{dx} = 1 + ke^x$

$\frac{d^2y}{dx^2} = ke^x$

$\therefore\ (1 - x)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = (1 - x)ke^x + x(1 + ke^x) - (x + ke^x)$

$= ke^x - kxe^x + x + kxe^x - x - ke^x = 0$

9 **a** $x = 2 \;\therefore\; y = e^2$
$\frac{dy}{dx} = e^x$, grad $= e^2$
$\therefore\; y - e^2 = e^2(x - 2)$
$[\, y = e^2(x - 1) \,]$

b $x = 3 \;\therefore\; y = \ln 3$
$\frac{dy}{dx} = \frac{1}{x}$, grad $= \frac{1}{3}$
$\therefore\; y - \ln 3 = \frac{1}{3}(x - 3)$
$[\, y = \frac{1}{3}x + \ln 3 - 1 \,]$

c $x = 0 \;\therefore\; y = -2$
$\frac{dy}{dx} = 0.8 - 2e^x$, grad $= -1.2$
$\therefore\; y = -1.2x - 2$

d $x = 1 \;\therefore\; y = 4$
$\frac{dy}{dx} = \frac{5}{x} - 4x^{-2}$, grad $= 1$
$\therefore\; y - 4 = x - 1$
$[\, y = x + 3 \,]$

e $x = 1 \;\therefore\; y = 1 - 3e$
$\frac{dy}{dx} = \frac{1}{3}x^{-\frac{2}{3}} - 3e^x$, grad $= \frac{1}{3} - 3e$
$\therefore\; y - (1 - 3e) = (\frac{1}{3} - 3e)(x - 1)$
$[\, y = (\frac{1}{3} - 3e)x + \frac{2}{3} \,]$

f $x = 9 \;\therefore\; y = \ln 9 - 3$
$\frac{dy}{dx} = \frac{1}{x} - \frac{1}{2}x^{-\frac{1}{2}}$, grad $= -\frac{1}{18}$
$\therefore\; y - (\ln 9 - 3) = -\frac{1}{18}(x - 9)$
$[\, y = \ln 9 - \frac{5}{2} - \frac{1}{18}x \,]$

10 **a** $x = e \;\therefore\; y = 1$
$\frac{dy}{dx} = \frac{1}{x}$, grad $= \frac{1}{e}$
$\therefore$ grad of normal $= -e$
$\therefore\; y - 1 = -e(x - e)$
$[\, y = e^2 + 1 - ex \,]$

b $x = 0 \;\therefore\; y = 7$
$\frac{dy}{dx} = 3e^x$, grad $= 3$
$\therefore$ grad of normal $= -\frac{1}{3}$
$\therefore\; y = 7 - \frac{1}{3}x$

c $x = 3 \;\therefore\; y = 10 + \ln 3$
$\frac{dy}{dx} = \frac{1}{x}$, grad $= \frac{1}{3}$
$\therefore$ grad of normal $= -3$
$\therefore\; y - (10 + \ln 3) = -3(x - 3)$
$[\, y = 19 + \ln 3 - 3x \,]$

d $x = 1 \;\therefore\; y = -2$
$\frac{dy}{dx} = \frac{3}{x} - 2$, grad $= 1$
$\therefore$ grad of normal $= -1$
$\therefore\; y + 2 = -(x - 1)$
$[\, y = -x - 1 \,]$

e $x = 1 \;\therefore\; y = 1$
$\frac{dy}{dx} = 2x + \frac{8}{x}$, grad $= 10$
$\therefore$ grad of normal $= -\frac{1}{10}$
$\therefore\; y - 1 = -\frac{1}{10}(x - 1)$
$[\, y = \frac{1}{10}(11 - x) \,]$

f $x = 0 \;\therefore\; y = -\frac{13}{10}$
$\frac{dy}{dx} = \frac{1}{10} - \frac{3}{10}e^x$, grad $= -\frac{1}{5}$
$\therefore$ grad of normal $= 5$
$\therefore\; y = 5x - \frac{13}{10}$

1 **a** $x = 0 \therefore y = \frac{1}{10}$

$\frac{dy}{dx} = \frac{2}{5} + \frac{1}{10}e^x$, grad $= \frac{1}{2}$

$\therefore$ grad of normal $= -2$

$\therefore y = -2x + \frac{1}{10}$

$20x + 10y - 1 = 0$

b $y = 0 \therefore x = \frac{1}{20}$

$(\frac{1}{20}, 0)$

2 **a** $x = 1 \therefore y = 5e$

$\frac{dy}{dx} = 5e^x - \frac{3}{x}$, grad $= 5e - 3$

$\therefore y - 5e = (5e - 3)(x - 1)$

$y = (5e - 3)x + 3$

b at Q, $x = 0 \therefore y = 3$

R is $(1, 0)$

area $= \frac{1}{2} \times (3 + 5e) \times 1$

$= \frac{1}{2}(5e + 3)$

3 **a** $\frac{dy}{dx} = 3 - \frac{1}{2}e^x$

SP: $3 - \frac{1}{2}e^x = 0$

$x = \ln 6$

$\therefore (\ln 6, 3\ln 6 - 3)$

b $\frac{d^2y}{dx^2} = -\frac{1}{2}e^x$

$x = \ln 6$: $\frac{d^2y}{dx^2} = -3$

$\therefore$ max

4 **a** at P, $x = 4 \therefore y = 6\ln 4 - 8$

$\frac{dy}{dx} = \frac{6}{x} - 2x^{-\frac{1}{2}}$, grad $= \frac{1}{2}$

$\therefore y - (6\ln 4 - 8) = \frac{1}{2}(x - 4)$

[$y = \frac{1}{2}x - 10 + 12\ln 2$]

b at Q, $y = 0 \therefore x = 20 - 24\ln 2$

at R, $x = 0 \therefore y = 12\ln 2 - 10$

area $= \frac{1}{2} \times (20 - 24\ln 2) \times (10 - 12\ln 2)$

$= (10 - 12\ln 2)^2$

5 **a** $\frac{dy}{dx} = 2 - \frac{1}{x}$, grad $= 1$

$\therefore y = x - 1$

b grad of normal $= -1$

$\therefore y = -(x - 1)$ [$y = 1 - x$]

at B, $x = 0 \therefore y = -1$

at C, $x = 0 \therefore y = 1$

mid-point of $(0, -1)$ and $(0, 1)$

$= (0, \frac{-1+1}{2}) = (0, 0)$

$\therefore$ mid-point of BC is the origin

c SP: $2 - \frac{1}{x} = 0$

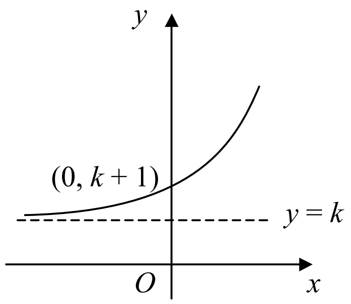
$x = \frac{1}{2}$

$\therefore y = 1 - 2 - \ln\frac{1}{2}$

$= -1 - \ln 2^{-1}$

$= \ln 2 - 1$

6 **a**



b $x = 2 \therefore y = e^2 + k$

$\frac{dy}{dx} = e^x$, grad $= e^2$

$\therefore y - (e^2 + k) = e^2(x - 2)$

[$y = e^2x - e^2 + k$]

c $(-1, 0) \therefore 0 = -e^2 - e^2 + k$

$k = 2e^2$

7 **a** $\frac{dy}{dx} = 6x - \frac{2}{x}$

at P, $6x - \frac{2}{x} = -1$

$6x^2 + x - 2 = 0$

$(3x + 2)(2x - 1) = 0$

$x > 0 \therefore x = \frac{1}{2}$

b $x = 1 \therefore y = 3$, grad = 4

$\therefore y - 3 = 4(x - 1)$

$[\ y = 4x - 1\ ]$

8 **a** $\frac{dy}{dx} = e^x$, grad at $P = e^p$

tangent: $y - e^p = e^p(x - p)$

$(0, 0) \therefore 0 - e^p = e^p(0 - p)$

$e^p(p - 1) = 0$

$e^p \neq 0 \therefore p = 1$

b $P(1, e)$, grad at $P = e$

$\therefore$ grad of normal $= -\frac{1}{e}$

$\therefore y - e = -\frac{1}{e}(x - 1)$

at Q, $y = 0 \therefore x = e^2 + 1$

$\therefore$ area $= \frac{1}{2} \times (e^2 + 1) \times e = \frac{1}{2}e(1 + e^2)$

9 **a** at P, $x = 0 \therefore y = 3$

$\frac{dy}{dx} = -e^x$, grad $= -1$

$\therefore$ grad of normal = 1

$\therefore y = x + 3$

b at Q, $y = 0 \therefore x = \ln 4$

grad at $Q = -4$

$\therefore y = -4(x - \ln 4)$ $[\ y = 8 \ln 2 - 4x\ ]$

c at R $x + 3 = -4(x - \ln 4)$

$5x = 4 \ln 4 - 3 = 8 \ln 2 - 3$

$x = \frac{1}{5}(8 \ln 2 - 3)$

$\therefore a = \frac{8}{5}$

d $b = -\frac{3}{5}$

10 $f'(x) = 36x^3 - \frac{16}{x}$

SP: $36x^3 - \frac{16}{x} = 0$

$x^4 = \frac{4}{9}$

$x^2 = -\frac{2}{3}$ [no solutions] or $\frac{2}{3}$

$x > 0 \therefore x = \sqrt{\frac{2}{3}}$

$\therefore$ decreasing for $0 < x \leq \sqrt{\frac{2}{3}}$

$k = \sqrt{\frac{2}{3}}$ or $\frac{1}{3}\sqrt{6}$