Mathematics
2018 Practice Paper
Paper 3 (Calculator)
Higher Tier

Time: 1 hour 30 minutes

You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser, calculator. Tracing paper may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided
– there may be more space than you need.
- **Calculators may be used.**
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.
- You must **show all your working.**

Information

- The total mark for this paper is 80
- The marks for **each** question are shown in brackets
– use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

1 Use your calculator to work out the value of $\dfrac{21.75 + \sqrt{98.1}}{0.192}$

write down all of the numbers on your calculator display

164.8674188

(Total for question 1 is 2 marks)

2 Frank is travelling from the USA to Germany.

Frank wants to book flights which cost $710 and a hotel which costs €45 per night for 12 nights.

The exchange rates are as follows:

$$£1 = €1.14$$
$$\$1 = €0.85$$

Frank can spend no more than £1000
Work out if Frank is able to book the flights and the hotel.

Flights:

$$710 \times 0.85 \quad = \quad €603.50$$

$$603.5 \div 1.14 \quad = \quad £529.39$$

Hotel:

$$45 \times 12 \quad = \quad €540$$

$$540 \div 1.14 \quad = \quad £473.68$$

$$529.39 + 473.68 \quad = \quad £1003.07$$

frank cannot afford the flights and hotel.

(Total for question 2 is 4 marks)

3 There are 30 sweets in a bag.
All of the sweets are either blue or red.
The ratio of blue sweets to red sweets is 2:1.

4 blue sweets are removed from the bag.

Find the ratio of the number of blue sweets now in the pack to the number red sweets now in the pack.
Give your answer in its simplest form.

$$\frac{30}{3} = 10$$

BLUE : RED

20 : 10

4 BLUE REMOVED:

16 : 10

8 : 5

8 : 5

(Total for question 3 is 3 marks)

4 (a) Write 0.000045 in standard form.

4.5×10^{-5}

(1)

(b) Work out the value of $(2.31 \times 10^{-2}) \div (6.37 \times 10^{-6})$
Give your answer in standard form correct to 3 significant figures.

3 6 2 6 . 3 7 ...

3 6 3 0   (3sf)

3.63 × 10³

3.63×10^{3}

(2)

(Total for question 4 is 3 marks)

5 Solve the simultaneous equations

$$5x + 3y = 8 \quad \times 4$$
$$4x - 2y = 13 \quad \times 5$$

$$20x + 12y = 32$$
$$20x - 10y = 65$$

$$22y = -33$$
$$y = -1.5$$

$$4x - 2(-1.5) = 13$$
$$4x + 3 = 13$$
$$4x = 10$$
$$x = 2.5$$

$x = \underline{2.5}$

$y = \underline{-1.5}$

(Total for question 5 is 3 marks)

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6 Change 90 km/h into m/s.
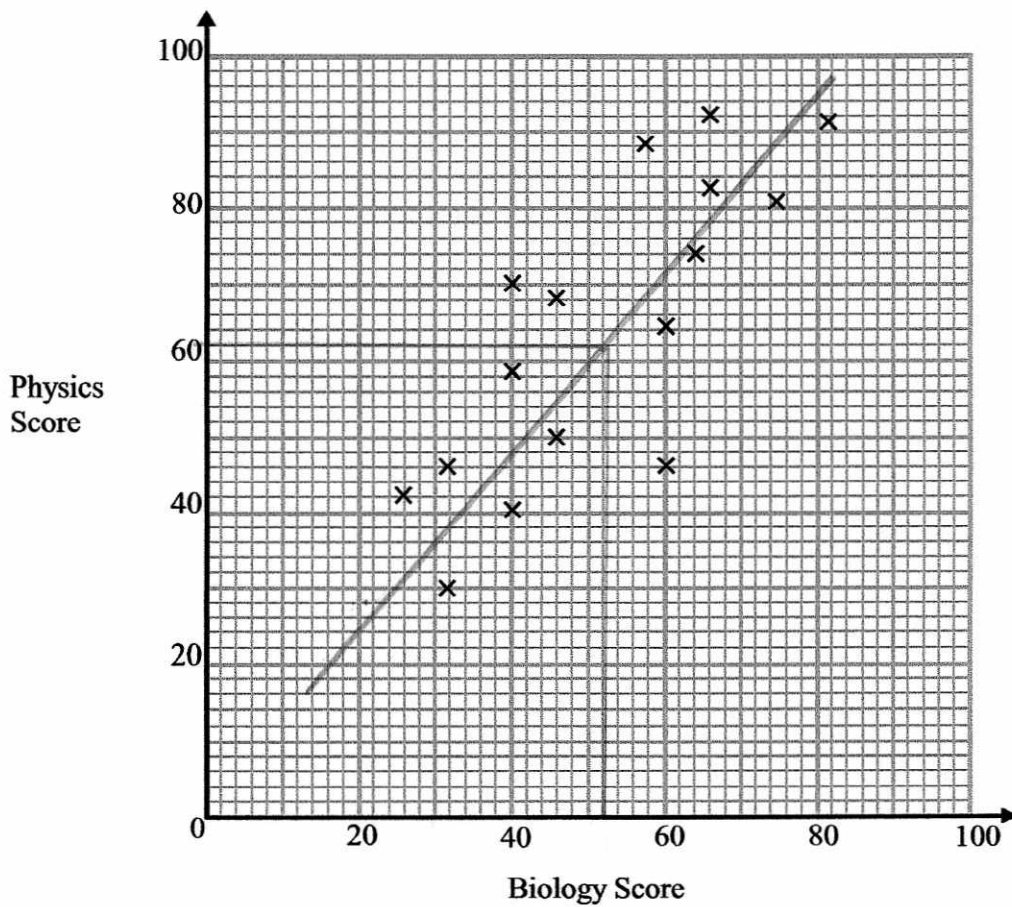
$$90 \ km / h$$

$\times 1000 \qquad 90000 \ m / h$

$\div 60 \qquad 1500 \ m / min$

$\div 60 \qquad 25 \ m / s$

$\underline{25}$ m/s

(Total for question 6 is 3 marks)

7   The scatter graph shows the scores of 15 students on their Biology and Physics tests.

(a) What type of correlation does the scatter graph show?

................positive................
(1)

(b) Another students scored 52 marks on their Biology test.
     Estimate the Physics score for this student.

.................62.................
(2)

60 — 65

(Total for question 7 is 3 marks)

8 David bought a new car.
Each year the car depreciates in value by 12%.

Work out the number of years it takes for the car to half in value.

$$100 \times 0.88^x = 50$$

$$\cancel{0.88^x = \frac{1}{2}}$$

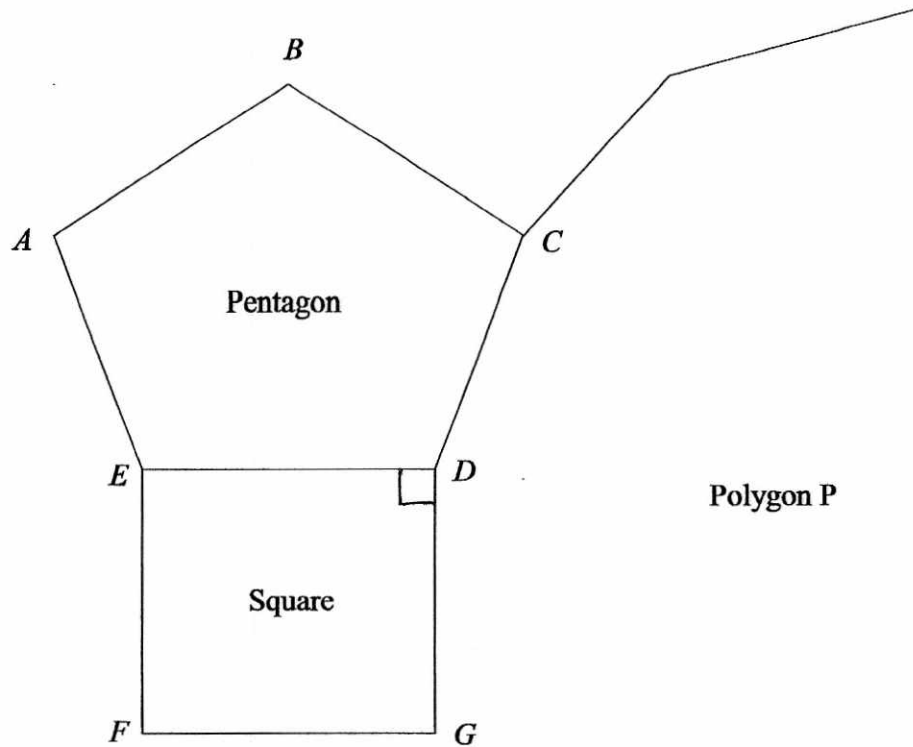$$100 \times 0.88^4 = 59.969536$$

$$100 \times 0.88^5 = 52.77319..$$

$$100 \times 0.88^6 = 46.44...$$

$$\uparrow$$

46% of its original
value.

.................6.................years

(Total for question 8 is 3 marks)

9

The diagram shows a regular pentagon, ABCDE, and a square, EDFG.

The lines CD and DG are both sides of another regular polgon, P.

How many sides does polygon P have?

You must show how you got your answer.

interior angle of square = $90°$

exterior angle of pentagon = $\frac{360}{5} = 72°$

interior angle of pentagon = $180 - 72 = 108°$

interior angle of P = $360 - 108 - 90 = 162°$

exterior angle of P = $180 - 162 = 18°$

$\frac{360}{18} = 20$ sides

..........20..........

(Total for question 9 is 4 marks)

10 The frequency table shows the speeds of 100 cars.

Speed (km/h)	Midpoint	Frequency	Mp × f
$0 < s \leqslant 20$	10 X	6	60
$20 < s \leqslant 40$	30 X	17 23	510
$40 < s \leqslant 60$	50 X	29 52	1450
$60 < s \leqslant 80$	70 X	25	1750
$80 < s \leqslant 100$	90 X	20	1800
$100 < s \leqslant 120$	110 X	3	330

5900

(a) Work out an estimate for the mean speed of the cars.

$$\frac{5900}{100}$$

.................59................km/h

(3)

(b) Write down the class interval that contains the median.

...........$40 < s \leqslant 60$...........

(1)

(Total for question 10 is 4 marks)

11 Cylinder A and Cylinder B are mathematically similar.
The ratio of the volume of Cylinder A to the ratio of Cylinder B is 8:27.

Cylinder A has a surface area of 108cm²
Work out the surface area of cylinder B.

scale factor cubed

Ratio ~~of~~ lengths   2 : 3

Ratio of surface areas   4 : 9

scale factor × $\frac{9}{4}$

$108 \times \frac{9}{4} = 243 \text{ cm}^2$

.........**243**.........cm²

(Total for question 11 is 3 marks)

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12 There are 52 cards in a deck.
Peter is going to give one card to Casper and one card to Kelly.

How many different ways are there of going this?

52×51

.....**2652**.....

(Total for question 12 is 2 marks)

13 The frequency table shows the time taken for 100 people to travel to an event.

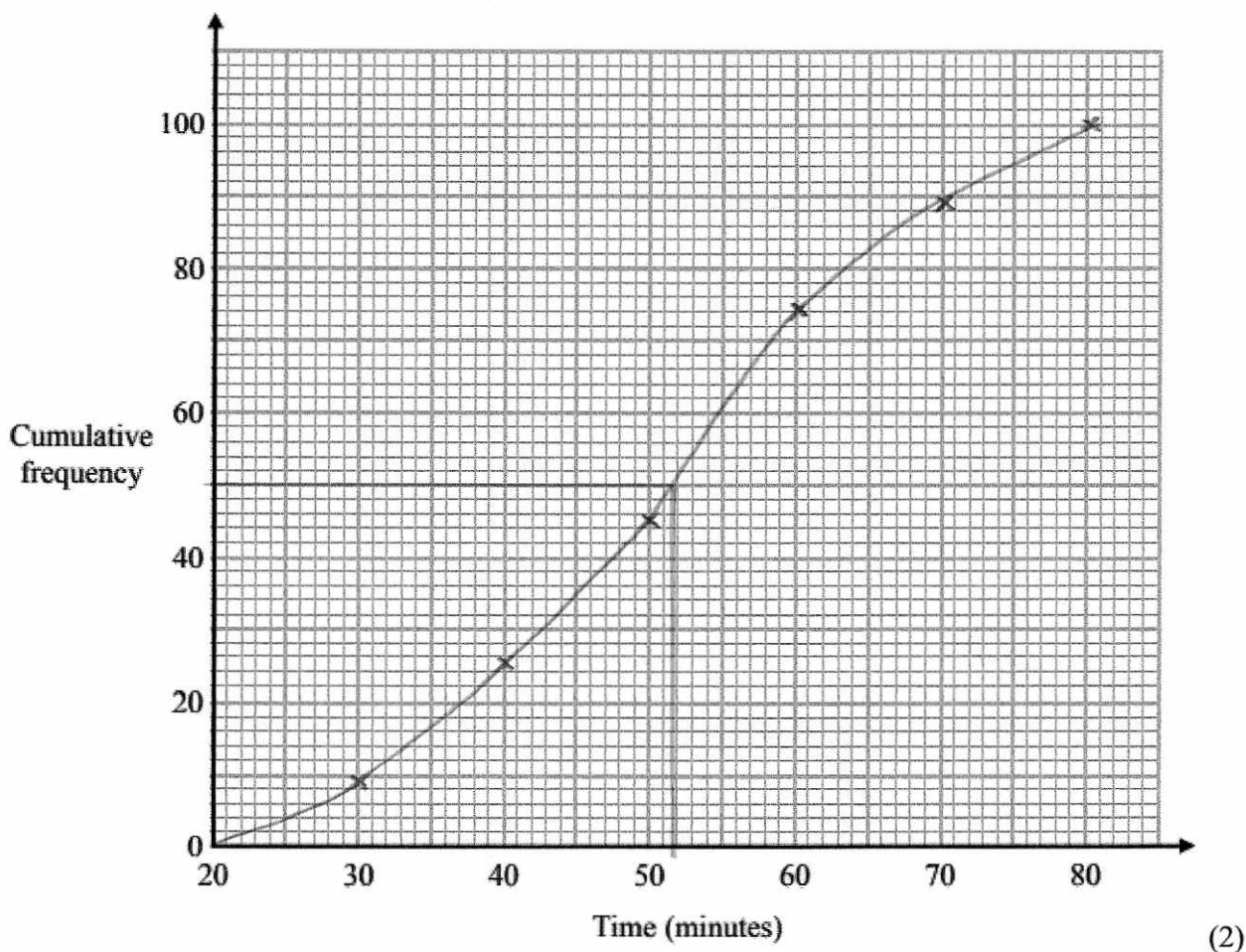| Time (minutes)	Frequency	C.F
$20 < t \leqslant 30$	9	9
$30 < t \leqslant 40$	16	25
$40 < t \leqslant 50$	20	45
$50 < t \leqslant 60$	29	74
$60 < t \leqslant 70$	15	89
$70 < t \leqslant 80$	11	100

(a) On the grid, plot a cumulative frequency graph for this information.

(2)

(b) Find an estimate for the median time taken.

..............52..............minutes

(1)

(Total for question 13 is 3 marks)

14 The diagram shows a sector of a circle with radius 6cm.
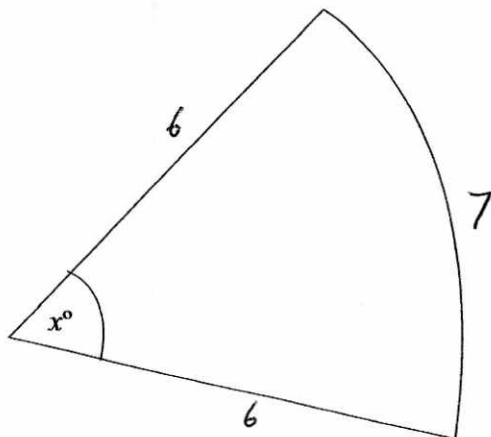
The sector has a perimeter of 19cm.
Work out the value of x.
Give your answer correct to one decimal place.

$$19 - 6 - 6 = 7$$

Arc length = 7 cm

$$\frac{x}{360} \times 2\pi(6) = 7$$

$$\frac{x}{360} = 0.185\ldots$$

$$x = 66.8 \quad (1dp)$$

66.8

15 By completing the square, find the coordinates of the turning point of the curve with the equation $y = x^2 + 8x + 3$

You must show all your working.

$$(x + 4)^2 - 16 + 3$$

$$(x + 4)^2 - 13$$

Min point: $(-4, -13)$

$(-4, -13)$

(Total for question 15 is 3 marks)

16 Make x the subject of the formula $a = \dfrac{x + 4}{x - 2}$
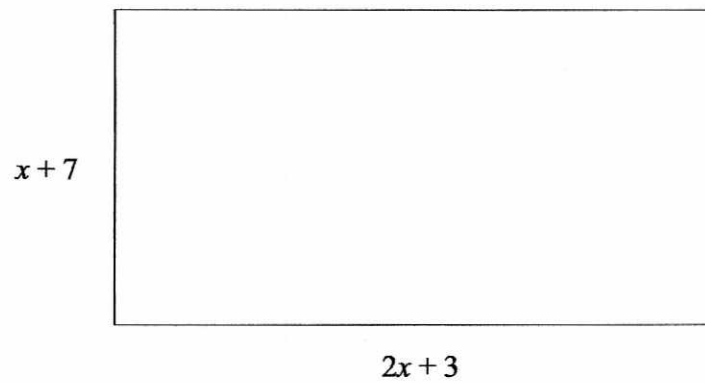
$$a(x - 2) = x + 4$$

$$ax - 2a = x + 4$$

$$ax - x = 2a + 4$$

$$x(a - 1) = 2a + 4$$

$$x = \frac{2a + 4}{a - 1}$$

$x = \dfrac{2a + 4}{a - 1}$

(Total for question 16 is 3 marks)

17

$x + 7$

$2x + 3$

The diagram shows a rectangle.

All measurements are in centimetres.

The area of the rectangle is 105 cm².
Find the value x.

$$(x + 7)(2x + 3) = 105$$

$$2x^2 + 3x + 14x + 21 = 105$$

$$2x^2 + 17x + 21 = 105$$

$$2x^2 + 17x - 84 = 0$$

$$(2x - 7)(x + 12) = 0$$

$$x = 3.5 \qquad x = -12$$

if $x = -12$ the lengths would be negative.

$$\therefore \quad x = 3.5$$

$x = $ ......3.5......

(Total for question 17 is 5 marks)

18 Here are the first 5 terms of a quadratic sequence.

$$an^2 + bn + c$$

<div align="center">

6   17   32   51   74

</div>

Find an expression, in terms of n, for the nth term of this sequence.

$a + b + c \longrightarrow$ 6   17   32   51

$3a + b \longrightarrow$ 11   15   19

$2a \longrightarrow$ 4   4

$2a = 4$ $3(2) + b = 11$ $2 + 5 + c = 6$

$a = 2$ $6 + b = 11$ $7 + c = 6$

$b = 5$ $c = -1$

$$2n^2 + 5n - 1$$

(Total for question 18 is 3 marks)

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19 Prove algebraically that $0.7\dot{3} \times 0.6\dot{3}$ can be written as $\dfrac{7}{15}$

$0.7\dot{3} = x$ $0.\dot{6}\dot{3} = y$

$7.\dot{3} = 10x$ $63.\dot{6}\dot{3} = 100y$

$73.\dot{3} = 100x$ $63 = 99y$

$66 = 90x$ $y = \dfrac{63}{99}$

$x = \dfrac{66}{90} = \dfrac{11}{15}$ $= \dfrac{7}{11}$

$$\dfrac{\cancel{11}}{15} \times \dfrac{7}{\cancel{11}} = \dfrac{7}{15}$$

(Total for question 19 is 4 marks)

20 (a) Show that the equation $x^3 + 4x = 1$ has a solution between $x = 0$ and $x = 1$.

$$x^3 + 4x - 1 = 0$$

when $x = 0$ $\quad (0)^3 + 4(0) - 1 = -1$

when $x = 1$ $\quad (1)^3 + 4(1) - 1 = 4$

change of sign $\therefore$ solution between 0 and 1.

(2)

(b) Show that the equation $x^3 + 4x = 1$ can be rearranged to give: $x = \dfrac{1}{4} - \dfrac{x^3}{4}$

$$4x = 1 - x^3$$

$$x = \frac{1}{4} - \frac{x^3}{4}$$

(1)

(c) Starting with $x_0 = 0$, use the iteration formula $x_{n+1} = \dfrac{1}{4} - \dfrac{x_n^3}{4}$ twice to find an estimate for the solution to $x^3 + 4x = 1$

Give answer to 3sf.

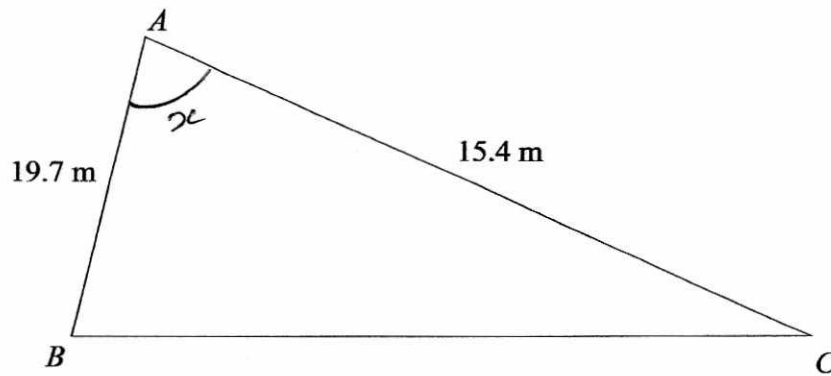$$x_1 = \frac{1}{4} - \frac{(0)^3}{4} = \frac{1}{4}$$

$$x_2 = \frac{1}{4} - \frac{Ans^3}{4} = 0.24609375$$

$$\underline{0.246}$$

(3)

(Total for question 20 is 6 marks)

21

A

19.7 m

x

15.4 m

B

C

The area of the triangle is 100m²
Calculate the perimeter of triangle *ABC*.
Give your answer to 3 significant figures.

$$\frac{1}{2} a \, b \, \sin C = 100$$

$$\frac{1}{2}(19.7)(15.4) \sin x = 100$$

$$\sin x = \frac{100}{\frac{1}{2}(19.7)(15.4)}$$

$$\sin x = 0.659239\ldots$$

$$x = \sin^{-1}(0.659\ldots)$$

$$x = 41.24187853$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$a^2 = (19.7)^2 + (15.4)^2 - 2(19.7)(15.4) \cos(41.24\ldots)$$

$$a^2 = 169.0069753$$

$$a = 13.0 \quad (3sf)$$

$$19.7 + 15.4 + 13.0 = 48.1 \text{ m}$$

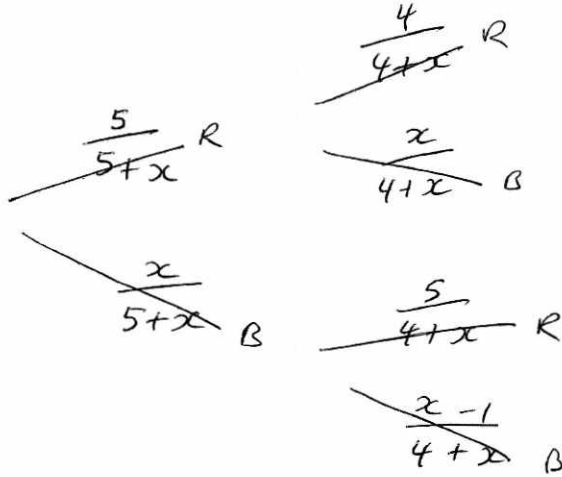48.1 ....m

(Total for question 21 is 5 marks)

22 There are 5 red counters and x blue counters in a bag.

2 counters are removed from the bag at random.

The probability that both the counters taken are red is $\dfrac{5}{33}$

Work out the value of x.

$\dfrac{4}{4+x}$ R

$\dfrac{5}{5+x}$ R

$\dfrac{x}{4+x}$ B

$\dfrac{x}{5+x}$ B

$\dfrac{5}{4+x}$ R

$\dfrac{x-1}{4+x}$ B

$$P(RR) = \frac{5}{33}$$

$$\frac{5}{5+x} \times \frac{4}{4+x} = \frac{5}{33}$$

$$\frac{\overset{4}{\cancel{20}}}{(5+x)(4+x)} = \frac{\cancel{5}^{1}}{33}$$

$$132 = (5+x)(4+x)$$
$$132 = 20 + 5x + 4x + x^2$$
$$132 = x^2 + 9x + 20$$

$$x^2 + 9x - 112 = 0$$
$$(x-7)(x+16) = 0$$
$$x = 7 \qquad x = -16$$

................**7**................

(Total for question 22 is 7 marks)

x cannot be negative $\therefore$ $x = 7$