1 **a** $r = 3$
$u_8 = 3 \times 3^7 = 6561$

b $r = \frac{1}{4}$
$u_8 = 1024 \times (\frac{1}{4})^7 = \frac{1}{16}$

c $r = -2$
$u_8 = 1 \times (-2)^7 = -128$

2 **a** $a = 1, r = 5$
$u_n = 5^{n-1}$

b $a = 3, r = -4$
$u_n = 3 \times (-4)^{n-1}$

c $a = 81, r = \frac{2}{3}$
$u_n = 81 \times (\frac{2}{3})^{n-1}$

3 **a** $a = 2, r = 2, n = 12$
$S_{12} = \frac{2(2^{12}-1)}{2-1} = 8190$

b $a = 640, r = \frac{1}{2}, n = 12$
$S_{12} = \frac{640[1-(\frac{1}{2})^{12}]}{1-\frac{1}{2}} = 1279\frac{11}{16}$

c $a = \frac{1}{6}, r = -3, n = 12$
$S_{12} = \frac{\frac{1}{6}[1-(-3)^{12}]}{1-(-3)} = -22\,143\frac{1}{3}$

4 **a** $S_8 = \frac{4(3^8-1)}{3-1} = 13\,120$

b $S_{14} = \frac{48[1-(\frac{1}{2})^{14}]}{1-\frac{1}{2}} = 95.994$

c $S_{12} = \frac{-[1-(-4)^{12}]}{1-(-4)} = 3\,355\,443$

d $S_{20} = \frac{200[1-(0.7)^{20}]}{1-0.7}$
$= 666.135$

e $S_{15} = \frac{120[1-(-\frac{3}{4})^{15}]}{1-(-\frac{3}{4})}$
$= 69.488$

f $S_{30} = \frac{-25[(1.2)^{30}-1]}{1.2-1}$
$= -29\,547.039$

5 **a** GP: $a = 3$
$r = 3, n = 9$
$S_9 = \frac{3(3^9-1)}{3-1}$
$= 29\,523$

b GP: $a = 64$
$r = 8, n = 6$
$S_6 = \frac{64(8^6-1)}{8-1}$
$= 2\,396\,736$

c GP: $a = 20$
$r = 2, n = 10$
$S_{10} = \frac{20(2^{10}-1)}{2-1}$
$= 20\,460$

d GP: $a = 0.8$
$r = 0.8, n = 8$
$S_8 = \frac{0.8[1-(0.8)^8]}{1-0.8}$
$= 3.329$ (3dp)

e GP: $a = 2$
$r = \frac{1}{6}, n = 10$
$S_{10} = \frac{2[1-(\frac{1}{6})^{10}]}{1-\frac{1}{6}}$
$= 2.400$ (3dp)

f GP: $a = -4$
$r = -4, n = 9$
$S_9 = \frac{-4[1-(-4)^9]}{1-(-4)}$
$= -209\,716$

g GP: $a = \frac{1}{16}$
$r = \frac{1}{2}, n = 17$
$S_{17} = \frac{\frac{1}{16}[1-(\frac{1}{2})^{17}]}{1-\frac{1}{2}}$
$= 0.125$ (3dp)

h GP: $a = -54$
$r = -3, n = 7$
$S_7 = \frac{-54[1-(-3)^7]}{1-(-3)}$
$= -29\,538$

6 **a** $r = 10 \div 2 = 5$
b $a \times 5 = 2 \;\therefore\; a = 0.4$
c $S_8 = \frac{0.4(5^8-1)}{5-1} = 39\,062.4$

7 **a** $a = 2, ar^3 = 54 \;\therefore\; r^3 = 54 \div 2 = 27$
$r = \sqrt[3]{27} = 3$
b $u_9 = 2 \times 3^8 = 13\,122$

8 **a** $r = 8 \div 24 = \frac{1}{3}$
b $a \times (\frac{1}{3})^2 = 24 \;\therefore\; a = 216$
c $S_{11} = \frac{216[1-(\frac{1}{3})^{11}]}{1-\frac{1}{3}} = 323.998$

9 **a** $a = 6, ar^2 = 24 \;\therefore\; r^2 = 24 \div 6 = 4$
$r = \pm 2$
b $r = 2,\; S_{15} = \frac{6(2^{15}-1)}{2-1} = 196\,602$

10 **a** $a = 768, ar^3 = -96$
$r^3 = -96 \div 768 = -\frac{1}{8}$
$r = \sqrt[3]{-\frac{1}{8}} = -\frac{1}{2}$
b $u_{10} = 768 \times (-\frac{1}{2})^9 = -1.5$

11 **a** $ar = 0.5,\; ar^4 = 32 \;\therefore\; r^3 = 32 \div 0.5 = 64$
$r = \sqrt[3]{64} = 4,\; a \times 4 = 0.5 \;\therefore\; a = 0.125$
b $0.125 \times 4^{n-1} < 10\,000 \;\therefore\; 4^{n-1} < 80\,000$
$(n-1) \lg 4 < \lg 80\,000$
$n < \frac{\lg 80000}{\lg 4} + 1$
$n < 9.14 \;\therefore\;$ 9 terms

12 **a** $\frac{a[(\frac{3}{2})^4 - 1]}{\frac{3}{2} - 1} = 130$

$a = 130 \div \frac{65}{8} = 16$

b $u_8 = 16 \times (\frac{3}{2})^7 = 273\frac{3}{8}$

c $\frac{16[(\frac{3}{2})^n - 1]}{\frac{3}{2} - 1} > 30\,000$

$(\frac{3}{2})^n > 938.5$

$n \lg \frac{3}{2} > \lg 938.5$

$n > \frac{\lg 938.5}{\lg 1.5}$

$n > 16.9 \ \therefore$ least $n = 17$

13 **a** $a + ar = a(1 + r) = 10.8$

$ar^2 + ar^3 = ar^2(1 + r) = 43.2$

$\therefore\ r^2 = 43.2 \div 10.8 = 4$

all terms +ve $\therefore\ r$ +ve $\therefore\ r = 2$

sub. $a = 10.8 \div 3 = 3.6$

b $S_{16} = \frac{3.6(2^{16} - 1)}{2 - 1} = 235\,926$

14 **a** $a = 12, r = 0.5$

$S_\infty = \frac{12}{1 - 0.5} = 24$

b $a = 270, r = \frac{1}{3}$

$S_\infty = \frac{270}{1 - \frac{1}{3}} = 405$

c $a = 25, r = -1.2$

no S_∞ as $r < -1 \ \therefore$ diverges

d $a = 216, r = \frac{2}{3}$

$S_\infty = \frac{216}{1 - \frac{2}{3}} = 648$

e $a = \frac{8}{25}, r = \frac{5}{4}$

no S_∞ as $r > 1 \ \therefore$ diverges

f $a = 500, r = -0.6$

$S_\infty = \frac{500}{1 - (-0.6)} = 312.5$

15 **a** $a = 0.9, r = 0.9$

$S_\infty = \frac{0.9}{1 - 0.9} = 9$

b $a = 3, r = \frac{1}{2}$

$S_\infty = \frac{3}{1 - \frac{1}{2}} = 6$

c $a = 1, r = -\frac{3}{4}$

$S_\infty = \frac{1}{1 - (-\frac{3}{4})} = \frac{4}{7}$

d $a = 32, r = 0.8$

$S_\infty = \frac{32}{1 - 0.8} = 160$

16 **a** $S_\infty = \frac{80}{1 - 0.2} = 100$

b $S_6 = \frac{80[1 - (0.2)^6]}{1 - 0.2} = 99.9936$

$S_\infty - S_6 = 0.0064$

17 **a** $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}$

b GP: $a = 1, r = \frac{1}{3}$

$S_\infty = \frac{1}{1 - \frac{1}{3}} = \frac{3}{2}$

18 **a** $\frac{a}{1 - 0.55} = 40$

$a = 0.45 \times 40 = 18$

b $18 \times (0.55)^{n-1} < 0.001$

$(n - 1) \lg 0.55 < \lg 0.0000556$

$n > \frac{\lg 0.0000556}{\lg 0.55} + 1$

$n > 17.4 \ \therefore$ smallest $n = 18$

19 **a** $u_1 = S_1 = 2^1 - 1 = 1$

$S_5 = 2^5 - 1 = 31,\ S_4 = 2^4 - 1 = 15$

$u_5 = S_5 - S_4 = 31 - 15 = 16$

b $S_{n-1} = 2^{n-1} - 1$

$u_n = S_n - S_{n-1} = (2^n - 1) - (2^{n-1} - 1)$

$= 2^n - 2^{n-1} = 2^{n-1}(2 - 1) = 2^{n-1}$

20 **a** $\frac{k}{k + 10} = \frac{k - 6}{k}$

$k^2 = (k + 10)(k - 6)$

$4k - 60 = 0$

$k = 15$

b $u_1 = 25, u_2 = 15 \ \therefore\ a = 25, r = 0.6$

$S_\infty = \frac{25}{1 - 0.6} = 62.5$

1 **a** $r = 20\frac{1}{4} \div 27 = \frac{3}{4}$

$a \times (\frac{3}{4})^2 = 27$

$a = \frac{16}{9} \times 27 = 48$

b $S_\infty = \frac{48}{1-\frac{3}{4}} = 192$

2 **a** $\frac{k+4}{k-8} = \frac{3k+2}{k+4}$

$(k + 4)^2 = (3k + 2)(k - 8)$

$k^2 - 15k - 16 = 0$

$(k + 1)(k - 16) = 0$

$k > 0 \ \therefore\ k = 16$

b $u_1 = 8,\ u_2 = 20 \ \therefore\ a = 8,\ r = \frac{5}{2}$

$u_6 = 8 \times (\frac{5}{2})^5 = 781\frac{1}{4}$

c $S_{10} = \frac{8[(\frac{5}{2})^{10} - 1]}{\frac{5}{2} - 1} = 50\,857.3$

3 **a** $ar = 75,\ ar^4 = 129.6$

$r^3 = 129.6 \div 75 = 1.728$

$r = \sqrt[3]{1.728} = 1.2$

$a = 75 \div 1.2 = 62.5$

b $u_{10} = 62.5 \times (1.2)^9 = 322.5$

c $S_{12} = \frac{62.5[(1.2)^{12} - 1]}{1.2 - 1} = 2473.8$

4 **a** $S_n = a + ar + ar^2 + \ldots ar^{n-2} + ar^{n-1}$

$rS_n = ar + ar^2 + ar^3 + \ldots ar^{n-1} + ar^n$

subtracting,

$S_n - rS_n = a - ar^n$

$(1 - r)S_n = a(1 - r^n)$

$S_n = \frac{a(1-r^n)}{1-r}$

b $\frac{2[1-(\sqrt{2})^n]}{1-\sqrt{2}} = 126(\sqrt{2} + 1)$

$1 - (\sqrt{2})^n = 63(\sqrt{2} + 1)(1 - \sqrt{2})$

$1 - (\sqrt{2})^n = 63(1 - 2)$

$(\sqrt{2})^n = 64$

$2^{\frac{1}{2}n} = 2^6$

$n = 12$

5 **a** $\frac{18}{1-r} = 15$

$\therefore\ 1 - r = \frac{18}{15} = 1.2$

$r = -0.2$

b $u_3 = 18 \times (-0.2)^2 = 0.72$

c $S_8 = \frac{18[1-(-0.2)^8]}{1-(-0.2)} = 14.9999616$

$S_\infty - S_8 = 0.000\,0384$

6 **a** $S_3 = 5(3^3 - 1) = 130$

$S_2 = 5(3^2 - 1) = 40$

$u_3 = S_3 - S_2 = 90$

b $S_{n-1} = 5(3^{n-1} - 1)$

$u_n = S_n - S_{n-1} = 5(3^n - 1) - 5(3^{n-1} - 1)$

$= 5[3^n - 3^{n-1}] = 5(3^n)[1 - \frac{1}{3}] = \frac{10}{3}(3^n)$

7 **a** $4 \times (1.25)^7 = 19.1$ mm (3sf)

b GP: $a = 4,\ r = 1.25$

$S_{20} = \frac{4[(1.25)^{20} - 1]}{1.25 - 1} = 1371.8$ mm

$\therefore$ length = 1.37 m (3sf)

8 **a** $ar = 30,\ ar^3 = 2.7 \ \therefore\ r^2 = 2.7 \div 30 = 0.09$

$r > 0 \ \therefore\ r = \sqrt{0.09} = 0.3$

$a = 30 \div 0.3 = 100$

b $S_\infty = \frac{100}{1-0.3} = 142.9$ (1dp)

9 **a** GP: $a = 27, r = 3$

$S_8 = \frac{27(3^8 - 1)}{3 - 1} = 88\,560$

b $\sum_{r=1}^{15} 2^r$: GP, $a = 2, r = 2$

$S_{15} = \frac{2(2^{15} - 1)}{2 - 1} = 65\,534$

$\sum_{r=1}^{15} 12r$: AP, $a = 12, d = 12$

$S_{15} = \frac{15}{2}[24 + (14 \times 12)] = 1440$

$\sum_{r=1}^{15} (2^r - 12r) = 65\,534 - 1440 = 64\,094$

10 **a** $a = 64,\ ar^2 - ar = 20$

$\therefore\ 64r^2 - 64r = 20$

$16r^2 - 16r - 5 = 0$

b $(4r + 1)(4r - 5) = 0$

$r = -\frac{1}{4}$ or $\frac{5}{4}$

c $r = -\frac{1}{4} \Rightarrow u_4 = 64 \times (-\frac{1}{4})^3 = -1$

$r = \frac{5}{4} \Rightarrow u_4 = 64 \times (\frac{5}{4})^3 = 125$

d $r = -\frac{1}{4} \Rightarrow S_\infty = \frac{64}{1 - (-\frac{1}{4})} = 51\frac{1}{5}$

11 **a** $u_8 = 4 \times (\frac{1}{2})^7 = \frac{1}{32}$

b $u_n = 4 \times (\frac{1}{2})^{n-1}$

$= 2^2 \times 2^{1-n}$

$= 2^{3-n}$

c $S_n = \frac{4[1 - (\frac{1}{2})^n]}{1 - \frac{1}{2}}$

$= 8(1 - 2^{-n})$

$= 8 - (2^3 \times 2^{-n})$

$= 8 - 2^{3-n}$

12 **a** $u_6 = 4 \times 3^6 = 2916$

b GP: $a = 12, r = 3$

$S_t = \frac{12(3^t - 1)}{3 - 1} = 6(3^t - 1)$

$\therefore\ 6(3^t - 1) > 10^{25}$

$3^t > \frac{10^{25}}{6} + 1$

$t \lg 3 > \lg(\frac{10^{25}}{6} + 1)$

$t > \frac{\lg(\frac{10^{25}}{6} + 1)}{\lg 3}$

$t > 50.8\ \therefore$ smallest $t = 51$

13 **a** $a + ar^2 = a(1 + r^2) = 150$

$ar + ar^3 = ar(1 + r^2) = -75$

$\therefore\ r = -75 \div 150 = -\frac{1}{2}$

$a = 150 \div \frac{5}{4} = 120$

b $S_\infty = \frac{120}{1 - (-\frac{1}{2})} = 80$

14 **a** $b - a = (3a + 4) - b$

$2b = 4a + 4$

$b = 2a + 2$

b $\frac{2a + 2}{a} = \frac{6a + 1}{2a + 2}$

$(2a + 2)^2 = a(6a + 1)$

$2a^2 - 7a - 4 = 0$

$(2a + 1)(a - 4) = 0$

a integer $\therefore a = 4$

sub. $b = 10$

15 **a** after 4^{th} bounce,

reaches $3 \times (0.6)^4 = 0.3888$ m

b total distance

$= h + 2[0.6h + (0.6)^2h + (0.6)^3h + \ldots]$

$= h + 2 \times S_\infty$ of GP, $a = 0.6h$, $r = 0.6$

$= h + \frac{2 \times 0.6h}{1 - 0.6}$

$= h + 3h = 4h$ metres