**1** **a** $= [2x^2 - x]_1^3$
$= (18 - 3) - (2 - 1)$
$= 14$

**b** $= [x^3 + 2x]_0^1$
$= (1 + 2) - (0)$
$= 3$

**c** $= [\frac{1}{2}x^2 - \frac{1}{3}x^3]_0^3$
$= (\frac{9}{2} - 9) - (0)$
$= -\frac{9}{2}$

**d** $= \int_2^3 (9x^2 + 6x + 1)\ dx$
$= [3x^3 + 3x^2 + x]_2^3$
$= (81 + 27 + 3) - (24 + 12 + 2)$
$= 73$

**e** $= [\frac{1}{3}x^3 - 4x^2 - 3x]_1^2$
$= (\frac{8}{3} - 16 - 6) - (\frac{1}{3} - 4 - 3)$
$= -12\frac{2}{3}$

**f** $= [8x - 2x^2 + x^3]_{-2}^4$
$= (32 - 32 + 64) - (-16 - 8 - 8)$
$= 96$

**g** $= [\frac{1}{4}x^4 - x^2 - 7x]_1^4$
$= (64 - 16 - 28) - (\frac{1}{4} - 1 - 7)$
$= 27\frac{3}{4}$

**h** $= [5x + \frac{1}{3}x^3 - x^4]_{-2}^{-1}$
$= (-5 - \frac{1}{3} - 1) - (-10 - \frac{8}{3} - 16)$
$= 22\frac{1}{3}$

**i** $= [\frac{1}{5}x^5 + 2x^3 - \frac{1}{2}x^2]_{-1}^2$
$= (\frac{32}{5} + 16 - 2) - (-\frac{1}{5} - 2 - \frac{1}{2})$
$= 23\frac{1}{10}$

**2** $\int_1^4 (3x^2 + ax - 5)\ dx = [x^3 + \frac{1}{2}ax^2 - 5x]_1^4$
$= (64 + 8a - 20) - (1 + \frac{1}{2}a - 5) = 48 + \frac{15}{2}a$
$\therefore\ 48 + \frac{15}{2}a = 18$
$a = -4$

**3** $\int_{-1}^k (3x^2 - 12x + 9)\ dx = [x^3 - 6x^2 + 9x]_{-1}^k$
$= (k^3 - 6k^2 + 9k) - (-1 - 6 - 9) = k^3 - 6k^2 + 9k + 16$
$\therefore\ k^3 - 6k^2 + 9k + 16 = 16$
$k(k^2 - 6k + 9) = 0$
$k(k - 3)^2 = 0$
$k \neq 0\ \ \therefore\ k = 3$

**4** **a** $= \int_1^3 (2 - x^{-2})\ dx$
$= [2x + x^{-1}]_1^3$
$= (6 + \frac{1}{3}) - (2 + 1)$
$= \frac{10}{3}$

**b** $= \int_{-2}^{-1} (6x + 4x^{-3})\ dx$
$= [3x^2 - 2x^{-2}]_{-2}^{-1}$
$= (3 - 2) - (12 - \frac{1}{2})$
$= -10\frac{1}{2}$

**c** $= [2x^{\frac{3}{2}} - 4x]_1^4$
$= (16 - 16) - (2 - 4)$
$= 2$

**d** $= \int_{-1}^2 (2x^3 - \frac{1}{2})\ dx$
$= [\frac{1}{2}x^4 - \frac{1}{2}x]_{-1}^2$
$= (8 - 1) - (\frac{1}{2} + \frac{1}{2})$
$= 6$

**e** $= [\frac{1}{2}x^2 - \frac{3}{2}x^{\frac{2}{3}}]_1^8$
$= (32 - 6) - (\frac{1}{2} - \frac{3}{2})$
$= 27$

**f** $= \int_2^3 (\frac{1}{3}x^{-2} - 2x)\ dx$
$= [-\frac{1}{3}x^{-1} - x^2]_2^3$
$= (-\frac{1}{9} - 9) - (-\frac{1}{6} - 4)$
$= -4\frac{17}{18}$

**5** $= \int_1^3 (3x^2 - 6x + 7)\ dx$
$= [x^3 - 3x^2 + 7x]_1^3$
$= (27 - 27 + 21) - (1 - 3 + 7) = 16$

**6** **a** $= \int_0^2 (x^2 + 2)\ \mathrm{d}x$

$= [\frac{1}{3}x^3 + 2x]_0^2$

$= (\frac{8}{3} + 4) - 0 = 6\frac{2}{3}$

**b** $= \int_{-2}^{1} (3x^2 + 8x + 6)\ \mathrm{d}x$

$= [x^3 + 4x^2 + 6x]_{-2}^{1}$

$= (1 + 4 + 6) - (-8 + 16 - 12) = 15$

**c** $= \int_2^4 (9 + 2x - x^2)\ \mathrm{d}x$

$= [9x + x^2 - \frac{1}{3}x^3]_2^4$

$= (36 + 16 - \frac{64}{3}) - (18 + 4 - \frac{8}{3}) = 11\frac{1}{3}$

**d** $= \int_{-1}^{0} (x^3 - 4x + 1)\ \mathrm{d}x$

$= [\frac{1}{4}x^4 - 2x^2 + x]_{-1}^{0}$

$= 0 - (\frac{1}{4} - 2 - 1) = \frac{11}{4}$

**e** $= \int_1^4 (2x + 3x^{\frac{1}{2}})\ \mathrm{d}x$

$= [x^2 + 2x^{\frac{3}{2}}]_1^4$

$= (16 + 16) - (1 + 2) = 29$

**f** $= \int_{-5}^{-1} (3 + 5x^{-2})\ \mathrm{d}x$

$= [3x - 5x^{-1}]_{-5}^{-1}$

$= (-3 + 5) - (-15 + 1) = 16$

**7** **a** $y = 0 \Rightarrow 4 - x^2 = 0$

$x^2 = 4$

$x = \pm 2$

$\therefore$ $(-2, 0)$ and $(2, 0)$

**b** $= \int_{-2}^{2} (4 - x^2)\ \mathrm{d}x$

$= [4x - \frac{1}{3}x^3]_{-2}^{2}$

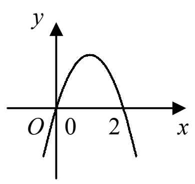
$= (8 - \frac{8}{3}) - (-8 + \frac{8}{3})$

$= 10\frac{2}{3}$

**8** **a** $6x - 3x^2 = 0$

$3x(2 - x) = 0$

$x = 0, 2$




area =

$\int_0^2 (6x - 3x^2)\,\mathrm{d}x$

$= [3x^2 - x^3]_0^2$

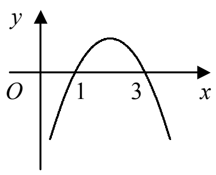
$= (12 - 8) - 0$

$= 4$

**b** $-x^2 + 4x - 3 = 0$

$-(x - 1)(x - 3) = 0$

$x = 1, 3$




area =

$\int_1^3 (-x^2 + 4x - 3)\,\mathrm{d}x$

$= [-\frac{1}{3}x^3 + 2x^2 - 3x]_1^3$

$= (-9 + 18 - 9)$

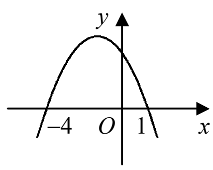
$\quad - (-\frac{1}{3} + 2 - 3)$

$= \frac{4}{3}$

**c** $4 - 3x - x^2 = 0$

$-(x + 4)(x - 1) = 0$

$x = -4, 1$




area =

$\int_{-4}^{1} (4 - 3x - x^2)\,\mathrm{d}x$

$= [4x - \frac{3}{2}x^2 - \frac{1}{3}x^3]_{-4}^{1}$

$= (4 - \frac{3}{2} - \frac{1}{3})$

$\quad - (-16 - 24 + \frac{64}{3})$

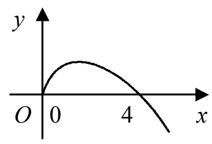
$= 20\frac{5}{6}$

**d** $2x^{\frac{1}{2}} - x = 0$

$x^{\frac{1}{2}}(2 - x^{\frac{1}{2}}) = 0$

$x^{\frac{1}{2}} = 0, 2$

$x = 0, 4$




area =

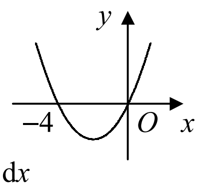
$\int_0^4 (2x^{\frac{1}{2}} - x)\,\mathrm{d}x$

$= [\frac{4}{3}x^{\frac{3}{2}} - \frac{1}{2}x^2]_0^4$

$= (\frac{32}{3} - 8) - 0$

$= \frac{8}{3}$

**9** **a** $x^2 + 4x = 0$
$x(x + 4) = 0$
$x = -4, 0$




**b** $= \int_0^2 (x^2 + 4x)\ dx$
$= [\frac{1}{3}x^3 + 2x^2]_0^2$
$= (\frac{8}{3} + 8) - 0 = 10\frac{2}{3}$

**10** **a** $x^2 + 2x - 15 = 0$
$(x + 5)(x - 3) = 0$
$x = -5, 3$
$\therefore$ (–5, 0) and (3, 0)

**b** $= [\frac{1}{3}x^3 + x^2 - 15x]_0^3$
$= (9 + 9 - 45) - 0 = -27$

**c** 27

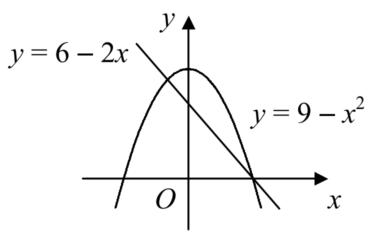
**11** **a** $x^3 - 5x^2 + 6x = 0$
$x(x - 2)(x - 3) = 0$
$x = 0, 2, 3$
$\therefore$ (0, 0), (2, 0) and (3, 0)

**b** $\int_0^2 (x^3 - 5x^2 + 6x)\ dx$
$= [\frac{1}{4}x^4 - \frac{5}{3}x^3 + 3x^2]_0^2$
$= (4 - \frac{40}{3} + 12) - 0 = \frac{8}{3}$
$\int_2^3 (x^3 - 5x^2 + 6x)\ dx$
$= [\frac{1}{4}x^4 - \frac{5}{3}x^3 + 3x^2]_2^3$
$= (\frac{81}{4} - 45 + 27) - \frac{8}{3} = -\frac{5}{12}$
total area $= \frac{8}{3} + \frac{5}{12} = 3\frac{1}{12}$

**12** **a** $x^2 - 3x + 4 = x + 1$
$x^2 - 4x + 3 = 0$
$(x - 1)(x - 3) = 0$
$x = 1, 3$
$\therefore$ (1, 2) and (3, 4)

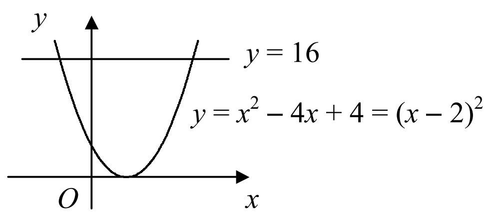
**b** area below curve
$= \int_1^3 (x^2 - 3x + 4)\ dx$
$= [\frac{1}{3}x^3 - \frac{3}{2}x^2 + 4x]_1^3$
$= (9 - \frac{27}{2} + 12) - (\frac{1}{3} - \frac{3}{2} + 4) = \frac{14}{3}$
area below line $= \frac{1}{2} \times 2 \times (2 + 4) = 6$
shaded area $= 6 - \frac{14}{3} = \frac{4}{3}$

**13** **a**



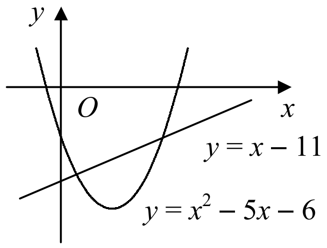

$9 - x^2 = 6 - 2x$
$x^2 - 2x - 3 = 0$
$(x + 1)(x - 3) = 0$
$x = -1, 3$
$\therefore$ intersect at (–1, 8) and (3, 0)
area below curve
$= \int_{-1}^3 (9 - x^2)\ dx$
$= [9x - \frac{1}{3}x^3]_{-1}^3$
$= (27 - 9) - (-9 + \frac{1}{3})$
$= 26\frac{2}{3}$
area below line
$= \frac{1}{2} \times 4 \times 8 = 16$
area between line and curve
$= 26\frac{2}{3} - 16 = 10\frac{2}{3}$

**b**




$x^2 - 4x + 4 = 16$
$x^2 - 4x - 12 = 0$
$(x + 2)(x - 6) = 0$
$x = -2, 6$
$\therefore$ intersect at (–2, 16) and (6, 16)
area below curve
$= \int_{-2}^6 (x^2 - 4x + 4)\ dx$
$= [\frac{1}{3}x^3 - 2x^2 + 4x]_{-2}^6$
$= (72 - 72 + 24) - (-\frac{8}{3} - 8 - 8)$
$= 42\frac{2}{3}$
area below line
$= 8 \times 16 = 128$
area between line and curve
$= 128 - 42\frac{2}{3} = 85\frac{1}{3}$

**c** $y = x^2 - 5x - 6 \Rightarrow y = (x + 1)(x - 6)$




$x^2 - 5x - 6 = x - 11$

$x^2 - 6x + 5 = 0$

$(x - 1)(x - 5) = 0$
$x = 1, 5$
$\therefore$ intersect at (1, −10) and (5, −6)
area above curve

$= -\int_1^5 (x^2 - 5x - 6)\ dx$

$= -[\frac{1}{3}x^3 - \frac{5}{2}x^2 - 6x]_1^5$

$= -[(\frac{125}{3} - \frac{125}{2} - 30) - (\frac{1}{3} - \frac{5}{2} - 6)]$

$= 42\frac{2}{3}$

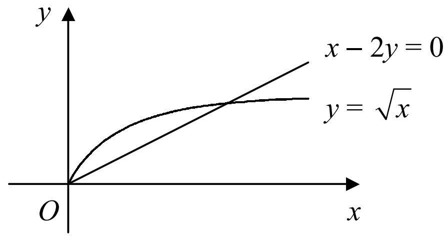
area above line

$= \frac{1}{2} \times 4 \times (10 + 6) = 32$

area between line and curve

$= 42\frac{2}{3} - 32 = 10\frac{2}{3}$

**d** $x - 2y = 0 \Rightarrow y = \frac{1}{2}x$




$x^{\frac{1}{2}} = \frac{1}{2}x$

$\frac{1}{2}x^{\frac{1}{2}}(2 - x^{\frac{1}{2}}) = 0$

$x^{\frac{1}{2}} = 0, 2$
$x = 0, 4$
$\therefore$ intersect at (0, 0) and (4, 2)
area below curve

$= \int_0^4 x^{\frac{1}{2}}\ dx$

$= [\frac{2}{3}x^{\frac{3}{2}}]_0^4$

$= \frac{16}{3} - 0 = \frac{16}{3}$

area below line

$= \frac{1}{2} \times 4 \times 2 = 4$

area between line and curve

$= \frac{16}{3} - 4 = \frac{4}{3}$

**1** **a** $f(x) = -[x^2 - 4x] + 3$

$= -[(x-2)^2 - 4] + 3$

$= -(x-2)^2 + 7$

$\therefore\ a = -1,\ b = -2,\ c = 7$

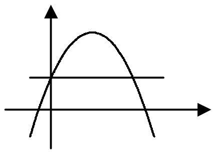
**b** (2, 7)

**c** intersect when

$3 + 4x - x^2 = 3$

$x(4 - x) = 0$

$x = 0, 4$



area below curve

$= \int_0^4 (3 + 4x - x^2)\ dx$

$= [3x + 2x^2 - \frac{1}{3}x^3]_0^4$

$= (12 + 32 - \frac{64}{3}) - 0 = \frac{68}{3}$

area below line

$= 4 \times 3 = 12$

area between line and curve

$= \frac{68}{3} - 12 = 10\frac{2}{3}$

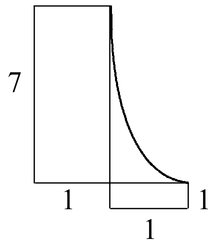
**2** **a** $= [-4x^{-2}]_1^2$

$= -1 - (-4)$

$= 3$

**b** $y = 1 \Rightarrow x = 2$

$y = 8 \Rightarrow x = 1$




shaded area

$= 3 - (1 \times 1) + (7 \times 1)$

$= 9$

**3** **a** $\frac{dy}{dx} = 5 - 4x$

grad = 1

$\therefore$ grad of normal = $-1$

$\therefore\ y - 3 = -(x - 1)$

$[y = 4 - x]$

**b** area below curve

$= \int_0^1 (5x - 2x^2)\ dx$

$= [\frac{5}{2}x^2 - \frac{2}{3}x^3]_0^1$

$= (\frac{5}{2} - \frac{2}{3}) - 0 = \frac{11}{6}$

normal meets $y$-axis at (0, 4)

area below line

$= \frac{1}{2} \times 1 \times (4 + 3) = \frac{7}{2}$

shaded area

$= \frac{7}{2} - \frac{11}{6} = \frac{5}{3}$

**4** **a** $\frac{4 - x^2}{x^2} = 0$

$4 - x^2 = 0$

$x^2 = 4$

$x > 0\ \therefore\ x = 2,\ P\,(2, 0)$

**b** $l$: $y - 0 = -3(x - 2)$

$y = 6 - 3x$

intersect when $\frac{4 - x^2}{x^2} = 6 - 3x$

$4 - x^2 = 6x^2 - 3x^3$

$3x^3 - 7x^2 + 4 = 0$

$x = 2$ is a solution $\therefore\ (x - 2)$ is a factor

$(x - 2)(3x^2 - x - 2) = 0$

$(x - 2)(3x + 2)(x - 1) = 0$

$x = 2$ (at $P$), $-\frac{2}{3}$, 1

$x > 0\ \therefore\ Q\,(1, 3)$

**c** area below curve

$= \int_1^2 (4x^{-2} - 1)\ dx$

$= [-4x^{-1} - x]_1^2$

$= (-2 - 2) - (-4 - 1) = 1$

area below line

$= \frac{1}{2} \times 1 \times 3 = \frac{3}{2}$

area between line and curve

$= \frac{3}{2} - 1 = \frac{1}{2}$