Write your name here:

Surname:	Other Names:

Mathematics
May/June 2017
Paper 3 (Calculator)
Higher Tier
Time: 1 hour 30 minutes

You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided – there may be more space than you need.
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.
- You must **show all your working out.**

Information

- The total mark for this paper is 80
- The marks for **each** question are shown in brackets – use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

P S E

14 Pritam, Sarah and Emily share some money in the ratios 3 : 6 : 4
Sarah gets $15 more than Emily.

Work out the amount of money that Pritam gets.

2 parts = $15

1 part = $7.50

3 x 7.50

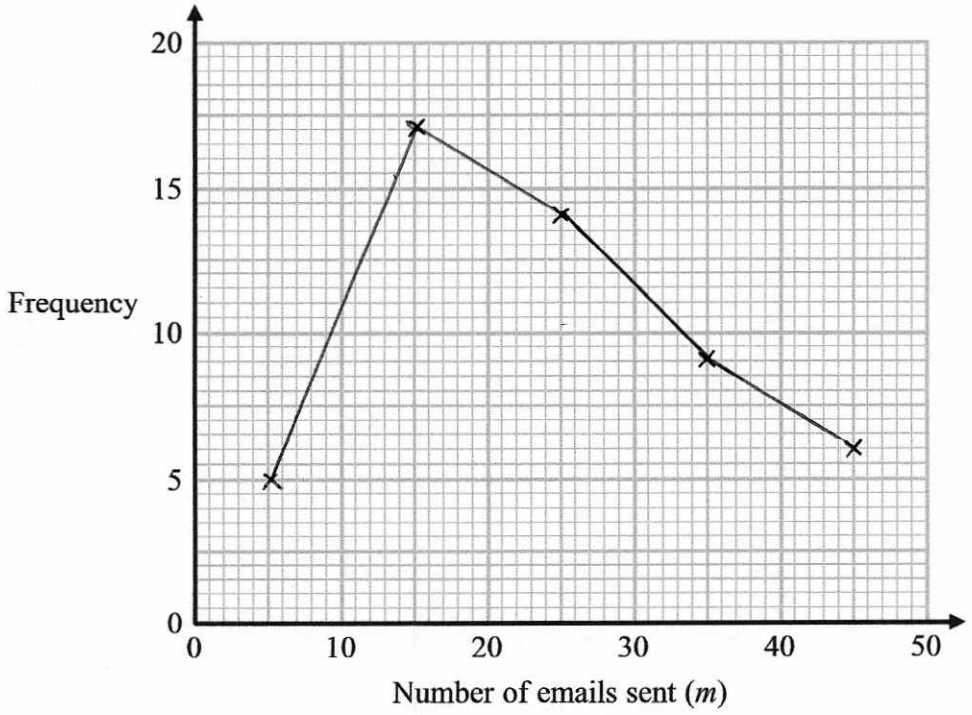
$ 22.50

(Total for Question 14 is 2 marks)

2 The frequency table gives information about the numbers of emails sent by 51 teachers on Monday.

Number of emails sent (m)	Frequency
$0 < m \leqslant 10$	5
$10 < m \leqslant 20$	17
$20 < m \leqslant 30$	14
$30 < m \leqslant 40$	9
$40 < m \leqslant 50$	6

(a) On the grid below, draw a frequency polygon for this information.



(2)

(b) Nalini says that at least a quarter of these teachers sent more than 30 emails.

Is Nalini correct?
You must explain your answer.

$\frac{15}{51}$ sent more than 30 emails

This is over $\frac{1}{4}$ (12.75) Yes.

(2)

(Total for Question 2 is 4 marks)

15 There are 130 adults at a language school.
Each adult studies one of French or Spanish or German.

96 of the adults are women.
12 of the women study French.
73 of the adults study Spanish.
55 of the women study Spanish.
9 of the men study German.

How many of the adults study French?

	FRENCH	SPANISH	GERMAN	TOTAL
MEN	7	18	9	34
WOMEN	12	55	29	96
TOTAL	19	73	38	130

19

(Total for Question 15 is 3 marks)

16

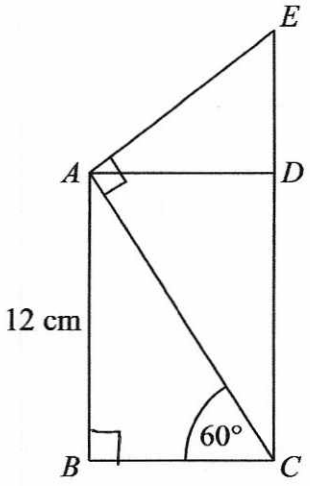
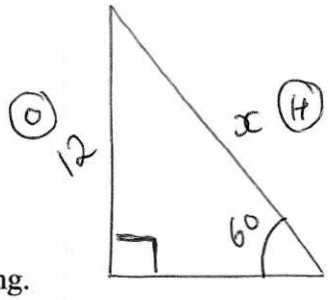


Diagram **NOT** accurately drawn

ABCD is a rectangle.
CDE is a straight line.

AB = 12 cm
Angle *ACB* = 60°
Angle *EAC* = 90°

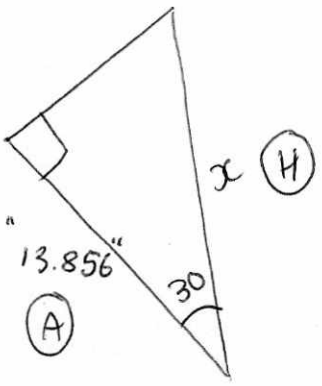
Calculate the length of *CE*.
You must show all your working.



$\sin(60) = \frac{12}{x}$

$x = \frac{12}{\sin(60)}$

$= 13.85640646$



$\cos(30) = \frac{"13.856"}{x}$

$x = \frac{"13.856"}{\cos(30)}$

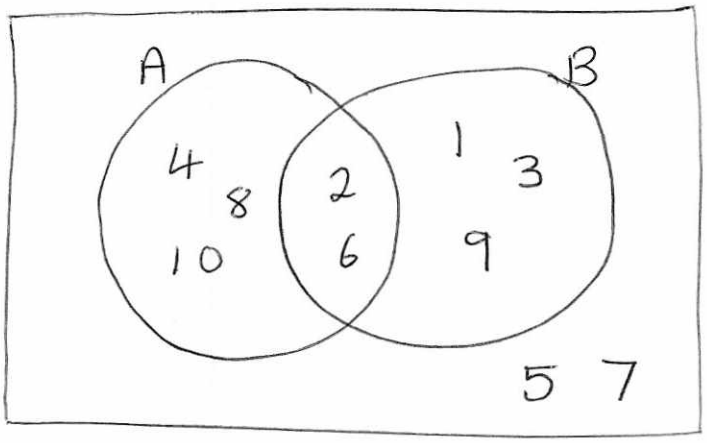
$= 16$

16 cm

(Total for Question 16 is 4 marks)

17 $\mathscr{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
$A = \{\text{multiples of } 2\}$
$A \cap B = \{2, 6\}$
$A \cup B = \{1, 2, 3, 4, 6, 8, 9, 10\}$

Draw a Venn diagram for this information.



(Total for Question 17 is 4 marks)

18 John changes £450 to euros.

The exchange rate is £1 = 1.16 euros.

(a) Change £450 to euros.

450×1.16

522 euros

(2)

When in Amsterdam, John uses his credit card to pay for a ring costing 850 euros.

He has to pay a bank charge of £3.50 for using his credit card in addition to the cost of the ring.

(b) Work out the total cost, in pounds (£), of the ring and the bank charge.

$850 \div 1.16 = £732.76$

$732.76 + 3.50 = £736.26$

£ 736.26

(3)

(Total for Question 18 is 5 marks)

19 Solve the simultaneous equations

$$5y - 4x = 8 \quad \times 1$$
$$y + x = 7 \quad \times 5$$

Show clear algebraic working.

① $5y - 4x = 8$

② $5y + 5x = 35$

② − ① $\quad 9x = 27$

$x = 3$

$y + 3 = 7$

$y = 4$

$x =$ 3

$y =$ 4

(Total for Question 19 is 3 marks)

20 In a box of pens, there are

three times as many red pens as green pens
and two times as many green pens as blue pens.

R G
3 : 1
G B
2 : 1

For the pens in the box, write down
the ratio of the number of red pens to the number of green pens to the number of blue pens.

R G G B
×2 3 : 1 2 : 1
6 : 2 2 : 1
6 : 2 : 1

6 : 2 : 1

(Total for Question 20 is 2 marks)

9 Sumeet records the times, in minutes, for 40 runners to finish a half marathon.

Information about these times is shown in the table.

Time (t minutes)	Frequency
$60 < t \leqslant 90$	10
$90 < t \leqslant 120$	14
$120 < t \leqslant 150$	9
$150 < t \leqslant 180$	5
$180 < t \leqslant 210$	2

75 × 10 = 750
105 × 14 = 1470
135 × 9 = 1215
165 × 5 = 825
195 × 2 = 390

Calculate an estimate for the mean time.

$\frac{4650}{40} = 116.25$

116.25 minutes

(Total for Question 9 is 4 marks)

10 Here is a plan of Martin's driveway.

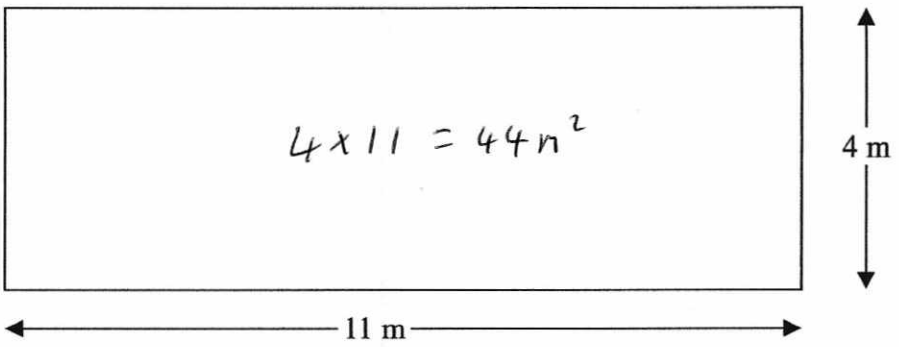


Diagram **NOT** accurately drawn

$4 \times 11 = 44 m^2$

Martin is going to cover his driveway with gravel.
The gravel will be 6 cm deep.
0.06m
Gravel is sold in bags.
There are 0.4 m^3 of gravel in each bag.
Each bag of gravel costs £38

Martin gets a discount of 30% off the cost of the gravel.

Work out the total amount of money Martin pays for the gravel.

Volume = $44 \times 0.06 = 2.64 m^3$

$\frac{2.64}{0.4} = 6.6$ bags (7 bags)

$7 \times 38 = £266$

0.3×266
30% of £266 = £79.80

Martin pays £266 − £79.80
= £186.20

£ 186.20

(Total for Question 10 is 5 marks)

11 Ravina wants to find an estimate for the number of birds in a sanctuary.

She catches a sample of 70 birds in the sanctuary and tags each of these birds.

These birds are then released back into the sanctuary.

Next day she catches a sample of 60 birds in the sanctuary.

Ravina had tagged 12 of these birds.

Work out an estimate for the number of birds in the sanctuary.

Write down an assumption you have made.

$$\frac{12}{60} = \frac{70}{x}$$

$$\frac{1}{5} \overset{\times 70}{=} \frac{70}{x}$$

$$\frac{70}{1} = 70$$

$$5 \times 70 = 350$$

350

None of the tags fell off.

(Total for Question 11 is 3 marks)

12 The diagram shows two regular hexagons, *OABCDE* and *OFGHIJ*.

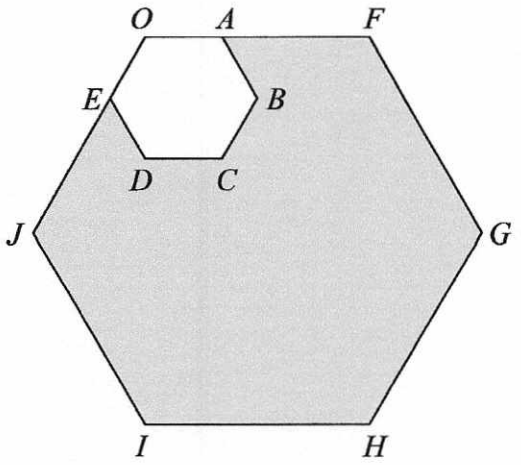


Diagram **NOT** accurately drawn

OAF and *OEJ* are straight lines.
OF = 3 *OA*.
The area of *OABCDE* is 4 cm^2.

Calculate the area of the shaded region.

Scale Factor (Length) $\times 3$

Scale Factor Area $\times 3^2$

$4 \times 3^2 = 36\text{ cm}^2$

Shaded Area $= 36 - 4$

32 cm^2

(Total for Question 12 is 3 marks)

13 There are 17 men and 26 women in a choir.
The choir is going to sing at a concert.

One of the men and one of the women are going to be chosen to make a pair to sing the first song.

(a) Work out the number of different pairs that can be chosen.

17×26

442

(2)

Two of the men are to be chosen to make a pair to sing the second song.

Ben thinks the number of different pairs that can be chosen is 136
Mark thinks the number of different pairs that can be chosen is 272

(b) Who is correct, Ben or Mark?
Give a reason for your answer.

$\frac{17 \times 16}{2} = 136$

Ben. Mark will have included each pair twice.

(1)

(Total for Question 13 is 3 marks)

14 (a) Factorise $4x^2 - 9$

$(2x+3)(2x-3)$

(1)

(b) Make m the subject of

$g - 3m = am + 5$

$+3m \quad +3m$

$g = am + 3m + 5$

$-5 \qquad\qquad -5$

$g - 5 = am + 3m$

$g - 5 = m(a+3)$

$m = \dfrac{g-5}{a+3}$

(3)

(Total for Question 14 is 4 marks)

15 The number of bees in a beehive at the start of year n is P_n.
The number of bees in the beehive at the start of the following year is given by

$$P_{n+1} = 1.05(P_n - 250)$$

At the start of 2015 there were 9500 bees in the beehive.

How many bees will there be in the beehive at the start of 2018?

$P_{2016} = 1.05(9500 - 250)$

$= 9712.5$

$P_{2017} = 1.05(9712.5 - 250)$

$= 9935.625$

$P_{2018} = 1.05(\text{Ans} - 250)$

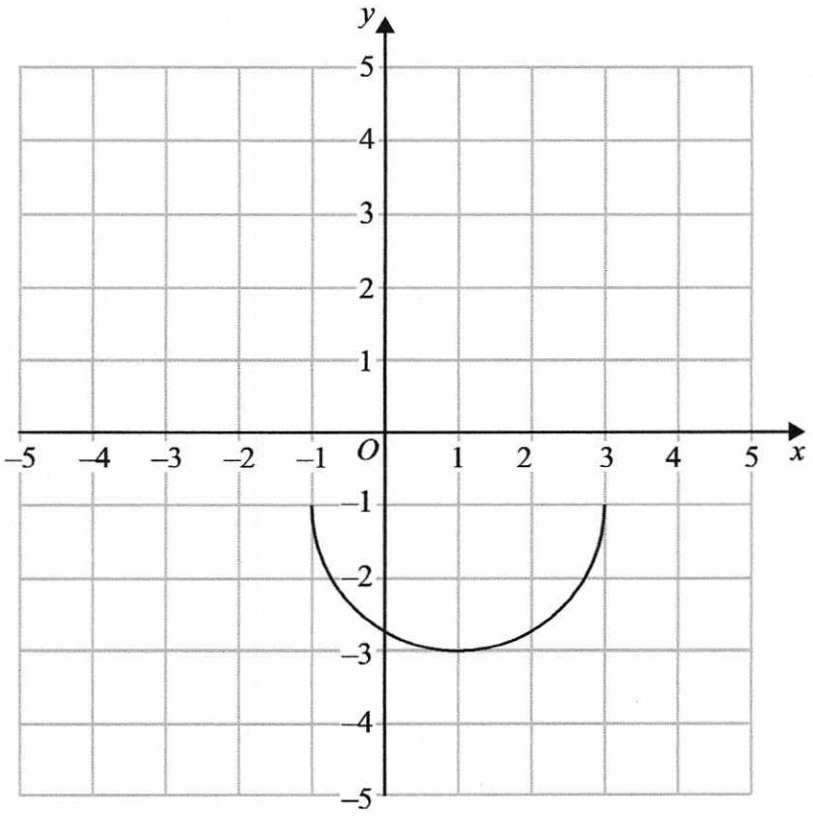
$= 10170$

(nearest bee)

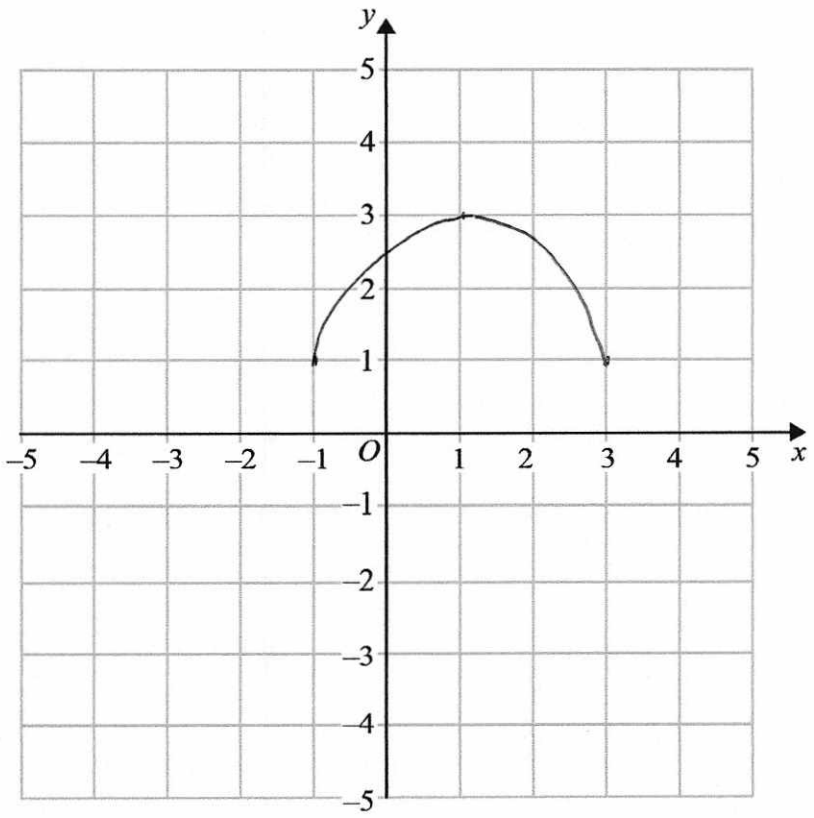
10170

(Total for Question 15 is 3 marks)

16 Here is the graph of $y = f(x)$.

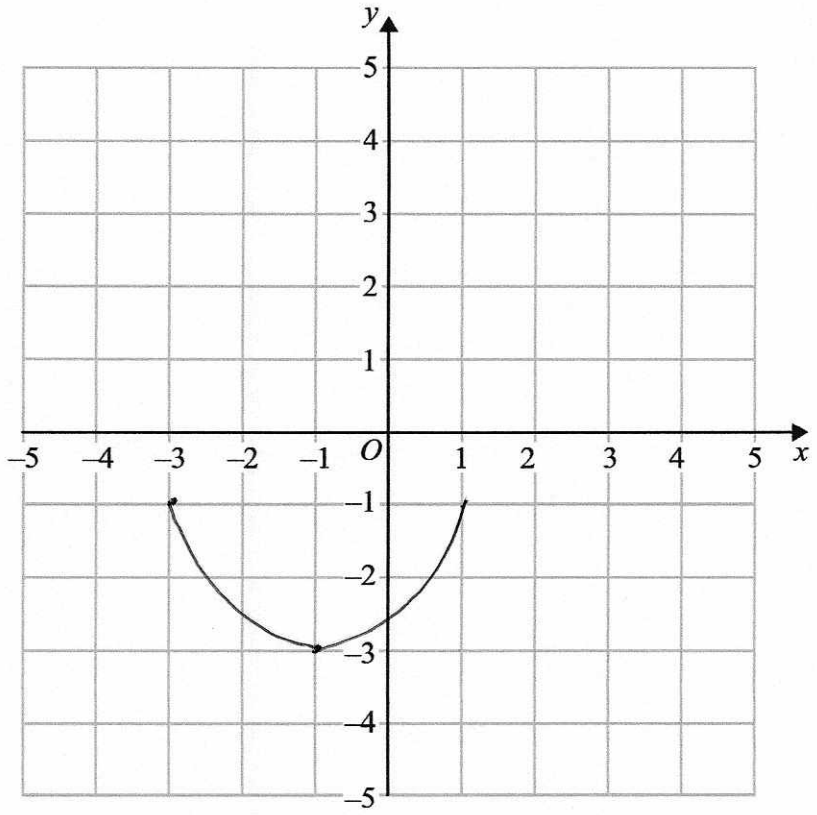


(a) On the grid below, draw the graph of $y = -f(x)$.



(2)

(b) On the grid below, draw the graph of $y = f(x + 2)$.



(2)

(Total for Question 16 is 4 marks)

17 (a) Write $2x^2 + 16x + 35$ in the form $a(x + b)^2 + c$ where a, b, and c are integers.

$2(x^2 + 8x) + 35$

$2((x+4)^2 - 16) + 35$

$2(x+4)^2 - 32 + 35$

$2(x+4)^2 + 3$

$2(x+4)^2 + 3$

(3)

(b) Hence, or otherwise, write down the coordinates of the turning point of the graph of $y = 2x^2 + 16x + 35$

$(-4, 3)$

(1)

(Total for Question 17 is 4 marks)

18 Simplify fully $(\sqrt{a} + \sqrt{4b})(\sqrt{a} - 2\sqrt{b})$

$\sqrt{4}\sqrt{b}$

$2\sqrt{b}$

$(\sqrt{a} + 2\sqrt{b})(\sqrt{a} - 2\sqrt{b})$

$a - 2\sqrt{a}\sqrt{b} + 2\sqrt{a}\sqrt{b} - 4b$

$a - 4b$

(Total for Question 18 is 3 marks)

19 The function f is defined as $f(x) = \frac{3}{4+x}$

(a) Find the value of f(1)

0.6 or $\frac{3}{5}$

(1)

(b) State which value of x must be excluded from any domain of f.

-4

(1)

The function g is defined as $g(x) = 5 + x$

(c) Given that $g(a) = 7$, find the value of a.

$g(a) = 5 + a = 7$

$5 + a = 7$

$a = 2$

$a =$ 2

(1)

(d) Calculate fg(1)

$g(1) = 5 + 1$

$= 6$

$f(6) = \frac{3}{4+6} = 0.3$

0.3

(2)

(e) Find fg(x)
Simplify your answer.

$fg(x) = \frac{3}{4+(5+x)}$

$= \frac{3}{9+x}$

fg(x) = $\frac{3}{9+x}$

(2)

(Total for Question 19 is 7 marks)

20

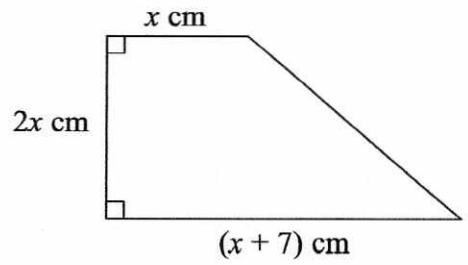


Diagram **NOT** accurately drawn

The diagram shows a trapezium.
The trapezium has an area of 17 cm^2

(a) Show that $2x^2 + 7x - 17 = 0$

$$\frac{x + x + 7}{2} \times 2x = 17$$

$$\left(\frac{2x + 7}{2}\right) 2x = 17$$

$$2x^2 + 7x = 17$$

$$2x^2 + 7x - 17 = 0$$

(3)

(b) Work out the value of x.
Give your answer correct to 3 significant figures.
Show your working clearly.

$a = 2 \quad b = 7 \quad c = -17$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(7) \pm \sqrt{(7)^2 - 4(2)(-17)}}{2(2)}$$

$= 1.65$ or -5.15

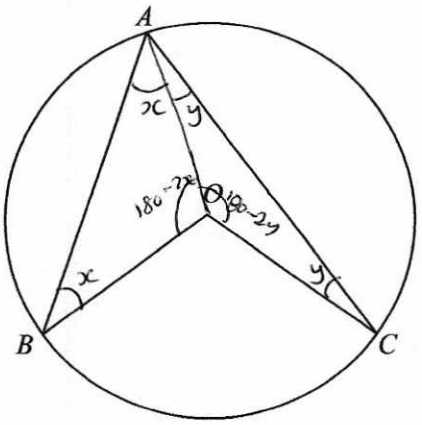
cannot be negative $\therefore x = 1.65$

$x =$ 1.65

(3)

(Total for Question 20 is 6 marks)

21 *A*, *B* and *C* are points on the circumference of a circle centre *O*.



Prove that angle *BOC* is twice the size of angle *BAC*.

Let $B\hat{A}O = x$ and $C\hat{A}O = y$

Triangles BAO and CAO are isosceles

$AOB = 180 - 2x$ $\quad AOC = 180 - 2y$

$BOC = 360 - (180 - 2x) - (180 - 2y)$

$= 360 - 180 + 2x - 180 + 2y$

$= \underline{2x + 2y}$

$BAC = \underline{\underline{x + y}}$

$\therefore BOC = 2(BAC)$

(Total for Question 21 is 4 marks)

TOTAL FOR PAPER IS 80 MARKS